Response verification book

Seeing through mathematics, Book 1
special edition

Jan Engen Harting Trimble Berger Cleveland Eneboe

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Response verification book

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Seeing through mathematics
special edition

Book 1

W. J. Gage Limited
Toronto

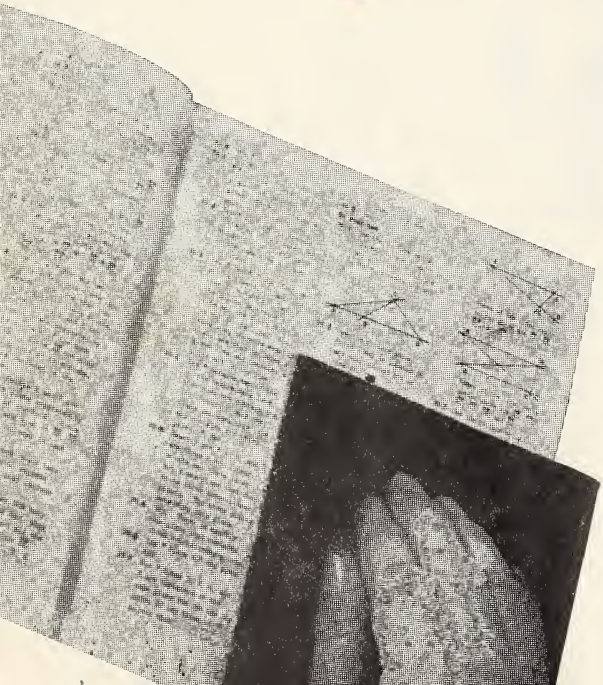
How to use this response verification book As you work through *Seeing Through Mathematics, Book 1,* you will notice two types of exercises, developmental and practice. The developmental exercises, identified by letters, make up the greater part of each lesson and have been designed to guide your thinking. The practice exercises, identified by numerals, have been designed to help you reinforce and maintain the ideas you have studied.

Your teacher will decide how this response book should be used. He may wish to use the developmental exercises for class discussion, in which case you may not need this response book. However, he may want you to study the developmental exercises by yourself. If you proceed in this way, you should make every effort to respond to each exercise clearly and completely before using this book to verify your response.

The practice exercises are entitled "On your own," "Keeping Skilful," and "Checking up," and are designed for written work. Your teacher may want you to complete an assigned block of work before verifying any responses.

If your teacher directs you to verify a response at a time, it is a good idea to cover all the responses in the column that are below the response you are concerned with at the moment. It is very difficult to avoid seeing other responses unless you use some means to conceal them. The picture shows one way to prevent unintentional "peeking."

Responses to the work entitled "Special challenge" are not included in this book. If you need help with these exercises, consult your teacher.



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Responses are keyed to the student's book by page numeral and exercise letter or numeral. For example, the response for exercise K on page 6 is labelled *6*-K; the response for the "On your own" exercise 17 on page 8 is labelled *8*-17. At the bottom of this page, you will see the words "Responses for pages 5-7." These words indicate that this page includes the responses for pages 5, 6, and 7 of the student's book. Each page of this key is labelled in this way. Also observe that when an exercise includes two or more questions, the responses to these questions are separated by an asterisk (*).

Lesson 1

5-A No. The information in the blank after "Age" should tell how old Steve Jones is.

5-B On blanks following "Name" and "Mathematics teacher"

5-C One is a group of mathematics students at Logan School, and the other is a group of mathematics teachers at Logan School: * The group of mathematics students

5-D Since 7 and VII are names for the same number, Grade 7 and Grade VII are names for the same grade. * Some examples of other names for seven are: 4 + 3, 12 - 5, and 7 × 1.

5-E There is no class period number 8 at Logan School

5-F Response depends on the number of class periods in your school. If there are six class periods, the numerals 1, 2, 3, 4, 5, or 6 might be written in the blank.

5-G 7, 8, and 9, or VII, VIII, and IX

5-H "My age is ——."

5-I "My favorite subject is——." * Mathematics, science, art, or the name of any other subject studied at Logan School

6-J Yes

6-K No. Richard's birthday is in June. * No

6-L Sentences B, E, and J * The other sentences express false ideas.

6-M If your birthday is in September, the sentence will express a true idea. * If your birthday is not in September, the sentence will express a false idea.

6-N If one or more of the students in the class have a birthday in September, then one or more of the sentences will express true ideas. * Some of the sentences will express false ideas unless all the students in class have birthdays in September.

6-O No * The sentence in D3 does not tell you who has a birthday in September. Sentence A in D4 does tell you who has a birthday in September.

7-P No * The sentence must contain the name of a letter of the alphabet instead of the star symbol before you can decide whether the idea expressed is true or false.

7-Q a, c, e, h, i, m, s, t

7-R 18 replacements

7-S No

7-A No * To decide whether the idea expressed is true or false, the sentence must contain the name of a number instead of "*n*."

7-B For the number 0, you would write the sentence, "0 is a number greater than 6." For each of the numbers 1 through 8, you would write similar sentences.

7-C The sentences obtained by using the numerals 7 and 8 express true ideas. * The others express false ideas.

7-D No. To decide whether the idea expressed is true or false, the sentence must contain the name of a number instead of "*y*."

7-E 8 is less than 1. * No * VIII is less than 1. * No

7-F No. You only obtain a different sentence that expresses the same true idea or the same false idea.

7-G No. Each of these numerals is a name of a number that is not less than one.

7-H 0 is less than 1. * Yes

7-I Yes

7-J When you replace the letter *s* by the numeral 1, you obtain the sentence, "1 is greater than 6 and less than 10." When you replace

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the letter *s* by the numerals from 2 through 10, you obtain similar sentences. * The numerals 7, 8, and 9 make sentences that express true ideas.

7-K Yes. There are many fraction numerals that will make sentences that express true ideas. Examples are 13/2, 29/4, and 17/2.

7-L x, ☆, n, $y * w$ is the placeholder.

8-M No. The sentence must contain the name of a number instead of n.

8-N n

8-O When you replace the placeholder by the numeral 10, you obtain the sentence, "10 is a number between 15 and 65." When you replace the placeholder by each of the other numerals, you obtain similar sentences.

8-P 20, 30, 40, 50, 60

8-Q 10, 70, 80, 90, 100

On Your Own

8-1 *

8-2 Responses will depend upon the names of the students that you choose. An example is "John writes with the left hand."

8-3 When the placeholder is replaced by the name of a student who writes with the left hand, the sentence will express a true idea.

8-4 When the placeholder is replaced by the name of a student who does not write with the left hand, the sentence will express a false idea.

8-5 Response will depend upon the answer to question 3.

8-6 The symbol used is z. Other responses will depend upon the names of the students used as replacements for the placeholder.

8-7 The symbol used is n.

8-8 The symbol used is ☆.

8-9 The symbol used is x.

8-10 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is less than 5." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences.

8-11 The sentences obtained by replacing the placeholder by the numerals 1, 2, 3, and 4 express true ideas.

8-12 The sentences obtained by replacing the placeholder by the numerals 5, 6, 7, and 8 express false ideas.

8-13 The numerals that make sentences that express true ideas are 1, 2, 3, and 4.

8-14 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is an even number." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences.* The sentences obtained by replacing the placeholder by the numerals 2, 4, 6, and 8 express true ideas. * The other sentences express false ideas.* The numerals that make sentences that express true ideas are 2, 4, 6, and 8.

8-15 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is the number of days in the week." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences. * The sentence obtained by replacing the placeholder by the numeral 7 expresses a true idea. * The other sentences obtained express false ideas. * The numeral that makes a sentence that expresses a true idea is 7.

8-16 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is greater than 7." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences. * The sentence obtained by replacing the placeholder by the numeral 8 expresses a true idea. * The other sentences obtained express false ideas. * The numeral that makes a sentence that expresses a true idea is 8.

8-17 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is less than 12." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences. * All of the sentences obtained express true ideas. *

The numerals that make sentences that express true ideas are 1, 2, 3, 4, 5, 6, 7, and 8.

8-18 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is a number less than 8 or equal to 8." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences. * All of the sentences obtained express true ideas. * The numerals that make sentences that express true ideas are 1, 2, 3, 4, 5, 6, 7, and 8.

8-19 When you replace the placeholder by the numeral 1, you obtain the sentence, "1 is an odd number." When you replace the placeholder by each of the numerals from 2 through 8, you obtain similar sentences. * The sentences obtained by replacing the placeholder by the numerals 1, 3, 5, and 7 express true ideas. * The other sentences express false ideas. * The numerals that make sentences that express true ideas are 1, 3, 5, and 7.

8-20 When you replace the placeholder by the numeral 0, you obtain the sentence, "0 is the number of months in a year." When you replace the placeholder by each of the numerals from 1 through 8, you obtain similar sentences.

8-21 No

8-22 Yes

Lesson 2

9-A There are many sets of objects in the picture. Some of these are: the set of pencils, the set of erasers, the set of pencil sharpeners, and the set of bookends.

9-B Four pencils * Four pens

9-C No * There are five other books including the notebook.

9-D A set of students in Miss Howe's class

10-E Yes. The names of students are also a collection of objects.

10-F There are many sets of objects shown in the picture. Some of them are: the set of objects used in sports, the set of helmets, the set of balls, the set of baseballs, the set of skates, the set of skis, and the set of golf clubs.

10-G The baseballs, the baseball bats, and the baseball gloves. (Consider the catcher's mitt as a baseball glove.)

10-H Yes * Yes

10-I 8. They are two baseball bats, two golf clubs, one tennis racket, two badminton rackets, one hockey stick.

10-J 4 * 2 * Yes

10-K Nine * Yes * Yes

11-L Yes * Yes. If each member of one set is also a member of a second set, then the first set is a subset of the second set.

11-M Yes * No. 11 is not a member of the set of numbers from 1 through 10.

11-N The numerals 6, 7, and 8

11-O The numbers 6, 7, and 8

11-A The set of the days of the week

11-B The set of odd numbers from 1 through 9

11-C The set whose members are two, four, six, eight, ten

11-D The set of the five Great Lakes

12-E Yes * The tabulations are different because the order in which the names of the members of the set are listed is different.

12-F No

12-G Yes. The two sets contain exactly the same objects. * The set whose members are one, two, three, four, five, six, seven, eight, nine, ten. You can read the second tabulation in the same way, or, the set whose members are three, five, six, nine, ten, seven, eight, one, two, four

12-I The set whose members are June, January, July

12-J The set whose members are Tom, Bill, Jack, Ed, Ken

12-K The set whose members are w, x, y, z

12-L The set whose members are book, chair, desk

12-M The set whose members are one, three, five, seven, nine

12-N The set whose members are ten, twenty, thirty, forty, fifty

12-O The set whose members are ninety-nine, seventy-five,

sixteen, one hundred two, one hundred fifteen, one hundred seventy-six

12-P The set whose members are eleven, one hundred one, one thousand one, ten thousand one

12-Q The tabulation should include the names of those students who sit in your row or at your table.

12-R {50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60}

12-S {50, 52, 54, 56, 58, 60}

12-T If you replace n with 1 your sentence will be: 1 is greater than 3 and less than 12. The other sentences will be similar to this. * {4, 5, 6, 7, 8, 9, 10, 11}

On your own

12-1 January, July, December, May, October

12-2 Macdonald, St. Laurent, King, Diefenbaker

12-3 Nickel, half-dollar, dime, cent

12-4 a, b, q, r

12-5 8, 4

12-6 The set of continents

12-7 The set of months each of which has just 30 days

12-8 The set of provinces that are Canada's western provinces

12-9 The set of the capital cities of Canada's Maritime provinces

12-10 The set of numbers from 0 through 3

12-11 The set of odd numbers from 1 through 9

12-12 The set of numbers from 21 through 29

12-13 The set of even numbers from 52 through 58

12-14 {drawing scale, foot ruler, yardstick}

12-15 {football, helmet}

12-16 {fishing rod}

Lesson 3

13-B No * Sentence A uses the symbols "3" and "4." Sentence B uses the symbols "3" and "four." Sentence C uses the symbols "three" and "four." Sentence D uses the symbols "III" and "IV."

13-C Each of the sentences B, C, and D also expresses the idea that the number three is less than the number four.

13-D Yes * Yes

13-E There are many sentences that express the same idea as each of the sentences in D1. Examples are: "three is less than 4" and "III is less than four."

13-G Yes * No * Yes

13-H Yes * The sentences do not contain the same symbols.

13-I Yes * Yes

13-J An example is: "Fifteen is less than twenty."

13-K Yes * The sentences do not contain the same symbols.

13-L Yes * Yes

13-M An example is: "Five is greater than 9."

14-N Sentences M and N * Yes

14-O Sentences L and O each express the statement: "The number 9 is less than the number 1." * Yes * No. Each of the sentences L and O express a false statement.

14-A No. The placeholder in "n is less than 8" must be replaced by a numeral in order that the idea expressed by the sentence is true or false. * No * Only when a sentence expresses an idea which is either true or false is the sentence a closed sentence.

14-B No * It does not express an idea which is either true or false.

14-C Yes. The sentences are different because the symbols used in the sentences are different.* The letter n * The letter x

14-D Sentence B also expresses the requirement that each number you are to find must be less than eight.

14-E Yes * Yes. "5 + 3" is another name for the number 8.

14-F There are many sentences that express the same requirement as each of the sentences in D6. Examples are: "y is less than 10 - 2," and "w is less than VIII." * Each of these sentences is an open sentence.

14-G The idea expressed by a closed sentence is either a true or false statement. The idea expressed by an open sentence is a requirement about each object that you are to find. A requirement is neither true nor false.

14-H The sentence in D7 is an

open sentence because it expresses a requirement about each number that you are to find. * 20

14-I Yes. The symbol "3 + *n*" expresses the sum of 3 and *n*.

14-J Sentences C and F

14-K Sentences D and E each express the requirement that each number you are to find must be greater than seventeen.

14-L There are many sentences that express the condition that each number you are to find must be less than the sum of five and eleven. Examples are: "*x* is less than the sum of 5 and 11," and "*x* is less than 5 + 11."

15-M Since you are thinking about a condition that is an idea, you should think of the idea expressed by the letter *y*. Therefore, think of *y* as a variable.

On your own

15-1 True statement
15-2 False statement
15-3 False statement
15-4 True statement
15-5 True statement
15-6 True statement
15-7 False statement
15-8 True statement
15-9 True statement
15-10 False statement
15-11 Open sentence
15-12 Closed sentence
15-13 Closed sentence
15-14 Closed sentence
15-15 Open sentence
15-16 Open sentence
15-17 Closed sentence
15-18 Closed sentence
15-19 Open sentence
15-20 Open sentence
15-21 Closed sentence
15-22 Closed sentence
15-23 Open sentence
15-24 Closed sentence
15-25 Open sentence
15-26 The sentences in exercises 12, 13, 17, 21, and 22 express true statements. * The sentences in exercises 14, 18, and 24 express false statements.
15-27 The sentences in exercises 11, 15, 16, 19, 20, 23, and 25 express conditions.
15-28 The sum of 4 and each number you are to find must equal 6.
15-29 Each number you are to find must be less than 36.
15-30 The difference of each number you are to find and 17 must be greater than 5.
15-31 The sum of 10 and 5 must be less than each number you are to find.
15-32 The sum of 3 and each number you are to find must equal 3.
15-33 25 must be less than the sum of each number you are to find and 7.
15-34 The product of 8 and 7 must be equal to the product of 7 and each number you are to find.
15-35 16 must be greater than the difference of 18 and each number you are to find.

Lesson 4

16-B The set whose members are one, three, five, seven, and nine * The set of odd numbers from 1 through 9

16-C When you replace *x* by 1, you obtain 1 + 6 = 9. You obtain a similar statement for each member of the set tabulated in D2. * Five statements are obtained. * Yes

16-D The statement expressed by sentence B

16-E Yes * When 3 is used as a replacement for *x*, a true statement is obtained.

16-F When each of the numbers 1, 5, 7, and 9 is used as a replacement for *x*, a false statement is obtained.

16-G {1, 3, 5, 7, 9}

16-H Yes * The members of this set were used as replacements for *x*.

16-I Just one true statement, 3 + 6 = 9, was obtained. 3 was used as a replacement for *x* to obtain this statement. Therefore, 3 satisfies *x* + 6 = 9.

17-J The set whose members are zero, one, two, three, four. Twelve minus *n* equals eight.

17-K Each member of the universe was used as a replacement for *n*. * Sentence E expresses a true statement. * Sentences A, B, C, and D each expresses a false statement.

17-L One * Because only one member of the universe can be used as a replacement for *n* to obtain a true statement

17-M The set whose members are zero, two, four, six, eight. Three plus *y* equals fourteen.

17-N 3 + 0 = 14. 3 + 2 = 14. 3 + 4 = 14. 3 + 6 = 14. 3 + 8 = 14.

17-O None * No true statements were obtained.

17-P Yes. When y is replaced by 11 the true statement, 3 + 11 = 14, is obtained.* One

17-A Three y equals twelve. Six minus two equals four. x minus twenty equals ninety. Five times six equals thirty. Zero plus five equals zero. Three plus n equals fifteen. * Sentences A, C, and F * Each of these sentences expresses a requirement.

17-B Each of the sentences B, D, and E

17-C Yes

17-D Yes

17-E No

17 F Yes. It is a true statement that includes the idea of equality. * No. It is not a true statement.

17-G One. The statement expressed by sentence E

17-H No * Yes * $3y = 12$ is not a statement and therefore cannot be an equation. $3(4) = 12$ is a true statement that includes the idea of equality and therefore is an equation.

18-I The set whose members are zero, one, two, three, four, five. x plus five equals eight. * 0 + 5 = 8. 1 + 5 = 8. 2 + 5 = 8. 3 + 5 = 8. 4 + 5 = 8. 5 + 5 = 8.

18-J 3 + 5 = 8.

18-K No. $x + 5 = 8$ is not a statement. * Yes. An equation was obtained when x was replaced by 3.

18-L {3} * Yes. Each member of {3} is also a member of the universe.

18-M Yes * Because {3} contains all the members of the universe that satisfy $x + 5 = 8$.

18-N $2x + 1 = 7$. * Sentences A through F

18-O {3} * Yes. Each member of {3} is also a member of the universe.

18-P Yes * The solution set of a condition is obtained by finding the replacements for the variable that make true statements. Since only members of the universe are used as replacements for the variable, each member of the solution set is also a member of the universe.

On your own

18-1 Equation

18-2 Condition for equality

18 3 Condition for equality

18-4 Equation

18 5 Condition for equality

18-6 Condition for equality

18-7 Equation

18-8 Condition for equality

18-9 Condition for equality

18-10 Equation

18-11 Condition for equality

18-12 Condition for equality

18-13 Equation

18-14 Condition for equality

18-15 3 + 1 = 10. 3 + 3 = 10. 3 + 5 = 10. 3 + 7 = 10.(T) 3 + 9 = 10.

18-16 Yes * $3 + x = 10$ is a condition that includes the idea of equality.

18-17 {7} contains all the members of the universe that satisfy the condition.

18-18 {10}

18-19 {10}

18-20 {5}

18-21 {15}

18-22 {20}

18-23 {0}

18-24 {10}

18-25 {15}

Lesson 5

19-A The set whose members are five, six, seven, eight, nine, ten * The set of numbers from five through ten

19-C Yes. Any capital letter may be used as a short name for a set.* A equals the set whose members are ten, twenty, thirty, forty, fifty.

19-D Yes. M and N are different names for the same set.

19-E Yes. {7, 9, 11} and {9, 11, 7} have exactly the same members. * Yes. R and T are the same set.

19-F Yes * S and T are the same set. In the tabulation {13, 14, 15, 13} a name for the number thirteen is listed twice but the members of {13, 14, 15, 13} are the same as the members of {13, 14, 15}.

19-G The letter U

19-H 0, 1, 2, 3, 4, and 5 * No * Only members of the universe may be used as replacements for the variable in a condition.

19-I No. 3 + 3 is greater than 6 is a false statement. * No. 3 + 5 is greater than

6 is a true statement. Therefore 5 also satisfies the condition.

19-J {4, 5} * The universe

20-A Seven statements * Yes

20-B Decide which of the statements are true. The members of the universe used to obtain true statements satisfy $x + 2 > 6$.

20-C The solution set of the condition * {5, 6}

20-D No. An equation is a true statement that includes the idea of equality. When you replace x in $x + 2 > 6$ by each member of the universe, you obtain the statements expressed in D6. None of these are equations. * No. The condition does not include the idea of equality.

20-E The set whose members are zero, one, two, three, four, five, six, seven, eight, nine, ten

20-F Eleven statements * When zero is used as a replacement for n in $2n < 9$, the statement obtained is expressed by the sentence $2(0) < 9$. When each of the other members of the universe is used as a replacement for n, the statements obtained are expressed by similar sentences. * Yes. Ten is a member of the universe. * Yes. Zero is a member of the universe.

20-G Five * Decide which statements are true. The members of the universe used to obtain true statements satisfy $2n < 9$.

20-H {0, 1, 2, 3, 4} * The solution set is a subset of the universe.

20-I No. An equation is a true statement that includes the idea of equality. None of the statements obtained by replacing n in $2n < 9$ by members of the universe are equations. * No. The condition does not include the idea of equality.

20-J A slant mark is drawn through the symbol for equality.

20-K Not equal to five

21-L The letter U

21 M Nine * No. No true statement obtained includes the idea of equality. * No. The condition does not include the idea of equality.

21-N No * $5 \neq 5$ is a false statement

21-O {0, 1, 2, 3, 4, 6, 7, 8}

21-P Eleven plus five equals sixteen. Three plus eight is less than twelve. Nineteen plus six is greater than sixteen. Twelve is not equal to nine. Four plus three is less than nine. Two times four is not equal to seven. Nine minus two equals seven. Nine times three is greater than twenty. * Yes * Each sentence expresses an idea that is either true or false. * All the statements expressed in D10 are true.

21-Q Sentences A and G

21-R The idea of "less than" * Yes

21-S The idea of "greater than" * Yes

21-T Sentences B, C, D, E, F, and H

21-U No. It is a true statement but it does not include the idea of "greater than," or "less than," or "not equal to." * No. It is a true statement but it does not include the idea of equality.

21-W Two times

21-X Yes

21-Y {0, 1, 2, 3, 4, 5, 6, 7}

21-Z Sentences A, B, C, D, E, F, G, and H. They express true statements that include the idea of "less than." Sentences I and J are not inequalities because they are false statements.

On your own

22-1 Inequality

22-2 Not an inequality

22-3 Not an inequality

22-4 Inequality

22-5 Inequality

22-6 Not an inequality

22-7 Equation

22-8 Inequality

22-9 Inequality

22-10 Equation

22-11 Inequality

22-12 Inequality

22-13 $5 + 2 = 7$.

22-14 $4 > 8 - 6$.

22-15 $16 + 3 < 20$.

22-16 $3 = 8 - 5$.

22-17 $3 \times 5 > 14$.

22-18 $12 < 4(6)$.

22-19 {6, 9, 10}

22-20 {6}

22-21 {0}

22-22 {0, 9, 10}

22-23 {0, 6, 9, 10}

22-24 {10}

22-25 {10}
22-26 {9, 10}
22-27 {0, 6, 9}
22-28 {0, 6, 9, 10}
22-29 {10}
22-30 {6}
22-31 {0}
22-32 {0}
22-33 {0}
22-34 {0}
22-35 {0}
22-36 {9, 10}

Lesson 6

22-A It is easier to describe this set than to tabulate the set by naming each member.
22-B The numerals 3 through 999
23-C 1001 members
23-D The set whose members are the natural numbers from fifty through one hundred
23-E The numerals 53 through 99
23-F 51 members
23-G Yes * Yes * If it is possible to tabulate a set by listing the name of each member, then you can count the members of the set.
23-H The three dots represent the numbers whose names are not listed in the tabulation.
23-I 2,000,000
23-J Yes * The number of members in the set is the natural number 2,000,000.
23-K Yes * The number of members in the set is the natural number 100,000,001
23-L No. For every natural number there is another natural number that is greater. * There is no end to the list of names of the members of the set of natural numbers.
23-M The three dots tell you that the natural numbers go on and on. * There is no last number in the set of natural numbers.
23-N 37 * Add 1 to 36
23-O Yes * Add 1 to each natural number to find the next natural number.
23-P No. * There are more members in the set of natural numbers than any natural number that you name.
23-Q No * The number of members in the set of natural numbers is not a natural number.
23-R Yes * {0, 2, 4, ...}
23-S Yes * {1, 3, 5, ...}
23-T Yes. The natural number 5 is the number of members in the set. * No
23-U No. The number of members in the set is not a natural number. * Yes. A set that is not finite is infinite.
23-V Yes. Zero is less than every other natural number.
24-A The set of natural numbers * Yes
24-B No. 5 does not satisfy $x + 4 > 9$. 5 + 4 is the same as 9, not greater than 9. * No. No number less than 5 satisfies $x + 4 > 9$.
24-C 6, 7, and 8.
24-D No. Any natural number greater than 5 satisfies $x + 4 > 9$.
24-E {6, 7, 8, ...}
24-F The set of natural numbers greater than 5 * Yes
24-G No. 5 does not satisfy $x + 4 < 9$. * No. No number greater than 5 satisfies $x + 4 < 9$.
24-H {0, 1, 2, 3, 4} * No * Yes. It contains 5 members.
24-I No * No
24-J {5} * One * Yes
24-K The sentences $x + 4 > 9$ and $x + 4 < 9$ * The sentence $x + 4 = 9$
24-L Yes
24-M Yes
25-N 3
25-O Yes
25-P Yes
25-Q {4, 5, 6, ...} * {0, 1, 2}
25-A s plus two is less than fifteen. * When you replace s by 0, you obtain the statement $0 + 2 < 15$. You obtain similar statements by replacing s by each of the other members of the universe. * {0, 1, 2, ..., 10}
25-B 11 and 12 are not members of the universe.
25-C 11 * 11
25-D Yes * Yes. When each member of one set is also a member of a second set, the first set is a subset of the second set.
25-E Yes * Yes. Each member of the universe is also a member of the solution set. * Yes. When one set is a subset of a second set and the second set is also a subset of the first set, the two sets are the same.
25-F $2 + 3 = 3 + 2$. * Yes

25-G 5,999,999 + 3 = 3 + 5,999,999. * Yes

25-H The set of natural numbers (N)

25-I Yes

25-J Yes

25-K If you replace x by 0, you obtain the statement 0 + 5 = 110. You obtain similar statements when you replace x by each of the other members of the universe. * None * None

25-L Yes. The solution set of the condition expressed in D11 has no members and a set that has no members is the empty set.

25-M { }

26-N U = N tells you that the universe is the set of natural numbers. * 1 + 2 = 2. * No * 1 + 10,000 = 10,000. * No

26-O No * Use the symbol for the empty set * The natural number 0 is the number of members in the empty set.

On your own

26-1 {1, 3, 5, ..., 39}

26-2 {11, 12, 13, ...}

26-3 {3, 5, 7}

26-4 The set of natural numbers from 0 through 34

26-5 The set of natural numbers from 3 through 30 that can be divided by 3 with a remainder of 0

26-6 The set of odd natural numbers greater than 10

26-7 {15, 16, 17, ...} * Infinite

26-8 {5} * Finite

26-9 {5} * Finite

26-10 {6, 7, 8, ...} * Infinite

26-11 {0, 1, 2, ...} * Infinite

26-12 { } * Finite

26-13 { } * Finite

26-14 {0, 1, 2, ...} * Infinite

26-15 { } * Finite

26-16 {1, 2, 3, ...} * Infinite

26-17 {9, 10, 11, ...} * Infinite

26-18 {0, 1, 2, 3, 4} * Finite

Lesson 7

26-A The set whose members are zero, one, two, three, four, five * Yes

26-B The set whose members are

26-C x is less than six. * The set whose members are all natural numbers that satisfy the condition that each number is less than six. * The set whose members are

26-D The set whose members are all x that satisfy the condition that each x is less than six

26-E The set whose members are all x that satisfy the condition that each x is less than six

27-F The set whose members are all x that satisfy the condition that each x is less than six

27-G The set whose members are all x such that each x is less than six

27-H $\{x \mid x + 4 < 7\}$

27-I $\{x \mid x < 10\}$

27-J $\{x \mid x > 100\}$

27-K Yes * Yes. The members of the set tabulated in D7 are all the members of N that satisfy $x + 4 < 20$.

27-L Yes. From exercise K you know that {0, 1, 2, ..., 15} is the solution set of $x + 4 < 20$. You also know that $\{x \mid x + 4 < 20\}$ is the solution set of $x + 4 < 20$.

27-M Yes * "$\{x \mid x + 4 < 20\}$" and "{0, 1, 2, ..., 15}" both name the solution set of $x + 4 < 20$.

28-N Yes. $\{x \mid 5 + x > 7\}$ is the set whose members are all x that satisfy the condition that $5 + x > 7$. * Yes. The members of {3, 4, 5, ...} are all members of N that satisfy $5 + x > 7$. This means that {3, 4, 5, ...} is the solution set of $5 + x > 7$. Therefore, "$\{x \mid 5 + x > 7\}$" and "{3, 4, 5, ...}" are names of the same set.

On your own

28-1 $\{x \mid x < 50\}$

28-2 $\{n \mid n \neq 3\}$

28-3 $\{y \mid y + 3 > 9\}$

28-4 $\{x \mid 2x < 20\}$. Other letters may be used instead of x.

28-5 $\{z \mid z + 1 \neq 4\}$

28-6 $\{x \mid x < 7\}$. Other letters may be used instead of x.

28-7 $\{r \mid r = 9\}$

28-8 $\{x \mid x + 3 > 8\}$

28-9 $\{x \mid x + x > 10\}$. Other letters may be used instead of x.

28-10 The set whose members are all x such that x is greater than two. The set whose members are all x that satisfy the condition that x is greater than two

28-11 The set whose members are all n such that n plus three is not equal to ten. The set whose members are

all n that satisfy the condition that n plus three is not equal to ten

28-12 The set whose members are all x such that x plus x is less than seventeen. The set whose members are all x that satisfy the condition that x plus x is less than seventeen

28-13 The set whose members are all z such that six times z equals eighteen. The set whose members are all z that satisfy the condition that six times z equals eighteen

28-14 $\{x \mid x + 3 > 20\}$

28-15 $\{y \mid y + 6 = 14\}$

28-16 $\{m \mid m + 3 < 10\}$

28-17 $\{r \mid r + 2 \neq 2\}$

28-18 {18, 19, 20, ...} * {8} * {0, 1, 2, 3, 4, 5, 6} * {1, 2, 3, ...}

28-19 False

28-20 True

28-21 False

28-22 True

28-23 {0, 1, 2, 3, 4, 5, 6}

28-24 {10, 11, 12, ...}

28-25 {10}

28-26 { }

28-27 {26, 27, 28, ...}

28-28 {0, 1, 2, ...}

Checking up

Test 1

28-1 10 > 3(0). True
10 > 3(1). True
10 > 3(5). False
10 > 3(10). False

28-2 5 ≠ 0. True
5 ≠ 1. True
5 ≠ 5. False
5 ≠ 10. True

28-3 8 + 0 = 17. False
8 + 1 = 17. False
8 + 5 = 17. False
8 + 10 = 17. False

28-4 12 - 0 < 3. False
12 - 1 < 3. False
12 - 5 < 3. False
12 - 10 < 3. True

28-5 {5}

28-6 {3, 4, 5}

28-7 {0, 1, 2, 4, 5}

28-8 {4, 5}

28-9 {4, 5}

28-10 {0, 2, 3, 4, 5}

28-11 { }

28-12 {0, 1, 2}

Test 2

28-13 8, 9, 10, 11

28-14 6, 8, 10, 12

28-15 13, 15, 17

28-16 A subset

28-17 Not a subset

28-18 Not a subset

28-19 A subset

Lesson 8

29-A Yes. Since points are objects and a collection of objects is a set, the points represented in picture C form a set of points.

29-B Picture D was obtained by first reproducing the dots in picture C and then placing another dot between each two dots. * Yes

29-C If your pencil is sharp enough, you can place many dots between any two dots. You should think of many points between any two points.

29-D Think of the set of points between two given points. This set is an infinite set. Picture E represents such a set. Therefore E represents an infinite set of points.

29-E Yes. The picture of each side of a rectangle is like picture E in D1 and represents an infinite set of points. * Yes

29-F A circle is a set of points. The picture of the circle in D2 should indicate to you that a circle is an infinite set of points. * A spiral is a set of points. The picture of the spiral in D2 should indicate to you that a spiral is an infinite set of points.

29-G You can make sketches of many different geometric figures. Most of these figures do not have names. Two examples of geometric figures are shown in the sketches below.

30-H Yes

30-I 4 dots * 4 points. Each dot represents one point.

30-J Yes

30-K Yes. Many of these dust particles are so small that you need a microscope to see them.

30-L Yes. If one dust particle suggests one point then many dust particles suggests many points.

30-M Yes. Each dust particle suggests a point, and so when more dust particles are released into the atmosphere, more points are suggested.

30-N You should think of space as an infinite set of points. You should think this because you know that

there are many points between any two points.

30-O Yes * The tip of the compass and the tip of the nail

30-A Yes. Each member of the illuminated dust particles is a member of the set of dust particles in the room.

30-B Yes. Space is the set of all points and any set of points is a subset of space.

31-C (Column 1) Yes * A line is a set of points and a set of points is a geometric figure.

31-D (Column 1) You should think of a tightly stretched wire as suggesting a line. * Many objects in your classroom should suggest lines. Examples could be the edge of a desk and a chalk tray.

31-F Yes * Yes * You know that there are many points between any two points. You also know that a line extends endlessly. This means that each line represented in D5 is an infinite set of points. It also means that any line is an infinite set of points.

31-G "Line c" or "c"

31-H No * Since a line extends endlessly you cannot say one line is longer than another line. * Line b contains different points than line c. Line b is also in a different position than line c.

31-I The name of the other line is ℓ_2 which is read "ℓ subscript two," "ℓ sub two," or, simply, "ℓ two."

31-J Yes * Yes. Any line extends endlessly in opposite directions.

31-A Yes. Each member of the set of illuminated dust particles is also a member of the set of dust particles in the room.

31-B Yes. Space is the set of all points and any set of points is a subset of space.

31-C (Column 2) Yes * A plane is a set of points and a set of points is a geometric figure.

31-D (Column 2) You should think of the surface of a mirror, if it is "flat," as suggesting a plane. * Many objects, such as the surface of calm water and the surface of a blackboard, suggest planes.

32-F Yes

32-G Point E

32-H {C, F} {A, E}

32-I Yes * Yes * Many more dots can be used to represent many more points. Since there are many points in a plane between any two points represented by dots and, also, since a plane extends endlessly, a plane is an infinite set of points.

32-J Since the dot that represents point B is not in a picture of either plane represented in D7, point B is not in either plane.

32-K Yes. All points are members of space. * No

32-L Yes * ℓ_2. Since the picture of ℓ_3 is not in a picture of either plane represented in D7, ℓ_3 is not a subset of either plane.

32-M Yes * You can picture as many lines as you want included in a plane. To picture many lines you will need to use very narrow pictures of lines.

32-N Yes. Even with a pencil having the sharpest tip possible, you cannot sketch pictures of all the lines included in a plane.

On your own

32-1 {B, C, D}

32-2 {A, E}

32-3 Yes * ℓ_2 and ℓ_3

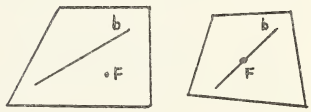
32-4 {ℓ_4}

32-5 {A, B, C, D, E}

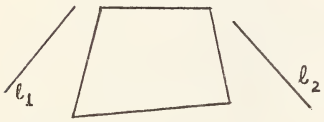
32-6 Yes. Space is the set of all points and therefore each member of {A, B, C, D, E} is a member of space. Any set of points is a subset of space.

32-7 Yes. Each member of {B, C, D} is a member of {A, B, C, D, E}.

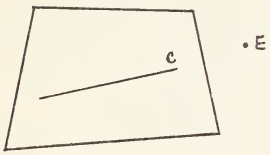
32-8 See the sketches below. First make a sketch of a plane. Line b may be included anywhere in the plane. You may locate F anywhere in the plane. The two sketches show that you may locate F in the line or not in the line.



32–9 See the sketch below. First make a sketch of a plane. Then you can include ℓ_1 and ℓ_2 anywhere but in this plane.



32–10 See the sketch below. First make a sketch of a plane. Then you can locate E anywhere but in this plane.



Lesson 9

33–A Points A and B

33–B Yes. A line is an infinite set of points. * Yes * Each member of {A, B} is also a member of ℓ_1.

33–C ℓ_1, ℓ_2, ℓ_3, ℓ_4, ℓ_5 * Yes. The dot representing point C is on the picture of each line.

33–D Yes * Think of making sketches of lines that contain C. You can make sketches of lines as close together as you can and there are still many lines containing C that you have not represented.

33–E No * There are lines that go through the plane suggested on page 33 of your book that also contain C.

33–F Only one line, ℓ_1

33–G Only one. Remember that "line" means "straight line." * No

33–H Yes. D8 represents 4 geometric figures that each contain E and F. It is impossible to make sketches of all the geometric figures that contain E and F.

33–I ℓ_1 * ℓ_2

33–J ℓ_1 * Yes. U and T are the same points as T and U.

33–K One. For any two different points there is exactly one line that contains both of these points.

34–L The double-headed arrow reminds you that a line extends endlessly in opposite directions.

34–M Yes. ℓ_6 is the only line that contains both G and H. * Yes. ℓ_6 is the only line that contains both H and J. * Since any two points determine a line, G and J must also determine ℓ_6.

34–N Yes. You can use the names of the two points that determine a line to name the line. * Yes. When naming a line by using the names of two points that determine the line, it does not make a difference in which order you write the names of these two points. * $\overleftrightarrow{HJ}$, $\overleftrightarrow{JH}$, $\overleftrightarrow{GJ}$, and $\overleftrightarrow{JG}$ are other names for ℓ_6.

34–O Yes. Q, R, and S are contained in the same line.

34–P Yes * Any two points are contained in the same line.

34–Q Q, R, and T are not contained in the same line.

34–A Yes * Yes

34–B No

34–C Yes

34–D Yes. If the stick is straight, it will touch the board all along its length.

34–E Yes

34–F Yes. Since each point in $\overleftrightarrow{AC}$ is also in the plane, $\overleftrightarrow{AC}$ is a subset of the plane. * Yes. Since each point in $\overleftrightarrow{CB}$ is also in the plane, $\overleftrightarrow{CB}$ is a subset of the plane.

34–G Yes. If a line contains two points of a plane, the line is included in the plane. * $\overleftrightarrow{GF}$ and $\overleftrightarrow{GE}$ * $\overleftrightarrow{HG}$, $\overleftrightarrow{HE}$, and $\overleftrightarrow{HF}$

35–A One line * For any two different points there is exactly one line that contains both of these points.

35–B Yes * Points A and B are in the plane suggested by the surface of the book cover. Since $\overleftrightarrow{AB}$ contains two points of this plane, $\overleftrightarrow{AB}$ is included in the plane.

35–C Yes. Points A and B are in each plane suggested by a leaf of the book. Since A and B are also in $\overleftrightarrow{AB}$, $\overleftrightarrow{AB}$ must be included in each plane.

35–D No * The book could have more leaves to suggest more planes that include $\overleftrightarrow{AB}$.

35–E Yes * No matter how many leaves the book had, they could not suggest all the planes that include $\overleftrightarrow{AB}$.

35–F A, B, C, D, E * C, D, E

35–G Only one

35-H One * One

35-I Yes * Point D is not in $\overleftrightarrow{AB}$. There is exactly one plane that includes a given line and contains a point not in the line.

35-J Yes * A, B, and D are not contained in the same line, and points not contained in the same line are noncollinear.

35-K Yes * Yes * Think of three points like A, B, and D or like A, B, and E. Two of the points determine a line and the third point is not in the line. You know that there is exactly one plane that includes a given line and contains a point not in the line.

35-L Yes * Because two of the points determine a line that does not contain the third point. Only one plane includes this line and contains the third point. * No

35-M Exactly one * A, B, and C are three noncollinear points and there is exactly one plane that contains three noncollinear points. The three points determine this plane.

36-N Yes. There is exactly one plane that contains three noncollinear points.

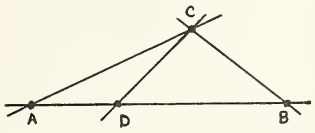
36-O Yes * Plane ACB and plane AEB

36-P Yes * Since D, F, G, and H are contained in the same plane you can use the names of any three of these points that are noncollinear to name the plane.

36-Q Yes

On your own

36-1 See the picture below. You may locate A and B in any positions. But the line must contain A and B.

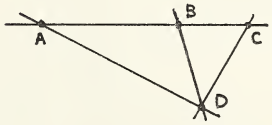


36-2 See the picture above. Point C may be located in any position but not in line AB.

36-3 See the picture in 36-1. You may locate point D anywhere in line AB.

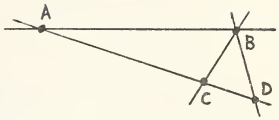
36-4 See the picture in 36-1.

36-5 See the picture below. The points represented in your picture may be located anywhere just so they meet the requirements in the exercise.

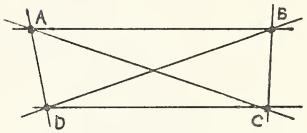


Names of the lines are $\overleftrightarrow{AD}$, $\overleftrightarrow{BD}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{AC}$. Each line has more than one name.

36-6 See the next two pictures. Notice that A, C, and D may be collinear or noncollinear. The points represented in your picture may be located anywhere just so they meet the requirements in the exercise.

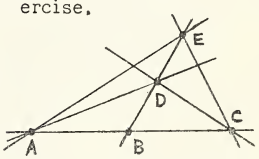


Names of the lines represented above are $\overleftrightarrow{AD}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, and $\overleftrightarrow{BD}$.



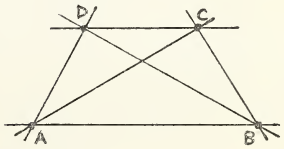
Names of the lines represented above are $\overleftrightarrow{AD}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{DB}$, and $\overleftrightarrow{BA}$.

36-7 See the picture below. The points represented in your picture may be anywhere just so they meet the requirements in the exercise.



Names of the lines are $\overleftrightarrow{AD}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{AC}$, and $\overleftrightarrow{BE}$.

36-8 See the picture below. The points represented in your picture may be anywhere just so they meet the requirements in the exercise.

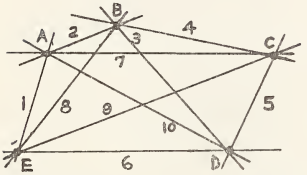


Names of the lines are $\overleftrightarrow{BD}$, $\overleftrightarrow{CA}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{DA}$.

36-9 Six

36-10 Ten lines are determined.

Below is a picture of the lines determined by 5 points, A, B, C, D, and E.



36-I1 Infinitely many lines can contain a given point.

36-I2 Exactly one line

36-I3 Infinitely many planes

36-I4 Infinitely many planes

36-I5 A, B, and C are non-collinear points.

36-I6 Four

36-I7 A, B, and C are collinear.

Lesson 10

37-A Yes

37-B Yes * Yes * Yes. Points that are contained in the same line are collinear.

37-C No. Points D, J, and H are not contained in the same line.

37-D Yes * If one point is between two other points, then the three points are collinear.

37-E {E, F, G} * No * D, H, and I are not collinear so I is not between D and H.

37-F Yes. The order of two points does not change the points that are between the two points.

37-G From D2 you know that M, N, O, and P are collinear because the dots representing these points are on the picture of one line.

37-H No * No * Yes

37-I Yes

37-J No

37-K No * No * For every three collinear points, just one of the points is between the other two.

37-L W is between S and V. For every three collinear points just one of the points is between the other two.

37-M Yes * Yes * Yes

37-N Yes * No matter how close together you place two dots, the dots can be made small enough so that you can place another dot between them. Points do not have size and can be located much closer together than dots. However, no matter how close together two points are located, you can always locate another point between them.

38-O Yes * Yes * Yes * For any two points, there is at least one point between them.

38-P Yes * Yes * Yes

38-Q Yes

38 R Yes * Yes

38-S Yes. More and more points can be located between two points in the manner represented in D3. Since you can never locate all the points between A and B, there are infinitely many points between A and B.

38-T The property that states that for any two points, there is at least one point between them. Suppose the two given points are A and B. The property states that there is a point between A and B. Name this point C. Now there are two more points between A and B. One of these points is between A and C and the other point is between C and B. In this way you can keep on locating points between A and B. Since you cannot locate all the points, there are infinitely many points between A and B.

38-A Yes * Yes. Because you are working with the set of points whose members are X and Z and all points between X and Z.

38-B No * No

38-C Yes * To name a segment, it does not matter in which order you write the names of the two endpoints.

38-D $\overline{AB}$, $\overline{CD}$, $\overline{EF}$. Each of these segments can be named in two ways. Other names for these segments are $\overline{BA}$, $\overline{DC}$, and $\overline{FE}$.

39-E (Column 1) $\overline{XZ}$ * Yes. Y is between X and Z. * No. W is not between X and Z. * Yes. $\overline{XZ}$ contains both X and Z.

39-F (Column 1) No * Yes * A line extends endlessly in opposite directions but a segment has endpoints.

39-G Yes * Each member of $\overline{XW}$ is also a member of $\overleftrightarrow{XW}$.

39-A $\overline{AB}$, $\overline{GH}$, $\overline{EF}$, $\overline{KL}$, $\overline{CD}$, $\overline{IJ}$. Remember that each segment has two names.

39-B The first sentence is read, "segment AB is congruent to segment GH." The other sentences are read in the

same way except for the names of the segments.

39-C No * No * $\overline{AB}$ and $\overline{GH}$ do not contain the same points and therefore are not the same. Since equality means "the same as," $\overline{AB} = \overline{GH}$ is not a true statement.

39-D You know that if a first number equals a second number then the second number equals the first number. In the same way, you are correct in thinking that, since $\overline{KL} \cong \overline{CD}$ is a true statement, $\overline{CD} \cong \overline{KL}$ is also a true statement.

39-E (Column 2) Yes. Since $\overline{KL} \cong \overline{IJ}$ is a true statement, $\overline{IJ} \cong \overline{KL}$ is also a true statement. * Yes * Yes

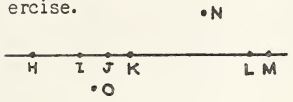
39-F (Column 2) You know that if a first number equals a second number and the second number equals a third number, then the first number equals the third number. In the same way, you are correct in thinking that, since $\overline{GH} \cong \overline{AB}$ and $\overline{AB} \cong \overline{EF}$ are true statements, $\overline{GH} \cong \overline{EF}$ is a true statement.

40-G Yes. The property you have just learned can also be expressed as follows: If $\overline{CD} \cong \overline{KL}$ and $\overline{KL} \cong \overline{IJ}$, then $\overline{CD} \cong \overline{IJ}$.

40-H If you think that you do not know whether or not $\overline{AB} \cong \overline{KL}$, you are thinking correctly. The statements expressed in D8 do not give enough information to decide whether or not $\overline{AB} \cong \overline{KL}$ is true.

On your own

40-1 See the picture below. The points represented in your picture may be located anywhere just so they meet the requirements in the exercise.



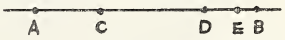
40-2 {I, J, K, L} * {J, K} * { }

40-3 {I, J, K} * {I, J, K}

40-4 No. If a point is between H and L it is also between L and H.

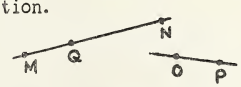
40-5 Betweenness applies only to collinear points.

40-6 See the picture below. The points represented in your picture may be located anywhere just so they meet the requirements in the exercise.



40-7 Yes * Infinitely many * For any two points, there is at least one point between them. Using this property you can keep on locating points indefinitely.

40-8 See the pictures below. The pictures of $\overline{MN}$ and $\overline{OP}$ may be placed in any position.



40-9 Yes. A segment is an infinite set of points.

40-10 M and N are the endpoints of $\overline{MN}$. O and P are the endpoints of $\overline{OP}$. * $\overline{NM}$ and $\overline{PO}$

40-11 See the picture in 40-8. In your picture, you may represent Q located anywhere in $\overline{MN}$. * Q is between M and N.

40-12 No * No

40-13 Yes * Since you know that $\overline{UV} \cong \overline{WX}$, you also know that $\overline{WX} \cong \overline{UV}$. You now know that if $\overline{WX} \cong \overline{UV}$ and $\overline{UV} \cong \overline{YZ}$, then $\overline{WX} \cong \overline{YZ}$.

40-14 Segment AB is congruent to segment CD.

40-15 Segment AB is congruent to segment EF.

40-16 Segment GH is congruent to segment IJ.

40-17 Segment KL is congruent to segment MN.

40-18 True statement

40-19 True statement

40-20 False statement

40-21 True statement

40-22 False statement

40-23 False statement

40-24 True statement

40-25 False statement

Lesson 11

41-A Yes

41-B {A, B, C}

41-C None

41-D One

41-E No * Since there is one point between A and C that is in the plane, point A and point C are not in the same half-space.

41-F Yes. Since each plane extends endlessly, every plane separates space into two half-spaces.

41-G Yes

41-H Yes

41-I {E, F, G}

41-J None

41-K One

41-L No. There is a point in ℓ_1 between F and G.

42-M Yes. Because every line extends endlessly in opposite directions

42-N Yes

42-O {H, I, J}

42-P No

42-Q Yes

42-R No. Since R is between I and J, I and J are not in the same half-line.

42-S Yes * Yes * Yes * Yes

42-T Yes * Because H is not between I and J

42-A ---

42-B Yes

42-C ---

42-D No * Yes

43-E ---

43-F ---

43-G Yes

43-H Yes

43-I $\overline{EF} \cong \overline{GH}$.

43-J No * No * $\overline{LM}$ is included in line LM. L separates line LM into two half-lines. Since you know that $\overline{JK} < \overline{LM}$, you know that there is a point R between L and M so that $\overline{JK} \cong \overline{LR}$. Now you know that $\overline{JK} \cong \overline{LM}$ is a false statement because point R is not the same point as M. You also know that $\overline{JK} > \overline{LM}$ is a false statement because M is not between L and R.

43-K If you think that $\overline{OB} < \overline{IJ}$ is a false statement, you are thinking correctly. * If you think that $\overline{OB} > \overline{IJ}$ is a true statement, you are thinking correctly. * You know that if one segment is less than a second segment, then the second segment cannot be less than the first segment. Using the same reasoning, since $\overline{IJ} < \overline{OB}$ is a true statement, $\overline{OB} < \overline{IJ}$ is a false statement. You also know that if one segment is less than a second segment, then the second segment must be greater than the first segment. Using the same reasoning, since $\overline{IJ} < \overline{OB}$ is a true statement, $\overline{OB} > \overline{IJ}$ is a true statement.

43-L If you think that $\overline{KL} < \overline{OB}$ is a true statement, you are thinking correctly. * Since $\overline{IJ}$ is congruent to $\overline{KL}$, you may replace $\overline{IJ}$ with $\overline{KL}$ in statements involving the ideas of "congruence," "is less than," or "is greater than."

43-M If you think that $\overline{IJ} < \overline{RS}$ is a true statement, you are thinking correctly. * If one segment is less than a second segment, and the second segment is less than a third segment, then the first segment must be less than the third segment. Since you know that $\overline{IJ} < \overline{OB}$ and also that $\overline{OB} < \overline{RS}$, then you know that $\overline{IJ} < \overline{RS}$.

On your own

43-1 Yes

43-2 Yes * No

44-3 Yes * No

44-4 No * Yes

44-5 Yes * No

44-6 Yes

44-7 No

44-8 Yes

44-9 No

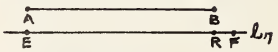
44-10 Yes

44-11 No

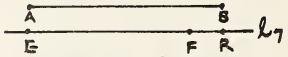
44-12 Yes

44-13 No

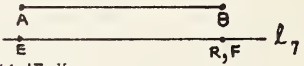
44-14 $\overline{AB} < \overline{EF}$ or $\overline{EF} > \overline{AB}$.



44-15 $\overline{AB} > \overline{EF}$ or $\overline{EF} < \overline{AB}$.



44-16 $\overline{AB} \cong \overline{EF}$ or $\overline{EF} \cong \overline{AB}$.



44-17 Yes

44-18 No * Yes

44-19 U and R are the same point.

44-20 Yes

44-21 Yes

44-22 Yes

44-23 Yes

44-24 Yes

44-25 No

44-26 No

44-27 Yes

44-28 Yes

44-29 No

44-30 $\overline{MO} > \overline{ST}$ is a true statement.

Lesson 12

45-A Plane ABC * Because the members of the universe are points

45-B Yes

45-C Yes. Because when X in $\overline{AX} \cong \overline{AB}$ is replaced by C, a true statement is obtained. * Yes. The solution set is made up of all the members of the universe

that satisfy the condition.

45-D Yes. Since $\overline{AC}$ is congruent to $\overline{AB}$, you may replace $\overline{AC}$ by $\overline{AB}$. You know that $\overline{AE} < \overline{AC}$. Therefore, replacing $\overline{AC}$ by $\overline{AB}$, you know that $\overline{AE} < \overline{AB}$. * Yes. You know that $\overline{AD} > \overline{AC}$. Therefore, replacing $\overline{AC}$ by $\overline{AB}$, you know that $\overline{AD} > \overline{AB}$.

45-E No. Because neither E nor D satisfy $\overline{AX} \cong \overline{AB}$

45-F P, T, H, S, M, Q

46-G You can replace $\overline{AH}$ by $\overline{AB}$ since you know that $\overline{AH} \cong \overline{AB}$. Therefore, $\overline{AN} < \overline{AB}$ and $\overline{AV} > \overline{AB}$. If $\overline{AN}$ is less than $\overline{AB}$, $\overline{AN}$ cannot be congruent to $\overline{AB}$. If $\overline{AV}$ is greater than $\overline{AB}$, $\overline{AV}$ cannot be congruent to $\overline{AB}$.

46-H If you think that K does not satisfy $\overline{AX} \cong \overline{AB}$, you are thinking correctly. * If you think that R does not satisfy $\overline{AX} \cong \overline{AB}$, you are thinking correctly.

46-I {B, C, P, T, H, S, M, Q} * You can locate many more points that are members of $\{X \mid \overline{AX} \cong \overline{AB}\}$.

46-J No * No

46-K No

46-L Yes

46-M No. A does not satisfy $\overline{AX} \cong \overline{AB}$. * No. Only those points that satisfy $\overline{AX} \cong \overline{AB}$ are in the circle.

46-N Yes * Point B * Yes. B satisfies $\overline{AX} \cong \overline{AB}$.

46-O Yes * Yes * Yes * One endpoint of $\overline{AC}$ is the centre of the circle and the other endpoint is in the circle so $\overline{AC}$ is a radius of the circle.

46-P Yes. Each point in a circle together with the centre of the circle determines a radius of the circle. * Since a circle contains infinitely many points and each of these points determines a radius, a circle has infinitely many radii.

47-Q Yes

47-R One

47-A (Column 1) Circle K * Point K

47-B (Column 1) $\overline{KC}$ and $\overline{KD}$

47-C Yes. The universe for X in $\overline{KX} \cong \overline{KC}$ is plane AKB.

47-D Yes * Yes

47-E Yes. Since $\overline{KD}$ is congruent to $\overline{KC}$, you may replace $\overline{KC}$ with $\overline{KD}$.

47-F You must know the name of the centre and the name of one radius, or a segment congruent to a radius.

47-G Yes. The endpoints of $\overline{AB}$ are points A and B. A and B are both in circle K. $\overline{DC}$ has both of its endpoints, C and D, in circle K.

47-H Yes. $\overline{CD}$ has both of its endpoints in circle K. A segment that has both of its endpoints in a circle is a chord of that circle.* Yes

47-I Yes. $\overline{CD}$ is a chord of circle K that contains the centre of circle K. A chord that contains the centre of a circle is a diameter of the circle. * No. $\overline{AB}$ is a chord of circle K that does not contain the centre and therefore is not a diameter.

47-J Four * $\overline{MA}$, $\overline{MB}$, $\overline{MC}$, $\overline{MD}$ * $\overline{AC}$, $\overline{BD}$, $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ * $\overline{AC}$, $\overline{BD}$

47-K Both endpoints of a chord are in the circle and only one endpoint of a radius is in the circle.

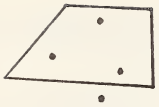
47-L Yes. The endpoints of $\overline{EF}$ are E and F. E and F are both in circle M. * Yes. Two points that are both in a circle determine a segment whose endpoints are both in the circle. * Yes. Since any two points in a circle determine a chord and a circle contains infinitely many points, a circle has infinitely many chords.

47-M Yes. Points M and G determine a line. Two points in the line, G and one other point, are in the circle. These two points determine a diameter. * Yes * Yes. Since a circle contains infinitely many points and each of these points is an endpoint of a diameter, a circle has infinitely many diameters.

47-N A sphere. The edge of a coin suggests a circle. If you spin the coin, the edge of the coin will suggest a sphere.

47-A (Column 2) Yes * Three points, whether they are collinear or noncollinear, are in the same plane. Since the three points are in the same plane, they are coplanar points.

47-B (Column 2) Your sketch should be similar to the following sketch.



48-C Yes * $\overline{RT}$

48-D Segment RX is less than segment RS. * Plane RSN

48-E $\overline{AR}$ * Yes * Yes. Because $\overline{RA} < \overline{RS}$ is a true statement.

48-F No * No * Since $\overline{RT} \cong \overline{RS}$ is a true statement, you know that $\overline{RT} < \overline{RS}$ is a false statement and since $\overline{RG} > \overline{RS}$ is a true statement, you know that $\overline{RG} < \overline{RS}$ is a false statement.

48-G J, I, H, A, R, E, D, K

48-H Yes * Yes

48-I Circle R is the solution set of $\overline{RX} \cong \overline{RS}$ while the interior of circle R is the solution set of $\overline{RX} < \overline{RS}$.

48-J Segment RX is greater than segment RS. * Yes * Since $\overline{RG} > \overline{RS}$ is a true statement, G is a solution of $\overline{RX} > \overline{RS}$.

48-K No * No * Since $\overline{RT} \cong \overline{RS}$ is a true statement, $\overline{RT} > \overline{RS}$ is a false statement. Since $\overline{RD} < \overline{RS}$ is a true statement, $\overline{RD} > \overline{RS}$ is a false statement.

48-L C, V, Q, P, G, F, W, M, N, B * Yes

48-M Circle R is the solution set of $\overline{RX} \cong \overline{RS}$ and the exterior of circle R is the solution set of $\overline{RX} > \overline{RS}$.

49-N No. A single point cannot satisfy both conditions like $\overline{RX} \cong \overline{RS}$ and $\overline{RX} < \overline{RS}$. * No. A single point cannot satisfy both of two conditions like $\overline{RX} > \overline{RS}$ and $\overline{RX} < \overline{RS}$.

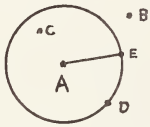
49-O Exterior of the circle * Yes. Consider the circle $\{X \mid \overline{AX} \cong \overline{AB}\}$. The universe is plane ABC. Each point in plane ABC must satisfy one of the conditions: $\overline{AX} \cong \overline{AB}$, $\overline{AX} < \overline{AB}$, or $\overline{AX} > \overline{AB}$.

49-P Yes

49-Q No

On your own

49-1 See the sketch below. You can locate B anywhere in the exterior of circle A.

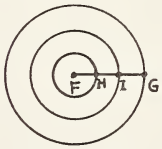


49-2 See the sketch above. You may locate C anywhere in the interior of the circle.

49-3 See the sketch in 49-1. You may locate D anywhere in the circle.

49-4 See the sketch in 49-1. You may locate E anywhere in the circle.

49-5 See the sketch below.



49-6 See the sketch above. You may locate H anywhere between F and G.

49-7 See the sketch in 49-5. You may locate I anywhere between H and G.

49-8 See the sketch in 49-5.

49-9 See the sketch in 49-5.

49-10 See the sketch in 49-5.

49-11 Infinitely many

49-12 Circle R. The centre is R. A radius is $\overline{RM}$.

49-13 Circle C. The centre is C. A radius is $\overline{BC}$.

49-14 Circle L. The centre is L. A radius is $\overline{KL}$.

49-15 Circle E. The centre is E. A radius is $\overline{ED}$.

49-16 $\{X \mid \overline{AX} \cong \overline{AB}\}$

49-17 $\{X \mid \overline{CX} \cong \overline{CD}\}$

49-18 $\{X \mid \overline{EX} \cong \overline{FG}\}$

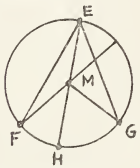
49-19 $\{X \mid \overline{HX} \cong \overline{HJ}\}$, or $\{X \mid \overline{HX} \cong \overline{HK}\}$

49-20 $\{X \mid \overline{LX} < \overline{LM}\}$

49-21 $\{X \mid \overline{NX} > \overline{NP}\}$, or $\{X \mid \overline{NX} > \overline{NQ}\}$

49-22 $\{X \mid \overline{RX} < \overline{TR}\}$

49-23 See the sketch below. E, F, and G may be located anywhere in the circle.



49-24 See the sketch in 49-23.

49-25 Yes

49-26 See the sketch in 49-23.

49-27 See the sketch in 49-23.

49-28 See the sketch in 49-23.

49-29 See the sketch in 49-23.

49-30 Yes * No

Checking up

Test 3

50-1 One

50-2 One

50-3 Point E and point G

50-4 B, E, and G

50-5 A and D

50-6 C and E

50-7 Two

50-8 B * F and G (E is the

boundary and is not in either half-line).

Test 4

50-9 K

50-10 Radii KN and KL are the only two represented in the display.

50-11 Chords LM and LN are the only two represented.

50-12 Diameter LN

50-13 H, K, and T

50-14 R, S, and V

50-15 L, M, and N

50-16 Segments KL and KN

Lesson 13

50-A Nine

50-B 1

50-C 2 * 2 * Point 2

51-D 5 * 3 is one more than 2.* 4 is one more than 3.

51-E ---

51-F 5, 6, 7, and 8 * Yes. The points represented by successive dots determine segments. These segments are congruent.

51-G Locate point 9 in the line and to the right of point 8 so that the segment determined by point 8 and point 9 is congruent to the segment determined by point 0 and point 1. * Locate point 10 in the line and to the right of point 9 so that the segment determined by point 9 and point 10 is congruent to the segment determined by point 0 and point 1.

51-H Yes. Each natural number is associated with a separate point and since there are infinitely many natural numbers, they are associated with infinitely many points.

51-I Yes

51-J The three small dots represent the points that are associated with natural numbers but whose dots are not shown in the display.

51-K Point 0 through point 37

51-L Point 39

51-M No. Points to the right of point 26 are associated with natural numbers that are greater than 26. The condition $x \leq 26$ requires that the members of $\{x \mid x < 26\}$ be less than 26. * Yes. Points to the left of point 26 are associated with natural numbers less than 26.

51-N Yes

51-O Yes

51-P 2 * Point 3 is located in the line and to the right of point 2 so that the segment determined by point 2 and point 3 is congruent to the segment determined by point 0 and point 1. * Point 4 is located in the line and to the right of point 3 so that the segment determined by point 3 and point 4 is congruent to the segment determined by point 0 and point 1.

52-A The set of natural numbers from 0 through 3

52-B Each of the numbers 1, 2, and 3 is a member of the set tabulated. * Yes. Each of the members of {0, 1, 2, 3} is also a member of the set of natural numbers.

52-C All the dots in the display * Yes * The encircled dots. These dots represent point 2, point 4, and point 6.

52-D The set whose members are the natural numbers from 0 through 10

52-E The set whose members are all x such that x is less than five * {0, 1, 2, 3, 4}

52-F All the dots * The universe is finite and every point that is associated with a member of the universe is represented by a dot in the graph.

52-G The encircled dots * 2 is a member of $\{x \mid x < 5\}$. * 7 is not a member of $\{x \mid x < 5\}$. * Yes. Each dot that is a member of the graph of $\{x \mid x < 5\}$ is also a member of the graph of the universe.

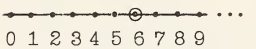
52-H The set of natural numbers * The graph of the set of natural numbers is a graph of an infinite set. * The condition that the sum of each replacement for x and 1 is less than 8

52-I The set of all x such that x plus one is less than eight * {0, 1, 2, 3, 4, 5, 6}

52-J The encircled dots * These are the dots in the graph of $\{x \mid x + 1 < 8\}$. * 7 is not a member of $\{x \mid x + 1 < 8\}$. * The graph of $\{x \mid x + 1 < 8\}$ is a subset of the graph of the universe.

53-K {6}

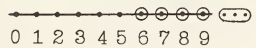
53-L See the graph below. Remember, you may choose any segment as the segment determined by point 0 and point 1.



53-M $x + 3 > 9$.

53-N 7 * There is no greatest solution.

53-O {6, 7, 8, ...} * Yes * See the graph below.

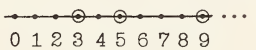


53-P {4, 5, 6, ...}

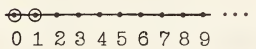
53-Q There are infinitely many conditions for which D14 shows the graph of the solution set. Some examples are: $x > 3$, $x + 1 > 4$, and $x - 1 > 2$.

On your own

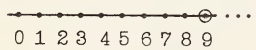
54-1



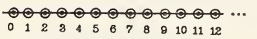
54-2



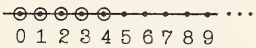
54-3



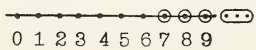
54-4



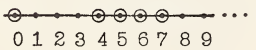
54-5



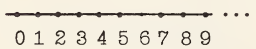
54-6



54-7



54-8



54-9 {7, 8, 9, ...}

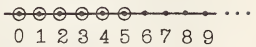
54-10 {3, 4, 5, 6, 7}

54-11 {0}

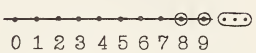
54-12 {2, 5}

54-13 {1, 3, 5, 7}

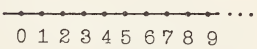
54-14 {0, 1, 2, 3, 4, 5}



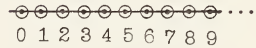
54-15 {8, 9, 10, ...}



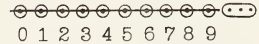
54-16 { }



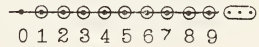
54-17 {0, 1, 2, ..., 9}



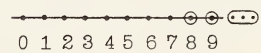
54-18 {0, 1, 2, ...}



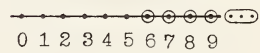
54-19 {1, 2, 3, ...}



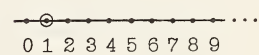
54-20



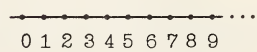
54-21



54-22



54-23



54-24 Examples are: $x > 5$, $x + 1 > 6$, and $x - 1 > 4$.

54-25 Examples are: $x < 6$, $x + 1 < 7$, and $x + 2 < 8$.

Checking up

Test 5

54-1	18,525	54-7	211,246
54-2	24,138	54-8	79,717
54-3	21,006	54-9	92,743
54-4	7850	54-10	80,012
54-5	13,582		
54-6	15,416		

Test 6

54-11	21,734	54-16	535,309
54-12	9084	54-17	46,695
54-13	28,024	54-18	6569
54-14	556,868	54-19	6331
54-15	106,435	54-20	295,769

Test 7

54-21	388,896	54-26	78,957
54-22	14,720	54-27	20,746
54-23	912	54-28	15,480
54-24	6540	54-29	21,054
54-25	23,850	54-30	33,630

Test 8

54-31	277,200	54-36	281,672
54-32	508,644	54-37	147,360
54-33	480,330	54-38	113,400
54-34	53,928	54-39	656,125
54-35	229,881	54-40	185,440

Test 9

54-41	96	54-46	112
54-42	94	54-47	318
54-43	35	54-48	159
54-44	50	54-49	240
54-45	64	54-50	509

Test 10

54-51	56	54-56	146
54-52	58	54-57	307
54-53	99	54-58	291
54-54	234	54-59	605
54-55	845	54-60	548

Lesson 14

On your own

56-1	861,867	56-5	34,268
56-2	7448	56-6	3,007,200
56-3	7515	56-7	53,756
56-4	341,796	56-8	265,740

56-9 302,162 56-11 365,310
56-10 60,990 56-12 162,240

Lesson 15

56-A Yes * Yes
56-B 300 * 200 * 210
56-C 4000 * 4570 * 4500
56-D 70 * 72,400
56-E 600 * 17,000
57-F 4 * 12 and 8 * 4
57-G Yes. The quotient in each case is 4.
57-H 27 and 9 * 8
57-I Yes. The quotient in each case is 8.
57-J 56 and 7 * 8
57-K Yes. The quotient in each case is 8.
57-L No
57-M 5 * 45 and 9 * 5
57-N Yes. The quotient in each case is 5.
57-O 8 * 2400 and 800
57-P Yes. The quotient in each case is 8.
57-Q No

On your own

57-1 400, 300, 370
57-2 900, 800, 820
57-3 500, 400, 430
57-4 9900, 9800, 9840
57-5 3200, 3100, 3180
57-6 19,400, 19,300, 19,370
57-7 800, 700, 770
57-8 4200, 4100, 4110
57-9 24,300, 24,200, 24,220
57-10 3000, 2400, 2470
57-11 7000, 6900, 6910
57-12 5000, 4500, 4530
57-13 8000, 7600, 7630
57-14 54,000, 53,200, 53,250
57-15 43,000, 42,000, 42,010
57-16 18,000, 17,800, 17,800
57-17 9000, 8900, 8900
57-18 3000, 2500, 2590
57-19 Divide by 100. The missing divisor is 8.
57-20 Multiply by 4. The missing dividend is 16.
57-21 Divide by 10. The missing dividend is 36.
57-22 Multiply by 10. The missing divisor is 70.
58-23 Divide by 8. The missing dividend is 9.
58-24 Divide by 100. The missing divisor is 8.
58-25 Multiply by 100. The missing dividend is 5400.
58-26 Divide by 10. The missing dividend is 8100.
58-27 Multiply by 10. The missing divisor is 30.
58-28 Divide by 4. The missing dividend is 8.
58-29 Multiply by 100. The missing divisor is 300.
58-30 Divide by 10. The missing dividend is 480.
58-31 Multiply by 8. The missing dividend is 63.
58-32 Divide by 100. The missing divisor is 4.

Lesson 16

On your own

60-1 97, remainder 27
60-2 137, remainder 0
60-3 307, remainder 34
60-4 446, remainder 36
60-5 201, remainder 194
60-6 408, remainder 8
60-7 273, remainder 300
60-8 508, remainder 8
60-9 263, remainder 3
60-10 137, remainder 17
60-11 5280, remainder 0
60-12 421, remainder 0
60-13 124, remainder 188
60-14 49, remainder 55

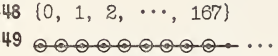
Keeping Skilful

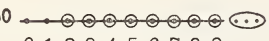
60-1 8955
60-2 6575
60-3 14,935
60-4 91,901
60-5 107,491
60-6 684,146
60-7 4858
60-8 466
60-9 2189
60-10 3696
60-11 601
60-12 10,573
60-13 2312
60-14 9499
60-15 4320
60-16 62,834
60-17 12,470
60-18 682,200
60-19 61,200
60-20 227,484
60-21 87, remainder 30
60-22 49, remainder 10
60-23 117, remainder 1
60-24 309, remainder 0
60-25 16, remainder 104
60-26 79, remainder 102
60-27 204, remainder 0
60-28 149, remainder 46
60-29 {6}
60-30 {0, 2, 4, 6, 8, 10, 12}
60-31 {8}
60-32 {0, 2, 4}
60-33 {4}
60-34 {2, 4, 6, 8, 10, 12}
60-35 { }
60-36 {6, 8, 10, 12}
60-37 {12}
60-38 {0, 2, 4, 6}
60-39 {14}
60-40 {0, 1, 2, ···, 7}
60-41 {38}
60-42 {8, 9, 10, ···}
60-43 { }
60-44 {8}
60-45 { }

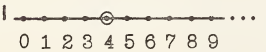
60-46 {0, 1, 2, ···}

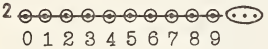
60-47 {0}

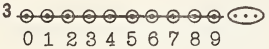
60-48 {0, 1, 2, ···, 167}

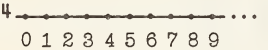
60-49 

60-50 

60-51 

60-52 

60-53 

60-54 

Lesson 17

62-A Because Sue spent some of the 15 quarters

62-B Because the number of quarters Sue had left equals 9

62-C Because 5 coins were put with, not taken from, the coins George had to begin with

62-D Because, after x is replaced by the number of coins George had at first, $x + 5$ and 39 are the same number

62-E Conditions for equality

62-F Yes

62-G No * The number of model cars that Ernie built during the summer

62-H Because the cars Ernie built during the summer were put with the 17 cars he had in May

62-I Yes

62-J Because after n is replaced by the number of model cars Ernie built during the summer, $17 + n$ and 32 are the same number

62-K No * The number of stamps Ann bought

62-L Because Ann used 14 of the postage stamps she bought

62-M Yes

62-N Because, after y is replaced by the number of postage stamps Ann bought, $y - 14$ and 18 are the same number

63-O The number of degrees the temperature was at noon

63-P Because, by noon the temperature had risen 7 degrees * Because, after replacing x by the number of degrees the temperature was at noon, $38 + 7$ and x are the same number

63-Q The number of tickets Nancy has left to sell

63-R Because, Nancy sold 14 of the tickets she had to begin with * Because after replacing n with the number of tickets Nancy had left to sell, $25 - 14$ and n are the same number

63-A 37 * 26

63-B The group of dimes and the group of nickels

63-C No * Yes

63-D The number of dimes

63-E 11

63-F Yes

63-G The number of objects in the larger group * You are asked to find the number of dimes and you know that Joe has more dimes than nickels.

63-H You cannot remove a group of nickels from a group of dimes. However, since Joe has 26 nickels, you may imagine a group of 26 dimes being removed from the larger group of dimes. * Yes

64-I A group of nickels and a group of dimes

64-J Yes * No

64-K The group of dimes * The group of nickels

64-L Yes

64-M The number of objects in the smaller group

64-N You cannot remove a group of nickels from 37 dimes so you imagine a group of dimes, equal in number to the group of nickels, being removed from the 37 dimes. * Yes * Yes

Sometimes you may obtain different conditions for problems than those expressed in this answer book. You must be able to explain how you obtained these conditions. Your teacher will decide if they are correct conditions for the problems.

You may express each condition of equality with either of two sentences. The condition $x + 3 = 43$ may be expressed by either the sentence $x + 3 = 43$ or $43 = x + 3$. This is also true for conditions that use the ideas of "not equal to" and "congruent to."

On your own

64-I x, the number of pounds Margaret weighed when she came home from camp.
$98 - 3 = x$.
Remember, you may use what-

ever letter you wish to express the variable.

64-2 x, the number of points Sam scored during the second quarter.
$6 + x = 14$.

64-3 x, the number of posters Judy needs to make.
$8 + x = 12$.

64-4 x, the number of feet higher Mt. McKinley is than Mt. King.
$20,320 - 17,130 = x$.

64-5 x, the number of votes in all that were cast in Newfoundland and Nova Scotia.
$92,858 + 394,130 = x$, or
$394,130 + 92,858 = x$.

64-6 x, the number of cents that Jerry spent.
$72 - x = 63$.

64-7 x, how many more Canadian post offices there were in 1958 than in 1868.
$12,500 - 3,638 = x$.

64-8 x, the number of lambs Mr. Martin had before he bought 40 lambs.
$x + 40 = 117$.

64-9 x, the number of doubles that Lou Gehrig hit.
$x - 18 = 34$.

64-10 x, the number of cents Helen had in her purse before she spent the 34 cents.
$x - 34 = 18$.

64-11 x, the number of pictures of opera singers Alice has.
$63 - x = 25$.

Lesson 18

66-A Because the model airplanes Allan and Pete have together is 13 + 10, not 13 - 10

66-B Because Margaret mailed some of the 27 post cards

66-C Conditions for inequality * In lesson 17 you made conditions for equality.

66-D No * For the number of boards Ken bought

66-E Yes

66-F Because Ken used some of the boards * Because Ken used 16 of the boards he had bought and x is the variable for the number of boards that he bought

66-G Because Ken had fewer than 5 boards left

66-H No * The number of snapshots Judy had in her album before she put in the 9 new ones

66-I Yes

66-J Because 9 new snapshots were put in the album * For the number of snapshots in the album after the 9 new ones were put in

66-K Because Judy had more than 50 snapshots in her album

66-L For the number of shells Ruth had

66-M Because Martha gave 18 shells to Rosemary * The number of shells Martha had after giving Rosemary 18 shells * Because after giving Rosemary 18 shells, Martha had fewer shells than Ruth

67-N The number of arrowheads George found

67-O George had 15 arrowheads and found some more. * The number of arrowheads George had after he found some more * Because after he found some arrowheads, he had more than 40

67-P The number of phonograph records Alice has

67-Q Because Sarah had 10 phonograph records and bought 5 more

67-R Because Sarah still has fewer records than Alice

67-S $10 + 5 < x$.

On your own

67-1 x, the number of dollars John had.
$7 + 3 > x$.
Remember, you may use whatever letter you wish to express the variable.

67-2 x, the number of buttons Ann sewed on her dress.
$24 - x > 12$.

67-3 x, the number of cents Virginia spent for the pencils.
$49 + x < 75$, or
$x + 49 < 75$.

67-4 x, the number of papers Don sold.
$29 + 33 < x$, or
$33 + 29 < x$.

67-5 x, the number of fish in the aquarium before Jane put in the 8 new fish.
$x + 8 > 19$.

67-6 x, the number of the population of Saskatchewan.
$x + 885,000 < 1,810,000$, or
$885,000 + x < 1,810,000$.

67-7 x, the number of feet of track Jay bought.
$17 + x > 31$.

67-8 x, the number of persons who attended the school play.
$300 - x < 12$.

67-9 x, the number of square miles that is the area

of British Columbia.
412,582 − 28,354 = x.

67-10 x, the number of dollars Miss Green had in her savings account at the beginning of the year.
$x + 300 > 850$.

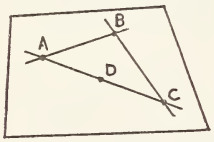
68-11 x, the number of model cars that John has.
$26 - x < 9$.
Notice that since you know that John has fewer model cars than Ralph, you use $26 - x$ in the condition instead of $x - 26$.

68-12 x, the number of dollars the students at Flint collected.
$195 + x < 425$, or
$x + 195 < 425$.

68-13 x, the number of acres Mr. Williams owned before he sold 1280 acres;
$x - 1280 > 2480$.

Keeping Skilful

68-1 See the sketch below. Your picture of a plane may have a different shape.



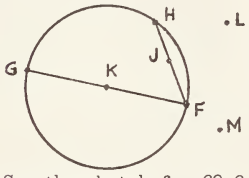
68-2 See the sketch for 68-1. Points A, B, and C may not be collinear. Otherwise they may be located anywhere in the plane.

68-3 Three lines. They are the lines determined by A and C, A and B, B and C. *
See the sketch for 68-1.

68-4 Point D may be located anywhere between A and C. See the sketch for 68-1.

68-5 Four lines. They are the lines represented in the sketch for 68-1 and the line determined by B and D.

68-6 See the sketch below. The centre of your circle may be located anywhere. Your circle may have a different radius than the one represented below.



68-7 See the sketch for 68-6. Points G and F may be located anywhere in circle K as long as K is in $\overline{FG}$.

68-8 See the sketch for 68-6. Point H may be located anywhere in circle K except at G and F. * Point J may be located anywhere between F and H.

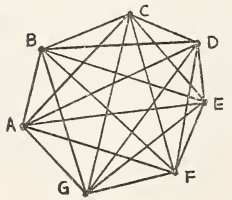
68-9 See the sketch for 68-6. Points L and M may be located anywhere in the exterior of circle K.

68-10 {F, G, H}

68-11 {J, K}

68-12 {L, M}

68-13 See the sketch below. The seven points are A, B, C, D, E, F, and G. * 21 lines
* See the sketch below for pictures of these lines.



Lesson 19

69-A {12}

69-B Yes. $15 + 12 = 27$ is a true statement.

69-C Yes

69-D The answer to both of these questions is yes.

69-E Yes. n is a variable for the number of new pennies Jane put in her collection.

69-F An example of a problem that can be solved by using the condition $15 + n = 27$ is: Bill had 15 model cars. After he made some more, he had 27 model cars. How many more model cars did he make?

69-G {9}

69-H Yes. $9 + 8 = 17$ is a true statement.

69-I Yes

70-J The answer to both of these questions is yes.

70-K An example of a problem that can be solved by using the condition $x + 8 = 17$ is: After Mrs. Morgan bought 8 more hamburger buns, she had 17 hamburger buns in all. How many hamburger buns did she have before she bought the 8?

70-L {24}

70-M Yes. When n is replaced by 24, the true statement $24 - 9 = 15$ is obtained.

70-N There were 24 elms in the park before the 9 were cut down.

70-O Phyllis had bought 24 yards of ribbon.

70-P If Phyllis had bought 24 yards of ribbon and had used 9 yards, would she then have 15 yards left?

Since the answer to this question is yes, you know that you have given the correct answer to the question asked in problem F

70–Q An example of a problem that can be solved by using the condition $n - 9 = 15$ is: Dick spent 9 cents and then had 15 cents left. How much money did Dick have before he spent the 9 cents?

71–R {19}

71–S Yes. When n is replaced by 19, the true statement $32 - 19 = 13$ is obtained.

71–T Tom had sold 19 tickets that day. Mrs. Ross had spent 19 dollars for the lamp.

71–U If Tom had 32 tickets to sell for the school play and after one day of sales he had 13 tickets left, had he sold 19 tickets that day? Since the answer to this question is yes, you know that you have given the correct answer to the question asked in problem G. If Mrs. Ross had saved 32 dollars to buy a lamp and after buying a lamp on sale, she had 13 dollars left, had she spent 19 dollars for the lamp? Since the answer to this question is yes, you know that you have given the correct answer to the question asked in problem H.

71–V An example of a problem that can be solved by using the condition $32 - n = 13$ is: Helen had 32 magazines. She gave some to her sister and then had 13 magazines left. How many magazines did Helen give to her sister?

71–A Because you know how many more baseball cards Don bought

71–B The number of baseball cards Don had after he bought 24 more

71–C You would need a group of 18 markers to represent the baseball cards Don had at first. You would need a group of 24 markers to represent how many more baseball cards Don bought.

71–D Because you should imagine that the 24 baseball cards Don bought are put with the 18 baseball cards he had at first * Addition

71–E {42}

71–F Yes. When w is replaced by 42, the true statement $18 + 24 = 42$ is obtained.

71–G 42

71–H If Tom had 18 baseball cards and he bought 24 more, would he then have 42 baseball cards? The answer to this question is yes.

71–I An example of a problem that can be solved by using $18 + 24 = w$ is: Yesterday Jane made 18 cupcakes in the morning. She made 24 cupcakes in the afternoon. How many cupcakes did she make altogether?

71–J Because Tim sold 49 of his 237 United States stamps

71–K The number of United States stamps Tim had left

71–L Because you want to find how many stamps Tim had left * Subtraction

71–M {188}

72–N Yes

72–O Tim had 188 United States stamps left. * If Tim sold 49 of his 237 United States stamps, would he have 188 United States stamps left? Yes

72–P An example of a problem that can be solved using $237 - 49 = n$ is: Mary is reading a book that has 237 pages. She has read 49 pages. How many pages does she have left to read before she finishes the book?

On your own

72–1 x, the number of degrees the temperature was at seven o'clock.
$28 - 15 = x$.
{13}. 13 degrees

72–2 x, the number of miles Mr. Brown travelled in all during the two weeks.
$798 + 1092 = x$.
{1890}. 1890 miles

72–3 x, the difference of the number of vessels that entered ports in Nova Scotia and the number of vessels that entered ports in New Brunswick.
$9942 - 6192 = x$.
{3750}.
3750 vessels

72–4 x, the number of pints of

strawberries Mrs. Reed sold.
$23 - x = 7$.
{16}. 16 pints

72-5 n, the number of pounds Elizabeth weighed a month ago.
$n - 98 = 4$.
{102}. 102 pounds

72-6 x, the number of dollars John had saved.
$x - 34 = 9$.
{43}. 43 dollars

72-7 x, the number of miles air distance from Chicago to Montreal.
$2489 - 1737 = x$, or
$1737 + x = 2489$.
{752}. 752 miles

72-8 x, the number of dollars Carolyn earned by baby-sitting.
$17 + x = 23$.
{6}. 6 dollars

72-9 x, the number of dollars Mr. Hubert had in his savings account before he withdrew the $75.
$x - 75 = 245$.
{320}. 320 dollars

72-10 x, the number of the year in which the Supreme Court of Canada was established.
$1867 + 8 = x$.
{1875}. The year 1875

72-11 x, the number of feet that is the height of Mauvoisin Dam.
$x - 592 = 188$.
{780}. 780 feet

72-12 x, the number of tickets the students had sold.
$1513 - x = 195$.
{1318}. 1318 tickets

72-13 x, the number of points the Oakwood team had scored before their game with Clark.
$x + 43 = 149$.
{106}. 106 points

72-14 x, the number of stamps Andy had before he bought the 75 new ones.
$x + 75 = 423$.
{348}. 348 stamps

72-15 x, the number of feet that is the height of the Horseshoe Falls.
$1200 - x = 1038$.
{162}. 162 feet

Lesson 20

73-A The set of counting numbers

73-B Yes * Yes

73-C ---

73-D 13 of the 30 markers are dimmed to show that you should think of 13 of 30 markers as removed. * No

73-E No * The sum of 13 and any number greater than 17 is greater than 30.

73-F Yes * Yes

73-G Yes. When a number less than 17 is used to replace x in $13 + x < 30$, a true statement is obtained.

73-H Yes * Because 0 is not a member of the universe

73-I Yes. When U = C, the solution set of $x < 17$ is the same as the solution set of $13 + x < 30$.

74-J (Column 1) Yes

74-K (Column 1) Yes

74-A The set of natural numbers

74-B Yes * Yes

74-C ---

74-D Subtraction * No.
$28 + 12 > 40$ is a false statement.

74-E No * When 12 is added to any number less than 28, the sum is less than 40.

74-F Yes. $29 + 12 > 40$ is a true statement.

74-G Yes. When 12 is added to any number greater than 28, the sum is greater than 40.

74-H Yes. Each member of {29, 30, 31, ···} satisfies $x + 12 > 40$.

74-I Yes

74-J (Column 2) The solution set of $x > 28$ is the same as the solution set of $x + 12 > 40$.

74-K (Column 2) Because the solution set is an infinite set

74-L {29, 30, 31, ···, 35}

75-M Yes

75-N The solution set of $x + 12 > 40$ is an infinite set and each member cannot be used to give a sensible answer to the problem. For example: 1,000,000 is a member of the solution set of $x + 12 > 40$ but it is not sensible to suppose that the temperature at noon was 1,000,000 degrees.

75-O 37

75-P 29 degrees

75-Q The set of natural numbers from 29 through 37 * {29, 30, 31, ···, 37}

75-R ---

75-S Because the new information for problem C is different from the new information for problem D

75-A The set of natural numbers

75-B Subtract * {17}

75-C No. $24 - 17 < 7$ is a false statement.

75-D No

75-E Yes

75-F Yes * 18

75-G If Mary made 24 popcorn balls for a party and after the party she had fewer than 7 left, were at least 18 popcorn balls eaten at the party? The answer to this question is yes, which verifies the answer to the problem.

75-H The set of natural numbers * Because n is the variable for the number of pages in the assignment and Jim had already read 9 pages. You must think of the 9 pages as being removed.

76-I Add * {24}

76-J No * $24 - 9 > 15$ is a false statement.

76-K No. If 9 is subtracted from any number less than 24, the result is less than 15.

76-L {25, 26, 27, ···} * 25

76-M Yes

76-N If, after Jim had read 9 pages of a reading assignment, he still had more than 15 pages to read, had he been assigned at least 25 pages? The answer to this question is yes, which verifies the answer to the problem.

On your own

76-1 x, the number of dollars Betty could have had in her account before she deposited the 7 dollars.
$x + 7 < 100$.
{0, 1, 2, ···, 92}.
92 dollars

76-2 x, the number of correct answers Ken could have had.
$60 - x > 4$.
{0, 1, 2, ···, 55}.
From 0 through 55 correct answers

76-3 x, the number of acres Mr. King could have plowed the second day.
$19 + x < 35$.
{1, 2, 3, ···, 15}. From 1 through 15 acres

76-4 x, the number of stamps that Terry can have.
$128 + 36 > x$.
{0, 1, 2, ···, 163}.
163 stamps

76-5 x, the number of place mats Sally could have made before she made the 8 for her mother.
$x + 8 < 20$.
{0, 1, 2, ···, 11}. From 0 through 11 place mats

76-6 x, the number of cents Phyllis could have had.
$82 - 35 > x$.
{0, 1, 2, ···, 46}. From 0 through 46 cents

76-7 x, the number of miles Mr. Owen's house can be from the Toronto airport.
$x + 962 < 1000$.
{1, 2, 3, ..., 37}. From 1 through 37 miles

76-8 x, the number of arrowheads Jack can have.
$45 - 29 > x$.
{0, 1, 2, ···, 15}.
From 0 through 15 arrowheads

76-9 x, the number of degrees the temperature could have been before it started to drop.
$x - 25 > 60$.
{86, 87, 88, ···}.
Since the highest temperature was 105 degrees, the answer is from 86 through 105 degrees.

76-10 x, the number of words Miss Jasper could have typed in the last five minutes.
$590 - x < 325$.
{266, 267, 268, ···, 590}.
266 words

76-11 x, the number of feet the greatest height of Mount Temple can be.
$19{,}850 - x = 8213$.
{0, 1, 2, ..., 11,636}.
11,636 feet

76-12 x, the number of points Ed could have scored after the first game of the season.
$36 - x > 6$.
{0, 1, 2, ···, 29}. From 0 through 29 points

76-13 x, the number of acres Mr. Hansen could have planted in oats.
$112 - x > 67$.
{1, 2, 3, ···, 44}. From 1 through 44 acres

Applying mathematics

77-1 x, total number of main-floor and balcony seats.
$823 + 518 = x$, or
$518 + 823 = x$.
{1341}.
1341 main-floor and balcony seats

77-2 x, number of papers John could have delivered.
$67 + x < 105$, or
$x + 67 < 105$.
{1, 2, 3, ···, 37}.
From 1 through 37 papers

77-3 x, number of envelopes

used to mail the annual report.
$500 - x = 245$.
{255}. 255 envelopes

77-4 x, number of hours Alice could have spent on homework the second week.
$8 - x > 3$.
{0, 1, 2, 3, 4}. From 0 through 4 hours

77-5 x, the number of dollars weekly salary Mr. Johnson received before his salary was increased.
$x + 19 = 103$.
{84}. $84

77-6 x, the number of miles the Mackenzie River is longer than the Peace River.
$2685 - 1195 = x$.
{1440}. 1440 miles

77-7 x, number of foreign stamps Phil could have had.
$201 - 84 > x$.
{0, 1, 2, ···, 116}.
116 stamps

77-8 x, the number of dollars Alex was given for his birthday.
$14 + x = 21$.
{7}. $7

77-9 x, the number of dollars Mrs. Jones had in her checking account before she wrote the check for $145.
$x - 145 = 278$.
{423}. $423

77-10 x, the number of pine trees the nursery owner sold.
$531 - x = 76$.
{455}. 455 pine trees

77-11 x, the number of pages the book can contain.
$x - 179 < 165$.
{179, 180, 181, ···, 343}.
343 pages

77-12 x, the number of dollars Mr. Butler paid the bank at the end of the year.
$1350 + 81 = x$.
{1431}. $1431

77-13 x, the number of dollars Charles spent for the flash attachment.
$51 - 44 = x$, or
$x + 44 = 51$, or
$44 + x = 51$.
{7}. $7

77-14 x, the number of miles the Richardson family could have driven the second day.
$421 - x > 295$.
{1, 2, 3, ···, 125}.
From 1 through 125 miles

77-15 x, the number of tomato plants Mr. Williams could have set out the second hour.
$46 + x < 140$.
{1, 2, 3, ···, 93}.
93 tomato plants.
Since Mr. Williams set out at least one plant the third hour, he set out less than 140 plants during the first two hours.

Lesson 21

78-A ---

78-B Yes * Yes

78-C The number of model cars Jerry gave to Frank * The number that is added to 16

78-D Because, in problem A, Frank had 16 model cars and Jerry gave him some more. In problem B, a number is added to 16.

78-E Because, after x is replaced, $16 + x$ becomes the same number as 33

78-F Subtraction

78-G {17}

78-H Yes

78-I Yes

78-J Yes * Yes

78-K Subtraction

78-L {7}

78-M No * Yes

78-N {0, 1, 2, ···, 6}

78-O Yes * Yes

78-P $85 - x = 47$.

78-Q An example is: What number can be subtracted from 85 so that the difference is 47?

On your own

79-1 $42 - 18 = x$.
{24}

79-2 $29 + x < 35$.
{0, 1, 2, 3, 4, 5}

79-3 $x + 6 = 25$.
{19}

79-4 $16(12) < x$, or
$12(16) < x$.
{193, 194, 195, ···}

79-5 $x - 37 = 14$.
{51}

79-6 $1134 \div 42 > x$.
{0, 1, 2, ···, 26}

79-7 $120 - x = 57$.
{63}

79-8 $x + 8 > 17$.
{10, 11, 12, ···}

79-9 $376 + 139 = x$.
{515}

79-10 $x + 16 = 12$.
{ }

79-11 $26 - x > 12$.
{0, 1, 2, ···, 13}

79-12 $147 - 15 < x$.
{133, 134, 135, ···}

79-13 $350(108) = x$.
{37,800}

79-14 $x + 13 > 15$.
{3, 4, 5, ···}

79-15 $95{,}676 \div 476 = x$.
{201}

79-16 $x - 6 < 9$.
{6, 7, 8, ···, 14}. The greatest number is 14.

79-17 $28 - x < 15$.
{14, 15, 16, ···, 28}.
The least number is 14.

Keeping Skilful

79-1 11,000
79-2 8943
79-3 111,503
79-4 196,169
79-5 100,000
79-6 40,767
79-7 25,601
79-8 159,751
79-9 34,270
79-10 41,867
79-11 49,799
79-12 58,385
79-13 29,900
79-14 7009
79-15 10,806
79-16 889
79-17 8261
79-18 39,005
79-19 2613
79-20 10,000
79-21 21,677
79-22 16,008
79-23 35,200
79-24 36,994
79-25 149,094
79-26 48, remainder 86
79-27 84, remainder 52
79-28 203, remainder 50
79-29 75, remainder 0
79-30 35, remainder 240
79-31 260, remainder 0
79-32 362, remainder 90
79-33 {1, 2, 3, 4, 5}
79-34 {1, 2, 3}
79-35 {1}
79-36 {4, 5}
79-37 {2}
79-38 {4, 5}
79-39 { }
79-40 { }
79-41 {5}
79-42 {1, 2, 3}
79-43 {1, 2, 3, 4, 5}
79-44 {27}
79-45 {9894}
79-46 {0, 1, 2, ···, 6}
79-47 { }
79-48 {5}
79-49 {0, 1, 2, ···, 2583}
79-50 {2}
79-51 {53, 54, 55, ···}
79-52 { }
79-53 {201, 202, 203, ···}
79-54 {0, 1, 2, ...}

Checking up

Test 11

80-1 geometry
80-2 diameter
80-3 line
80-4 collinear
80-5 tabulation
80-6 equation
80-7 universe
80-8 numeral

Test 12

80-9 False
80-10 True
80-11 False
80-12 False
80-13 False
80-14 True
80-15 False
80-16 False
80-17 True
80-18 False

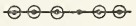
Test 13

80-19 empty set
80-20 member
80-21 $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$
80-22 subset
80-23 solution set
80-24 false
80-25 ten
80-26 true
80-27 counting

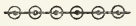
Test 14

80-28 {0, 1}

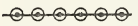
80-29 {0, 1, 2, 4, 5}



80-30 {0, 1, 2, 3, 4, 5}



80-31 {0, 1, 2, 3, 4, 5}



Test 15

81-32 (c) $1 + 4 < 5$.
81-33 (d) the counting numbers greater than 5000.
81-34 (b) >
81-35 (c) $\neq$
81-36 (c) $7 - 3 < 6$.
81-37 (c) 4
81-38 (b) congruent

Test 16

81-39 $\overline{KD}$, $\overline{KL}$, $\overline{KR}$
81-40 $\overline{DR}$, $\overline{RL}$
81-41 $\overline{DR}$
81-42 D, L, R
81-43 B, C, M
81-44 A, K, S

Test 17

81-45 x, the number of post cards Ann could have had before she bought the 15.
$x + 15 < 31$.
{0, 1, 2, ···, 15}.
From 0 through 15 post cards

81-46 x, the number of members

by which the Canadian House of Commons had increased from 1867 to 1961.

81–47 x, the number of magazines Sue could have thrown away.
$57 - x > 39$.
{1, 2, 3, ···, 17}. From 1 through 17 magazines

81–48 x, the number of pounds Kay weighed a month ago.
$101 - x = 4$.
{97}. 97 pounds

81–49 $71 - x > 13$.
{0, 1, 2, ···, 57}. The numbers from 0 through 57

81–50 $x + 482 = 620$.
{138}. The number 138

Test 18

81–51 3231

81–52 4830

81–53 5929

81–54 198,256

81–55 423,936

Test 19

81–56 880

81–57 79

81–58 409

81–59 68

81–60 608

Lesson 22

82–A ---

82–B Yes * Yes * Chile

82–C The set whose members are Peru, Chile * Yes * Yes

82–D No

82–E Brazil does not border on the Pacific. * Paraguay does not border on the Pacific.

82–F A and B were used to furnish members for the set {Peru, Chile}. Those objects that belong to both set A and set B are members of {Peru, Chile}.

82–G No * No * Yes * 6 is a member of D but not of C. 3 is a member of C but not of D. 4 is a member of both set C and set D.

83–H Because each of the members of {1, 3, 6} is not a member of both set C and set D

83–I Yes * Because {2, 4} contains all the objects that are members of both set C and set D and {2, 4} contains no other objects

83–J The intersection of the set whose members are one, two, three, four and the set whose members are two, four, six equals the set whose members are two, four. * The members of $\{1, 2, 3, 4\} \cap \{2, 4, 6\}$ are 2 and 4. The members of {2, 4} are also 2 and 4. * Yes

83–K Yes * "$\{1, 2, 3, 4\} \cap \{2, 4, 6\}$" and "$C \cap D$" are names for the same set.

83–L Yes * Yes * The objects that belong to both C and D are the same objects that belong to both D and C. Therefore, the intersection of C and D is the same set as the intersection of D and C. This also means that $C \cap D = D \cap C$ is a true statement.

83–M {6, 7, 8, ···} * {0, 1, 2, ···, 9} * Set E

83–N No * No * 5 is a member of F but not E. 10 is a member of E but not F. The intersection of E and F contains only those objects that are members of both set E and set F.

83–O {6, 7, 8, 9} * Yes. $E \cap F$ has 4 members.

84–P {7, 8, 9, ···} * {3, 4, 5, ···}

84–Q {7, 8, 9, ···} * Yes. The number of members in $K \cap L$ is not a natural number.

84–R {0, 1, 2, ···, 8} * 7 and 8

84–S Yes

84–T No * No

84–U They have no common members.

84–V Yes * Since no objects are members of both I and J, the intersection of I and J contains no members.

84–A The portion of the diagram that represents A is completely in the portion of the diagram that represents U. This indicates that each member of set A is also a member of the universe.

84–B Yes. Because each Canadian citizen who is a registered voter is, of course, a Canadian citizen.

84–C Some examples are: The set of automobiles manufactured in 1950, the set of sports cars, and the set of automobiles made in Canada.

84–D U, A, B

84–E Yes. Each member of B is also a member of U.

84–F No * Yes * Two sets that have no common members are disjoint sets.

85–G Yes * $A \cap B$ has no members.

85-H Some examples are: The set of odd natural numbers, the set of natural numbers greater than 10, the set whose only member is 5.

85-I Their intersection is the empty set.

85-J Each member of B is also a member of U.

85-K Yes * Each person that is a registered voter living in Ontario is a member of both set A and set B and each member of both set A and set B is a registered voter living in Ontario.

85-L The portion of the diagram that represents A ∩ B has both horizontal and vertical shading.

85-M The set of all registered voters living in Quebec.

85-N Some examples are: The set of natural numbers less than 10, the set of natural numbers greater than 10, the set of odd numbers less than 10.

85-O Their intersection is not the empty set.

85-P Because each registered voter living in Manitoba is a Canadian citizen who is a registered voter.

85-Q The portion of the diagram that represents set B is completely in the portion of the diagram that represents set A.

85-R Yes * The portion of the diagram that represents A ∩ B has both vertical and horizontal shading.

85-S Yes * Because each member of both set A and set B is a member of set B, and each member of set B is a member of both set A and set B. In D13 the portion of the diagram that represents A ∩ B is shaded in the same way as the portion of the diagram that represents B.

85-T Some examples are: The set of boys in the school band, the set of girls in the school band, and the set of students who play clarinets in the school band.

86-U Each set described

86-V If one set is a subset of a second set, the intersection of the two sets is the first set.

86-W The intersection of a set with itself is the set itself. * The common members of a set and itself are all the members of the set.

86-X Yes * Since each member of the intersection of two sets is also a member of each of the two sets, you know that the intersection of two sets is a subset of each of the two sets.

86-Y The empty set * A set and the empty set have no common members.

On your own

86-1 {12, 24}

86-2 {0, 2, 4, ···}

86-3 { }

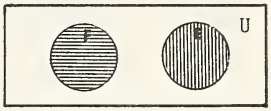
86-4 C = {0, 1, 2, ···, 7}.
D = {5, 6, 7, ···}.
C ∩ D = {5, 6, 7}.

86-5 C = {27}.
D = {0, 1, 2, ···, 34}.
C ∩ D = {27}.

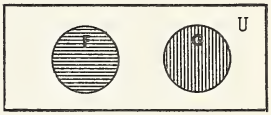
86-6 C = {0, 1, 2, ···, 6}.
D = {5, 6, 7, ···}.
C ∩ D = {5, 6}.

86-7 C = {0, 1, 2, ···, 7}.
D = {12, 13, 14, ···}.
C ∩ D = { }.

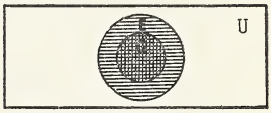
86-8



86-9



86-10



Lesson 23

86-A Yes * No

86-B Mr. Cox

86-C Yes * Yes * Miss Evans

86-D The set whose members are Mr. Adams, Miss Evans, Mr. Wilson, Mr. Cox * Yes * No

86-E Sets S and M were used to furnish the members for the set tabulated in D2. Each teacher that is a member of either S or M, or of both S and M, is a member of {Mr. Adams, Miss Evans, Mr. Wilson, Mr. Cox}.

87-F ---

87-G No * {Mrs. Robbins, Miss Schroeder, Mr. Johnson, Mr. Barnes, Miss Hansen, Mr. Adams, Miss Evans, Mr. Wilson}

87-H No * {Mrs. Robbins, Miss Schroeder, Mr. Johnson,

Mr. Barnes, Miss Hansen, Mr. Adams, Miss Evans, Mr. Cox}

87-I Yes * Yes * Yes * Yes * 8

87-J 7 does not belong to either C or D, or to both C and D. 9 does not belong to either C or D, or to both C and D.

87-K {5, 6, 8, 10}

87-L The union of the set whose members are five, ten and the set whose members are six, eight, ten equals the set whose members are five, six, eight, ten * Yes * Because {5, 10} ∪ {6, 8, 10} has exactly the same members as {5, 6, 8, 10}

87-M Yes. "{5, 10} ∪ {6, 8, 10}" and "C ∪ D" are different names for the same set.

87-N Yes * Yes * The objects that belong to either D or C, or to both D and C are the same as the objects that belong to either C or D, or to both C and D. This means that the union of D and C is the same set as the union of C and D. Therefore, C ∪ D = D ∪ C is a true statement.

88-O {0, 1, 2, ···, 5} * {11} {9}

88-P No * Because they have no common members

88-Q No * Yes * Yes * 6 does not belong to either E or F, or to both E and F. 0 belongs to E. 11 belongs to F.

88-R {0, 1, 2, 3, 4, 5, 11} * No. E ∪ F has 7 members.

88-S Yes * Yes

88-T Yes

88-U Yes

88-V {0, 1, 2, ···, 10} * {4, 5, 6, ···}

88-W Yes * No * {0, 1, 2, ···} * No

88-X Yes * Yes

88-Y Yes * 4, 5, 6, 7, 8, 9, 10

88-Z {4, 5, 6, ···, 10} * Yes * Because each member of K ∩ L is also a member of K ∪ L

88-A By the picture of the rectangle and its interior * By the pictures of the circles and their interiors

88-B Yes * Yes

88-C Yes * Because there is no city in Canada that is located both in the Prairie Provinces and in Ontario

88-D Yes * Because the union of two sets contains all those objects that are in either of the two sets, or in both sets

88-E By the pictures of both circles and their interiors. In other words, by all of the diagram that has either horizontal or vertical shading

88-F Some examples are: The set of even numbers less than 20, and the set of natural numbers greater than 20.

88-G Each union is a set that contains all the members of A and all of the members of a set that you described.

88-H Yes

88-I The members of A ∪ B are all the cities located in the Prairie Provinces and all the cities that are provincial capitals * By the portion of the diagram that has horizontal shading and the portion that has vertical shading and the portion that has both horizontal and vertical shading

88-J Some examples are: The set of even numbers greater than 6, and the set of counting numbers less than 20.

89-K Each union is a set that contains all the members of a set that you described and all the members of set A.

89-L The portion of the diagram that represents set B is completely in the portion of the diagram that represents set A. The portion of the diagram that represents set B has both horizontal and vertical shading.

89-M Yes * All cities in Alberta are located in the Prairie Provinces and so each member of B is also a member of A.

89-N A ∪ B is the set of all cities either in Alberta or in the Prairie Provinces, or both. * By the portion of the diagram that has either horizontal shading or vertical shading, or both

89-O Yes * A contains all the cities that are in either A or B, or in both A and B. Since A does not contain any other members, A is the union of A and B.

89-P Some examples are: The set of odd numbers less

than 10, and the set of odd numbers greater than 1000.

89–Q Set A

89–R Their union is the same as the second set.

89–S Yes * Yes * Yes * Each member of A is also a member of A ∪ B. Therefore A is a subset of A ∪ B. Each member of B is also a member of A ∪ B. Therefore, B is a subset of A ∪ B. Each member of A ∩ B is also a member of A ∪ B. Therefore, A ∩ B is a subset of A ∪ B.

89–T The given set * The given set contains all the members that are in the given set or the empty set, or both the given set and the empty set.

89–U The set itself * You know that the union of two sets cannot contain any members that are not in either set, or in both sets. You also know that the union of two sets must contain all the members of each set. Therefore, the union of a set with itself must be the set.

89–V Each of the sets must be the empty set.

89–W The two sets are the same set.

On your own

89–1 {bluebird, crow, hawk, owl, eagle, vulture}

89–2 {3, 5, 6, 9, 10, 12, 15}

90–3 {0, 1, 2, ···}

90–4 {1, 2, 5, 9}

90–5 {0, 1, 2, ···}

90–6 {1, 5, 7, 9, 11, 13}

90–7 C = {0, 1, 2, ···, 6}.
D = {2, 3, 4, ···}.
C ∩ D = {2, 3, 4, 5, 6}.
C ∪ D = {0, 1, 2, ···}.

90–8 C = {0, 1, 2, ···, 16}.
D = {15}.
C ∩ D = {15}.
C ∪ D = {0, 1, 2, ···, 16}.

90–9 C = {9, 10, 11, ···}.
D = {0, 1, 2, ···, 8}.
C ∩ D = { }.
C ∪ D = {0, 1, 2, ···}.

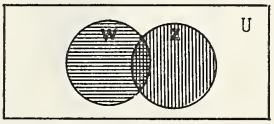
90–10 C = {14, 15, 16, ···}.
D = {14, 15, 16, ···}.
C ∩ D = {14, 15, 16, ···}.
C ∪ D = {14, 15, 16, ···}.

90–11 C = {0, 1, 2, ···, 13}.
D = {13, 14, 15, ···}.
C ∩ D = {13}.
C ∪ D = {0, 1, 2, ···}.

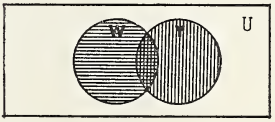
90–12 R = {0}.
S = {5, 6, 7, ···}.
T = {0, 1, 2, ···, 5}.
R ∩ S ∩ T = { }.
R ∪ S ∪ T = {0, 1, 2, ···}.

90–13

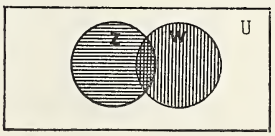
90–14



90–15

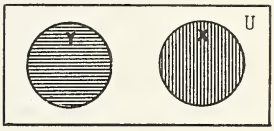


90–16



90–17

90–18



Lesson 24

90–A $x + 3 > 7$. $6 + x < 14$. *
{0, 1, 2, ···, 10} *
$\{x \mid x + 3 > 7\}$ =
{5, 6, 7, ···, 10}.
$\{x \mid 6 + x < 14\}$ =
{0, 1, 2, ···, 7}.

90–B ---

91–C The set whose members are all x such that x plus three is greater than seven and six plus x is less than fourteen

91–D 5, 6, 7

91–E Yes * Yes. {5, 6, 7} and $\{x \mid x + 3 > 7 \wedge 6 + x < 14\}$ have the same members.

91–F Yes * Each of the two sets is a solution set of a compound condition. The two compound conditions consist of the same two simple conditions. Therefore, $\{x \mid x + 3 > 7 \wedge 6 + x < 14\}$ and $\{x \mid 6 + x < 14 \wedge x + 3 > 7\}$ have the same members.

91–G ---

91–H The intersection of the solution set of x plus three is greater than seven and the solution set of six plus x is less than

fourteen is the set whose members are five, six, seven. * Yes * The intersection of $\{x \mid x + 3 > 7\}$ and $\{x \mid 6 + x < 14\}$ is the set whose members are 5, 6, 7. {5, 6, 7} is the set whose members are 5, 6, 7. The sentence expresses a true statement because both sets are the same set.

92-I Yes * They contain exactly the same members.

92-J x is less than nine and x plus two is less than eight. * $x < 9$. $x + 2 < 8$.

92-K 0, 1, 2, 3, 4, 5

92-L 0, 1, 2, 3, 4, 5 * Yes

92-M 0, 1, 2, 3, 4, 5 * Yes

92-N Sets U, A, B and sets like $A \cap B$ and $A \cup B$ that can be obtained from U, A, B * The picture of the rectangle and its interior

92-O The portion of the diagram that has both horizontal and vertical shading * The same portion of the diagram

92-P Because the intersection of the solution sets of two simple conditions contain those members, and only those members, that are in both solution sets. Since each member of a solution set of a condition satisfies the condition, the intersection of the solution sets of two simple conditions contain those members, and only those members, that satisfy both simple conditions. Now you know that the solution set of a compound condition that includes "and" also contains those members, and only those members, that satisfy both simple conditions. Therefore, the solution set of the compound condition and the intersection of the solution sets of the simple conditions are the same set.

92-A Yes

92-B Yes

92-C By seeing which dots are both encircled and enclosed by squares

92-D The dots that are encircled form the graph of $\{x \mid x < 9\}$. The dots that are enclosed by squares form the graph of $\{x \mid x + 2 < 8\}$. The dots that are both encircled and enclosed by squares form the graph of $\{x \mid x < 9 \wedge x + 2 < 8\}$.

93-E

93-F The portion of the diagram that has both horizontal and vertical shading. In this case it is the same as the portion of the diagram that represents B.

93-G In D11, the dots in the graph of $\{x \mid x < 9\}$ are encircled. The dots in the graph $\{x \mid x < 9 \wedge x + 2 < 8\}$ are also encircled. In the Venn diagram you made for exercise E, the portion of the diagram that represents $\{x \mid x < 9\}$ should have a certain type of shading. The portion of the diagram that represents $\{x \mid x < 9 \wedge x + 2 < 8\}$ should also have this type of shading as well as the shading used to represent $\{x \mid x + 2 < 8\}$. * In D11, all the dots that are enclosed by squares are also encircled. In the Venn diagram, the entire portion that represents $\{x \mid x + 2 < 8\}$ should have both kinds of shading used in the diagram.

93-H x is less than two and x plus one is greater than four. * 0, 1

93-I None

93-J None

93-K { }

93-L {0, 1, 2, ···} * Yes

93-M Solutions greater than 8

93-N {9, 10, 11, ···} * Yes

93-O The solution set of $x + 2 = 16$

93-P {14}

On your own

93-1 {9, 10, 11, 12, 13}

93-2 {10}

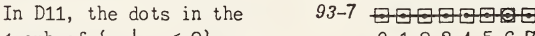
93-3 { }

93-4 {0, 1, 2, ···, 8}

93-5 {15, 16, 17, ···}

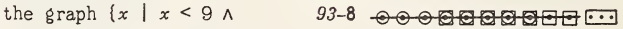
93-6 {101, 102, 103, ···}

93-7



{6}

93-8



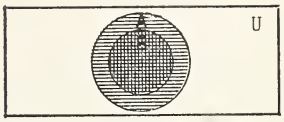
{3, 4, 5, 6, 7}

93–9

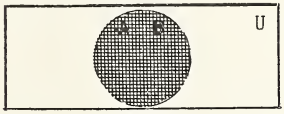
0 1 2 3 4 5 6 7 8 9

{ }

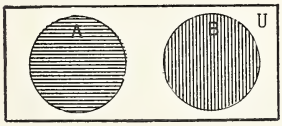
93–10



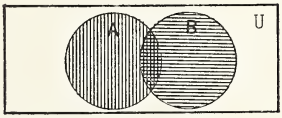
If A and B are the same set, the Venn diagram should be similar to the one below.



94–11



94–12



94–13 Venn diagram of exercise 94–12

94–14 Venn diagram of exercise 94–11

94–15 Venn diagram of exercise 93–10

94–16 Venn diagram of exercise 94–12

Keeping Skilful

94–1 18,228

94–2 15,225

94–3 76,296

94–4 238,392

94–5 249,920

94–6 72,900

94–7 131, remainder 20

94–8 64, remainder 8

94–9 208, remainder 0

94–10 379, remainder 0

94–11 47, remainder 0

94–12 22, remainder 42

94–13 345, remainder 380

94–14 307, remainder 0

94–15 $F = \{2, 3, 4, \cdots\}$.
$G = \{0, 1, 2, 3\}$.
$F \cap G = \{2, 3\}$.
$F \cup G = \{0, 1, 2, \cdots\}$.

94–16 $F = \{27\}$.
$G = \{26, 27, 28, \cdots\}$.
$F \cap G = \{27\}$.
$F \cup G = \{26, 27, 28, \cdots\}$.

94–17 $F = \{7, 8, 9, \cdots\}$.
$G = \{0, 1, 2, \cdots, 8\}$.
$F \cap G = \{7, 8\}$.
$F \cup G = \{0, 1, 2, \cdots\}$.

94–18 $F = \{0, 1, 2, \cdots, 7\}$.
$G = \{18\}$.
$F \cap G = \{\ \}$.
$F \cup G = \{0, 1, 2, 3, 4, 5, 6, 7, 18\}$.

94–19 $F = \{39\}$.
$G = \{39\}$.
$F \cap G = \{39\}$.
$F \cup G = \{39\}$.

94–20 $F = \{81, 82, 83, \ldots\}$.
$G = \{6, 7, 8, \cdots\}$.
$F \cap G = \{81, 82, 83, \cdots\}$.
$F \cup G = \{6, 7, 8, \cdots\}$.

Lesson 25

94–A Because she needed 10 hamburger buns for lunch and she did not have enough * Yes

94–B Because you again refer to the same number, the number of buns she could have had before she bought more

94–C Yes

94–D Yes

94–E Each member of the set satisfies $x + 6 > 12$.

95–F No * Because you know that each member of $\{x \mid x + 6 > 12\}$ does not satisfy the other simple condition for the problem

95–G You know Mrs. Ames had fewer than 10 hamburger buns to begin with.

95–H {7, 8, 9}

95–I The intersection

95–J Yes

95–K Yes * Yes.

95–L Are 8 buns fewer than 10 buns? Yes. If Mrs. Ames had 8 buns and bought 6 more, would she then have more than 12 buns? Yes. This verifies 8 buns as an answer. Are 9 buns fewer than 10 buns? Yes. If Mrs. Ames had 9 buns and bought 6 more, would she then have more than 12 buns? Yes. This verifies 9 buns as an answer.

95–M Yes * The number that can be George's record for consecutive push-ups

95–N Yes * x is again a variable for the number that can be George's record for consecutive push-ups.

95–O $x < 14 \wedge x - 1 > 9$.

95–P {11, 12, 13, ···}
{0, 1, 2, ···, 13}

95–Q The intersection

95–R {11, 12, 13}

95–S George's record can be 11, 12, or 13 consecutive push-ups.

95–T Are 11 consecutive push-ups fewer than 14 consecutive push-ups? If George's record is 11 consecutive push-ups and one day he missed this record by 1, did he still do more than 9 push-ups? Since the answer to each

of these questions is yes, 11 push-ups is an answer to the problem. The other two answers can be verified in the same way.

95-A Yes * Yes * The number greater than 5 that can be added to 6 so that the sum is less than 16 *
$x > 5 \wedge 6 + x < 16$.

95-B {0, 1, 2, ···, 9} * No

95-C 6, 7, 8, 9

95-D {6, 7, 8, 9}

96-E $s + 3 = 40 \wedge s - 15 = 25$.

96-F {37}

96-G No

96-H { }

On your own

96-1 x, the number of model cars Pete can have.
$x < 13 + 9 \wedge x > 13$, or
$x < 9 + 13 \wedge x > 13$.
{14, 15, 16, ···, 21}.
From 14 through 21 model cars

96-2 x, the number of years Bill's age can be now.
$x + 3 < 21 \wedge x - 5 > 9$.
{15, 16, 17}.
15, 16, or 17 years

96-3 x, the number of magazine subscriptions Henry could have sold in May.
$x < 25 \wedge 62 + x > 81$.
{20, 21, 22, 23, 24}.
From 20 through 24 subscriptions

96-4 x, the number of miles Mr. Ray could have driven last week.
$x < 1790 \wedge x + 175 > 1900$.
{1726, 1727, 1728, ···, 1789}.
From 1726 through 1789 miles

96-5 x, the number of tomato plants Mr. Hill could have left to set out.
$144 - x > 96 \wedge x > 12$.
{13, 14, 15, ···, 47}.
From 13 through 47 tomato plants

96-6 x, the number of postage stamps Mrs. Johnson could have used. $x > 27 \wedge 60 - x > 24$. {28, 29, 30, ···, 35}. From 28 through 35 stamps

96-7 x, the number of pairs of overshoes Mr. Jones could have had left.
$315 - x > 269 \wedge x > 27$.
{28, 29, 30, ···, 45}.
From 28 through 45 pairs of overshoes

96-8 $2950 + x = 4010 \wedge 3000 - x = 1940$.
{1060}

96-9 $x > 406(850) \wedge 279,879 + x < 600,000$, or
$x > 850(406) \wedge 279,879 + x < 600,000$.
{ }

96-10 $x > 5010 - 3465 \wedge 4276 + x < 5826$.
{1546, 1547, 1548, 1549}

96-11 $x > 68,068 \div 187 \wedge 576 + x < 1000$.
{365, 366, 367, ···, 423}

96-12 $x + 1496 = 2130 \wedge 373 + x = 1000$.
{ }

96-13 $x - 9568 = 3627 \wedge 1708 + x = 14,903$.
{13,195}

Applying Mathematics

97-1 x, the number that is the difference in the areas of Lake Huron and Lake Michigan.
$23,010 - 22,4000 = x$.
{610}. 610 square miles

97-2 x, the number of points Dan could have scored during the last three quarters of the game.
$5 + x < 24$.
{0, 1, 2, ···, 18}.
From 0 through 18 points

97-3 x, the number of votes cast by the eighth and ninth grades together.
$627 - 276 = x$, or
$276 + x = 627$, or
$x + 276 = 627$.
{351}.
351 votes

97-4 x, the number of coins Joe could have had at first.
$98 - 19 > x$.
{0, 1, 2, ···, 78}.
From 0 through 78 coins

97-5 x, the number of years Jim's age can be.
$x - 13 < 6$, or
$x > 13 \wedge x - 13 < 6$.
{14, 15, 16, 17, 18}.
From 14 through 18 years old

97-6 x, the number of telephones there could have been in Alberta that year.
$166,629 + 122,205 < x$,
or $122,205 + 166,629 < x$.
{288835, 288836, 288837, ...}.
288,835 telephones

97-7 x, the number of years that Janet's age can be now.
$x + 7 > 18 \wedge x - 6 < 9$.
{12, 13, 14}.
12, 13, or 14 years old

97-8 x, the number of European coins Dave has.
$x - 9 = 17$.
{26}.
26 European coins

97-9 x, the number of pounds the Discoverer I satellite weighed.
$1700 - x = 1455$.
{245}.
245 pounds

97-10 x, the number of history books Miss Jones could have had before she obtained the 9 books.
$x < 35 \wedge x + 9 > 37$.
{29, 30, 31, 32, 33, 34}.
From 29 through 34 history books

97-11 x, the number that could be the number of persons of voting age who did not vote in the federal election. $5{,}701{,}000 + x < 8{,}402{,}000$ or $x + 5{,}701{,}000 < 8{,}402{,}000$.
{0, 1, 2, ..., 2, 700, 999}.
2,700,999 persons of voting age

97-12 x, the number of feet the length of the aircraft carrier can be.
$x - 1031 < 71$, or
$x > 1031 \wedge x - 1031 < 71$.
{1032, 1033, 1034, ···, 1101}.
1101 feet

97-13 $x < 48 \div 4 \wedge 13 - x > 3$.
{0, 1, 2, ···, 9}

97-14 $x < 243 - 195 \wedge 64 + x > 112$.
{ }

97-15 $x < 137 + 48 \wedge 211 - x > 19$, or
$x < 48 + 137 \wedge 211 - x > 19$.
{0, 1, 2, ···, 184}

97-16 $x > 42(19) \wedge 124 + x > 912$, or $x > 19(42) \wedge 124 + x > 912$.
{799, 800, 801, ···}

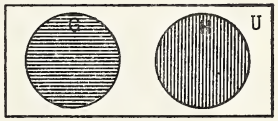
Checking up

Test 20

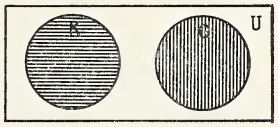
98-1 {2, 4}
98-2 {0, 1, 2, 3, 4, 5, 6}
98-3 { }
98-4 {0, 1, 2, 3, 4, 5, 6}
98-5 { }
98-6 {3}
98-7 {1, 3, 5}
98-8 {1, 3, 5}
98-9 {1, 3, 5}

Test 21

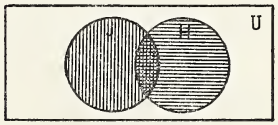
98-10



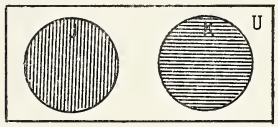
98-11



98-12



98-13



98-14

98-15

Test 22

98-16 {0, 1, 2, 3, 4, 5}
98-17 {4, 5, 6, ···}
98-18 {4, 5}
98-19 ... 0 1 2 3 4 5 6 7 8 9
98-20 ... 0 1 2 3 4 5 6 7 8 9
98-21 ... 0 1 2 3 4 5 6 7 8 9
98-22 {0, 1, 2}
98-23 From the graphs for 98-21

Lesson 26

99-A Plane WXY and plane RST. Remember that it does not matter in which order the letters are used to name a plane. Plane WXY, plane XWY, plane XYW, plane YWX, plane WYX, and plane YXW are the same plane. * Yes

99-B Yes * Sets that have no common members are disjoint sets.

99-C Yes * Because two parallel planes have no points in common and sets that have no common members are disjoint

99-D Photograph F

99-E The intersection is the empty set.

99-F The intersection is the empty set.

99-G Plane ABD and plane ABC. Remember that you can use the names of any three points in a plane to name the plane. * Yes * Yes

100-H If a line contains two points of a plane, the line is included in the plane. Line AB contains points A and B of plane ABC, so line AB is included in plane ABC. Line AB also contains points A and B of plane ABD, so

line AB is included in plane ABD.

100-I Yes. Each point in line AB is a common member of plane ABC and plane ABD. * Yes. Geometric figures that have common members are said to intersect.

100-J Photograph C

100-K Line AB

100-L One. There is exactly one plane that contains three noncollinear points. * If E was a common member of both plane ABD and plane ABC then two planes would contain the three noncollinear points A, B, and E.

100-M Because they have common members * Because the intersection of two planes that meet is a line and therefore the three points must be members of the same line

100-N The intersection is line GH.

100-A The empty set

100-B {M}

101-C (Column 1, top) If a line contains two points of a plane, the line is included in the plane. * ℓ_3

101-D (Column 1) Photograph E

101-E (Column 1) Photograph H

101-F (Column 1) Photograph B

101-A ℓ_1, ℓ_2, ℓ_3 * An infinite set of points * Yes

101-B Plane EFG * Yes. Plane EFG includes ℓ_1 and ℓ_2.

101-C (Column 1, bottom) Yes * The empty set

101-D (Column 2) Photograph D

101-E (Column 2) No * No * Parallel lines are included in the same plane. Since ℓ_1 and ℓ_3 are not included in the same plane, they are not parallel.

101-F (Column 2) Photograph A

101-G No * You do not know whether or not $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are included in the same plane and, therefore, you do not know whether or not they are parallel.

101-H The empty set * The empty set

102-I ℓ_4 and ℓ_5 are skew lines and their intersection is the empty set.

102-J ℓ_6 and ℓ_7 are parallel.

102-K No * No

102-L Point S * No * Yes

102-M It is the set containing just one point, S. * Yes

102-N Photograph G

102-O A set containing just one point

102-P One

On your own

102-1 Line AC

102-2 Line AD

102-3 Plane ABC and plane ABD

102-4 {A}

102-5 Three

102-6 Line EF and line AB, as well as line DE and line AB, are pairs of skew lines.

102-7 Line DE and line AF could be a pair of parallel lines.

102-8 {A}

102-9 Line AB

103-10 {R, T}

103-11 {J}

103-12 {}

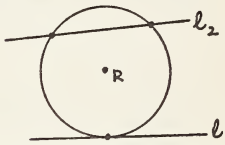
103-13 The set of all points between points R and T

103-14 {}

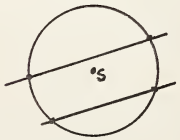
103-15 {}

103-16 ℓ_3

103-17 See the drawing below. * ℓ_1 intersects circle R in one point. To sketch a line that intersects the circle in just one point, you may locate that point anywhere in the circle. * ℓ_2 intersects circle R in two points. To sketch a line that intersects the circle in two points, you may locate those points anywhere in the circle.



103-18 See the drawing below. * You may make a sketch of any line that intersects the circle in two points. Then you may make a sketch of any other line that intersects the circle in two points and is parallel to the first line.

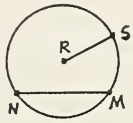


103-19 Yes * {}

103-20 No * {Y}

103-21 One * No

103-22 See the drawing below. * You may locate point S anywhere in circle R. * {S}



103-23 See the drawing above. You may locate M and N anywhere in circle R. * {M, N}

103-24 {T}

Keeping Skilful

103-1 {1, 3}
103-2 {5, 7, 9, 11}
103-3 {1, 3, 5, 7, 9, 11}
103-4 {7}
103-5 {1, 3, 5, 7}
103-6 {11}
103-7 {9}
103-8 {3}
103-9 {3, 5, 7, 9, 11}
103-10 {7}
103-11 {1, 3, 5, 7, 9}
103-12 {1, 3, 5, 7}
103-13 {1, 3, 5, 7, 9, 11}
103-14 {1, 3, 5, 7, 9}
103-15 {1, 3, 5, 7, 9, 11}
103-16 { }
103-17 {1827}
103-18 {0, 1, 2, ..., 15}
103-19 {0, 1, 2, ..., 34991}
103-20 {0}
103-21 {0, 1, 2, ..., 8}
103-22 {13093}
103-23 {58, 59, 60, ...}
103-24 {9}
103-25 {0, 1, 2, ...}
103-26 {0}
103-27 {12, 13, 14}
103-28 {28, 29, 30, ..., 37}
103-29 {0, 1, 2, ..., 16}
103-30 {39, 40, 41, ..., 123}
103-31 {12, 13, 14}
103-32 {1135, 1136, 1137, ..., 1142}
103-33 {319}
103-34 { }
103-35 {19}
103-36 {93, 94, 95, ...}

Lesson 27

104-A Yes * Yes * A is not between T and U. A is not between R and S.
104-B Yes * Yes
104-C The set of points that contains A and all the points in the line to the left of A
104-D C, E, G, I
104-E A * B
104-F The symbol reminds you that a ray extends endlessly in one direction.
104-G Ray CD, ray EF, ray GH, ray IJ * $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{GH}$, $\overrightarrow{IJ}$
105-H No * Each ray extends endlessly in one direction.
105-I The set of points consisting of Q and all points in ℓ_1 to the left of Q
105-J P * Q
105-K No * They do not contain the same points. They have different endpoints and extend in opposite directions.
105-L Segment PQ
105-M Yes * Each ray consists of P and all points in ℓ_1 to the right of P.
105-N Ray PS
105-A $\overrightarrow{EF}$ and $\overrightarrow{ED}$ * $\overrightarrow{HI}$ and $\overrightarrow{HG}$
105-B The set that contains all the points in $\overrightarrow{EF}$ together with all the points in $\overrightarrow{ED}$. The set that contains all the points in $\overrightarrow{HI}$ together with all the points in $\overrightarrow{HG}$.
105-C For the angle represented at the left, the sides are $\overrightarrow{BC}$ and $\overrightarrow{BA}$ and the vertex is B. For the angle represented in the middle, the sides are $\overrightarrow{EF}$ and $\overrightarrow{ED}$ and the vertex is E. For the angle represented at the right, the sides are $\overrightarrow{HI}$ and $\overrightarrow{HG}$ and the vertex is H.
105-D Yes * Each point contained in a side of an angle is also contained in the angle.
105-E Yes. The vertex of an angle together with two other points, a point in each side of the angle are noncollinear points. Any three noncollinear points are contained in the same plane. Since this plane contains two points of each side, both sides are included in the plane. * Yes
105-F No * No * Since the sides of an angle are rays, and a ray extends endlessly in one direction, you cannot say that one side of an angle is longer than the other side or that one angle has longer sides than another angle.
105-G The symbol reminds you that an angle is the union of two rays that have a common endpoint but that do not form a line.
106-H Yes

106–I ∠DEF and ∠FED. ∠GHI and ∠IHG

106–J No * They have different vertices and different sides.

106–A $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AB}$

106–B A * $\overrightarrow{AD}$

106–C Yes

106–D ∠GFH and ∠HFE. ∠KJL and ∠IJL

107–E ∠HFG * ∠KJL * ∠EFH * ∠IJL

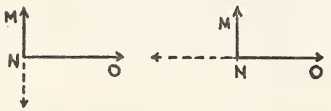
107–F You know that if $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. In the same way, if ∠ABC ≅ ∠DEF, then ∠DEF ≅ ∠ABC.

107–G You know that if $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. In the same way if ∠GHI ≅ ∠JKL and ∠JKL ≅ ∠MNO, then ∠GHI ≅ ∠MNO.

107–H Yes. ∠ABD and ∠DBC have a common vertex and a common side and the sides that are not common form a line. * ∠DBC * ∠ABD

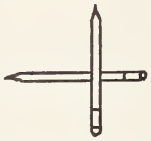
107–I ∠TSR * Yes * Yes * Each angle of a linear pair is a supplement of the other angle of the pair. A right angle is congruent to its supplement and an angle that is congruent to its supplement is a right angle.

107–J The drawings below illustrate how the picture of either one of the rays may be extended to make a sketch of the supplement of ∠MNO. * Yes. A right angle is congruent to its supplement.



107–K $\overrightarrow{RC}$, $\overrightarrow{RF}$, $\overrightarrow{RD}$, $\overrightarrow{RE}$ * ∠CRF, ∠FRD, ∠DRE, ∠ERC

108–L Hold two pencils so that they touch each other as shown below.



108–M There are many pairs of objects in your classroom that suggest perpendicular lines. Examples are the hands of a clock at three o'clock and a window frame.

On your own

108–1 True statement

108–2 False statement

108–3 True statement

108–4 False statement

108–5 False statement

108–6 True statement

108–7 True statement

108–8 Yes * Yes

108–9 The sides of ∠AOC are $\overleftrightarrow{OC}$ and $\overrightarrow{OA}$. The vertex is O.

108–10 The sides of ∠COB are $\overleftrightarrow{OC}$ and $\overleftrightarrow{OB}$. The vertex is O.

108–11 Yes * Yes

108–12 You must know whether or not the sides that are not common form a line.

108–13 Q * $\overleftrightarrow{QL}$

108–14 Yes * Yes

108–15 ∠LQN and ∠NQK, ∠NQK and ∠KQM, ∠KQM and ∠MQL * The common side of ∠NQK and ∠KQM is $\overleftrightarrow{QK}$. The common side of ∠KQM and ∠MQL is $\overleftrightarrow{QM}$.

108–16 Yes * ∠QVW and ∠QVU

108–17 ∠RVU

108–18 ∠QVU

108–19 ∠QVW and ∠QVU

108–20 Yes * Yes * Yes

108–21 ∠IRH and ∠HRJ

Lesson 28

109–A Three. Each pair of points determines one segment.

109–B Yes * The union of three sets contains all the objects that belong to the three sets.

109–C Yes

109–D Side AB, side BC, side AC * Vertex A, vertex B, vertex C

109–E No * There is exactly one plane that contains three noncollinear points. Therefore, all three vertices of a triangle are in only one plane. Since a line that contains two points of a plane is included in that plane, the sides of a triangle are included in plane determined by the vertices.

109–F Yes * Yes * Only one triangle is determined by three noncollinear points and it does not matter in which order you use the names of these points to name the triangle.

109–G No * ∠ABC contains points that are not members of ΔABC.

110–A $\overline{GH} > \overline{GI}$ is a true statement. * None

110–B Yes * ΔMNO

110–C ΔMNO

110–D ΔGHI

110–E ΔJKL and ΔMNO

110–F ΔMNO

110–G Yes * A triangle that has three congruent sides must have two congruent sides.

110-H Yes * Each equilateral triangle is also an isosceles triangle.

110-I No * Those isosceles triangles with only two congruent sides are not members of the set of all equilateral triangles.

110-J See the drawings below.

Equilateral triangle | Isosceles triangle | Scalene triangle

110-K – – –

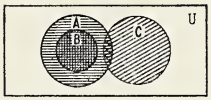
110-L Yes. A triangle with one right angle is a right triangle.

110-M No. WXYZ is not a triangle.

110-N Any figure that has more than one right angle is not a triangle.

111-O A right triangle may be an isosceles triangle. * A right triangle can not be an equilateral triangle.

111-P See the diagram below. * $A \cap B$ is the set of all equilateral triangles.



111-Q See the diagram above. * $B \cap C$ is the empty set.

111-R C ∩ D is the set of all scalene triangles having one right angle.

111-A – – –

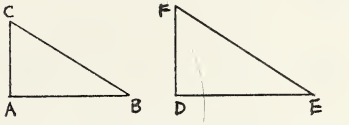
111-B Yes * Yes

111-C Yes * Yes

111-D If $\Delta IJK \cong \Delta LMN$ is a true statement, then $\Delta LMN \cong \Delta IJK$ is a true statement.

111-E If one triangle is congruent to a second triangle, and the second triangle is congruent to a third triangle, then the first triangle is congruent to the third triangle. Therefore, $\Delta IJK \cong \Delta OPQ$ is a true statement.

111-F The drawing below may be used to illustrate that the three sides of one triangle need not be congruent to the three sides of the other.



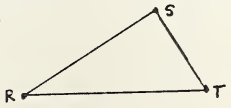
$\angle ABC \cong \angle DEF$. $\angle ACB \cong \angle DFE$. $\angle CAB \cong \angle FDE$.

111-G The three angles of one triangle are congruent to the three angles of the other triangle.

On your own

112-1 3 * A triangle

112-2 See the sketch below. You may locate R, S, and T anywhere in the plane suggested by your paper as long as they are non-collinear. * Sides are $\overline{RS}$, $\overline{ST}$, and $\overline{RT}$. * Vertices are R, S, and T.



112-3 $\overline{RS}$, $\overline{ST}$, $\overline{RT}$

112-4 Yes

112-5 Yes

112-6 No

112-7 Yes * No * A triangle with two congruent sides is an isosceles triangle. Since only two sides are congruent, the triangle is not an equilateral triangle.

112-8 $\angle EFD \cong \angle FDE$ is a true statement.

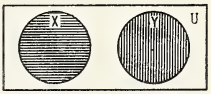
112-9 Yes * Yes * A triangle with three congruent sides is both an equilateral triangle and an isosceles triangle.

112-10 $\angle HIG \cong \angle IGH$ is a true statement. * $\angle IGH \cong \angle GHI$ is a true statement. * $\angle HIG \cong \angle GHI$ is a true statement.

112-11 Yes * A triangle with no two sides congruent is a scalene triangle.

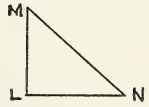
112-12 $\angle KLJ \cong \angle LJK$ is a false statement. * $\angle LJK \cong \angle JKL$ is a false statement. * $\angle KLJ \cong \angle JKL$ is a false statement.

112-13 See the diagram below. * { }



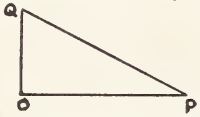
112-14 No * It is impossible for an equilateral triangle to have a right angle.

112-15 See the drawing below.



112-16 Yes * Isosceles

112-17 See the drawing below.



112-18 Yes * A right triangle

112-19 See the drawing below. * No

112-20 See the drawing above. * No

112-21 No

112-22 $\triangle RST \cong \triangle UVW$.

112-23 Yes

112-24 $\triangle RST \cong \triangle XYZ$ is a true statement.

112-25 $\overline{BD} \cong \overline{EG}$, $\angle CDB \cong \angle FGE$, and $\angle DBC \cong \angle GEF$ are true statements.

112-26 $\overline{EG} \cong \overline{HJ}$, $\overline{FE} \cong \overline{IH}$, and $\angle GEF \cong \angle JHI$ are true statements.

Lesson 29

113-A None * Because there is no point in ℓ_1 between A and B

113-B One * Because there is a point in ℓ_1 between B and C

113-C Interior of circle R. Exterior of circle R.

113-D Yes * R

113-E A line * The circle * The circle

113-F The empty set

113-G The entire plane that includes the circle

113-H $\overleftrightarrow{KL}$

113-I Yes * Yes * Each point in ray KJ is also in line KJ. Each point in ray KL is also in line KL.

113-J Yes * By pictures of horizontal lines

113-K By the portion of the drawing that has pictures of both horizontal and vertical lines

114-L M * R and N * J, K, L

114-M The intersection contains one point.

114-N Triangle ADF * 3 * 3

114-O Angle DAF * Angle ADF * Angle AFD

114-P Point V * Point V

114-Q Point V * A, D, F

114-R The intersection contains one point.

115-S The empty set

115-T The entire plane that includes the triangle

115-A Yes * A geometric figure all of whose points are coplanar is called a plane figure.

115-B No * No * Yes * Yes * To make a sketch of a segment, you do not start and finish at the same place. The same is true when you make a sketch of an angle. To make a sketch of either a triangle or a circle, you need not lift your pencil from the paper, you will not cross a pencil mark that you have already made, and you will start and finish at the same place.

115-C Each figure represented in G, H, J, and L intersects itself. To make a sketch of the figure represented in either I or K, you do not start and finish at the same place.

115-D Two. The simple closed curve is the boundary of each of these regions.

115-E It is not a simple closed curve, not an angle, and not a line, because each of these figures separates the plane into two regions.

115-F The intersection contains at least one point.

115-G The empty set * The empty set

115-H The entire plane that includes the simple closed curve

116-I Yes

116-J Two

116-K Yes * A triangle is a simple closed curve that is the union of segments and therefore is a polygon.

116-L The vertices are A, B, C, D, E. The sides are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$. * The vertices are F, G, H, I, J, K. * The sides are $\overline{FG}$, $\overline{GH}$, $\overline{HI}$, $\overline{IJ}$, $\overline{JK}$, $\overline{KF}$.

116-M T * A, B, C, D, E, F, G, H, I, J, K

116-N $\overline{LM}$, $\overline{MN}$, $\overline{NR}$, $\overline{RL}$

On your own

116-1 F and H

116-2 A, B, C, D

116-3 F

116-4 J

116-5 {C, D}

116-6 A, B, C, D, E, K

116-7 F

116-8 A, B, G, H, K, J

117-9 H

117-10 C, D, E, F

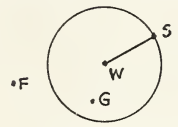
117-11 Six * $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FA}$

117-12 $\angle ABC$, $\angle BCD$, $\angle CDE$, $\angle DEF$, $\angle EFA$, $\angle FAB$

117-13 {M, N}
117-14 T
117-15 None
117-16 A, B, C, D, E, F, M, N
117-17 S, T, C, D, R
117-18 A, B, E, F, S, R
117-19 None * Yes
117-20 One

Keeping Skilful

117-1 See the drawing below. You may locate W anywhere in the plane.



117-2 See the drawing above. You may locate S anywhere in the circle.

117-3 See the drawing for 117-1. You may locate G anywhere in the interior of circle W. You may locate F anywhere in the exterior of circle W.

117-4 $\{X \mid \overline{WX} \cong \overline{WS}\}$. U = plane SWG.

117-5 $\{X \mid \overline{WX} < \overline{WS}\}$. U = plane SWG.

117-6 $\{X \mid \overline{WX} > \overline{WS}\}$. U = plane SWG.

117-7 The intersection contains the one point in circle W that is between F and G.

117-8 $\overline{AB} < \overline{AC}$.

117-9 $\overline{AB} > \overline{AC}$.

117-10 $\overline{AB} \cong \overline{AC}$. This is also a special case where one segment has two names. Because $\overline{AB}$ and $\overline{AC}$ name the same segment, $\overline{AB} = \overline{AC}$ is a true statement.

117-11 Six

Checking up

Test 23

118-1 vertex
118-2 isosceles
118-3 scalene
118-4 line
118-5 triangle
118-6 parallel
118-7 ray

Test 24

118-8 meet, or intersect
118-9 compound condition
118-10 Venn
118-11 set A
119-12 skew
119-13 ABCDE
119-14 no, or zero
119-15 subset
119-16 two

Test 25

119-17 (d) $\cap$
119-18 (b) $\wedge$
119-19 (c) are parallel
119-20 (c) $\overleftrightarrow{KJ} \cup \overleftrightarrow{KL}$
119-21 (a) the supplement of
119-22 (b) segments
119-23 (c) 1

Test 26

119-24 ray AE and ray AC
119-25 ray AC
119-26 Angle GAE
119-27 Angle BAE
119-28 Angle BAG, angle GAC, angle CAF, angle FAB
119-29 {R}
119-30 {D, K}
119-31 { }
119-32 V and L

Test 27

120-33 {8, 9, 10, ..., 14}
120-34 {13, 14, 15, 16}
120-35 {0, 1, 2, ..., 10}
120-36 {7, 8, 9, ..., 24}
120-37 {8, 9, 10, ...}
120-38 { }
120-39 {0}
120-40 { }

Test 28

120-41 x, number of points Jerry scored.
$x > 18 \wedge x < 18 + 9$.
{19, 20, 21, ..., 26}.
From 19 through 26 points

120-42 x, number of old pennies Pat had before he sold 15 to Harold.
$x < 75 \wedge x - 15 > 47$.
{63, 64, 65, ..., 74}.
From 63 through 74 pennies

120-43 x, number of books Ray read during the summer.
$x < 19 \wedge x + 9 > 26$.
{18}.
18 books

120-44 $x < 78 \wedge x > 19 + 31$.
{51, 52, 53, ..., 77}

120-45 $x < 9 \wedge 15 + x > 20$.
{6, 7, 8}

120-46 $x < 26 \wedge 347 - x > 315$.
{0, 1, 2, ..., 25}

120-47 $x > 13 \wedge 98 + x < 111$.
{ }

Checking up

Test 29

120-1 true
120-2 infinite
120-3 circle or sphere
120-4 line
120-5 subset
120-6 plane
120-7 collinear
120-8 endpoints
120-9 universe

Test 30

120-10 {1, 2}
120-11 {1, 3, 4, 5, 6, 7, 8, 9, 10}
120-12 {10}
120-13 {7, 8, 9, 10}
120-14 {2, 3}
120-15 {5, 6, 7, 8, 9, 10}

120-16 {1, 2, 3, ..., 9}

Test 31

120-17 (c) an equation

120-18 (b) a condition for inequality

121-19 (b) variable

121-20 (a) a finite set

121-21 (a) point

Test 32

121-22 x, number of golf balls Phil had before he found the 7.
$x + 7 = 23$.
{16}. 16 golf balls

121-23 x, number of autographs that Betty has.
$x - 57 = 24$.
{81}. 81 autographs

121-24 x, number of picture postcards Joyce had before she mailed the 17.
$x > 21 \wedge x - 17 < 9$.
{22, 23, 24, 25}.
From 22 through 25 postcards

121-25 $43 + x > 51$.
{9, 10, 11, ...}

121-26 $x > 5 \wedge 24 - x > 13$.
{6, 7, 8, 9, 10}

121-27 $x < 300 - 172 \wedge 280 + x > 355$.
{76, 77, 78, ..., 127}

Test 33

121-28 190 *121*-34 15

121-29 1368 *121*-35 109

121-30 10,664 *121*-36 85

121-31 288,687 *121*-37 820

121-32 80,730

121-33 47

Lesson 30

122-A {Paris, Ottawa, Madrid}

122-B {Norway, France, Spain, Canada}

122-C No * The open sentence in D2 expresses a condition, not a statement. A condition is neither true nor false.

122-D Two * One

122-E Because Paris is a member of set A and the members of set A are to be used as replacements for m * Because Spain is not a member of set A

122-F Conditions do not express either true or false ideas.

122-G Because Spain is a member of set B and the members of set B are to be used as replacements for n * Because Paris is not a member of set B

122-H No * The sentence in D4 expresses a condition, not a statement. A condition is neither true nor false.

122-I Yes

122-J Yes * No

123-K Set A

123-L Set B

123-M – – –

123-N Yes * Yes

123-O A – (Paris, Norway).
B – (Paris, France).
C – (Paris, Spain).
D – (Paris, Canada).
E – (Ottawa, Norway).
F – (Ottawa, France).
G – (Ottawa, Spain).
H – (Ottawa, Canada).
I – (Madrid, Norway).
J – (Madrid, France).
K – (Madrid, Spain).
L – (Madrid, Canada).
* The name of the replacement for m

123-P B, H, K

123-Q (Paris, France), (Ottawa, Canada), (Madrid, Spain)

123-R The symbol (Paris, Norway) is read "The ordered pair whose first component is Paris and whose second component is Norway." The symbol {Paris, Norway} is read "The set whose members are Paris, Norway." * (Paris, Norway) is an ordered pair having two components while {Paris, Norway} is a set having two members.

123-A x and y

123-B – – –

124-C Two

124-D Yes

124-E – – –

124-F For the statement $2 < 4$, (2, 4) was used to make replacements. Similar replacements were made for each of the other statements expresses in D9.

124-G $1 < 2$, $1 < 4$, $1 < 6$, $2 < 4$, $2 < 6$, $3 < 4$, $3 < 6$, $4 < 6$. * (1, 2), (1, 4), (1, 6), (2, 4), (2, 6), (3, 4), (3, 6), (4, 6)

124-H $x + x = 4$ has one variable. $x + y = 4$ has two variables. * $x + x = 4$ requires you to find a number which added to itself gives a sum of 4. $x + y = 4$ requires you to find any two numbers whose sum is 4.

On your own

124-1 (Robert Louis Stevenson, *Treasure Island*)

124-2 (Mary, guitar)

124-3 (15°, 8 P.M.)

124-4 (Larry, 141)

124-5 (1, 13)

124-6 $6 + 2 = 5$. $6 + 2 = 10$. $6 + 2 = 15$. $6 + 4 = 5$. $6 + 4 = 10$. (T) $6 + 4 = 15$. $6 + 6 = 5$. $6 + 6 = 10$. $6 + 6 = 15$.

124-7 $2 + 5 < 5$. $2 + 5 < 10$. (T) $2 + 5 < 15$. (T) $4 + 5 < 5$. $4 + 5 < 10$. (T) $4 + 5 < 15$. (T) $6 + 5 < 5$. $6 + 5 < 10$. $6 + 5 < 15$. (T)

124-8 $2 + 4 = 5$. $2 + 4 = 10$. $2 + 4 = 15$. $4 + 4 = 5$. $4 + 4 = 10$. $4 + 4 = 15$. $6 + 4 = 5$. $6 + 4 = 10$. (T) $6 + 4 = 15$.

124-9 $5 > 2 + 1$. (T) $10 > 2 + 1$. (T) $15 > 2 + 1$. (T) $5 > 4 + 1$. $10 > 4 + 1$. (T) $15 > 4 + 1$. (T) $5 > 6 + 1$. $10 > 6 + 1$. (T) $15 > 6 + 1$. (T)

124-10 (1, 3), (4, 12), (3, 9), (2, 6), (8, 24)

124-11 (8, 5), (21, 18), (12, 9), (32, 29), (4, 1)

Applying mathematics

125-1 x, number of dollars Miss Larson withdrew from her savings account.
$1405 - x = 890$.
{515}. \$515

125-2 x, number of points Leo scored.
$x + 17 < 32$, or $17 + x < 32$.
{1, 2, 3, ..., 14}.
From 1 through 14 points

125-3 x, number of new dinner plates the restaurant owner bought.
$x < 150 \wedge 315 + x > 434$.
{120, 121, 122, ..., 149}.
From 120 through 149

125-4 x, number of handbills Charles had to deliver.
$x - 117 < 43$.
{117, 118, 119, ..., 159}.
159 handbills

125-5 x, number of buttons Susan had collected before her aunt gave her 18.
$x + 18 = 205$.
{187}.
187 buttons

125-6 x, number of miles Mr. Foster drove the second week.
$547 + x > 913$.
{367, 368, 369, ...}.
367 miles

125-7 x, number of private pilots in Canada.
$x - 2548 = 6495$.
{9043}.
9043 private pilots

125-8 x, number of suits the merchant had before he sold the 89.
$x > 210 \wedge x - 89 < 131$.
{211, 212, 213, ..., 219}.
From 211 through 219 suits

125-9 x, number of cents Walter spent.
$92 - x > 34$.
{1, 2, 3, ..., 57}.
From 1 through 57 cents

125-10 x, number of years that is Mr. Roberts' age now.
$x - 9 < 39 \wedge x + 15 > 56$.
{42, 43, 44, ..., 47}.
From 42 through 47 years old

125-11 x, number of automobiles registered in 1923.
$1,193,847 - x = 680,772$.
{513075}.
513,075 automobiles

125-12 x, number of degrees that was the temperature at 10 A.M.
$71 - x < 16 \wedge x < 60$.
{56, 57, 58, 59}.
From 56 through 59 degrees

125-13 $62 - x > 53$.
{0, 1, 2, 3, ..., 8}

125-14 $x - 39 = 27 \wedge 39 + x = 105$.
{66}

125-15 $148(203) = x$.
{30044}

125-16 $x < 49 + 86$.
{0, 1, 2, ..., 134}

125-17 $7224 + x > 7401$.
{178, 179, 180, ...}

125-18 $x < 13(18) \wedge 76 + x > 295$.
{220, 221, 222, ..., 233}

125-19 $x > 75 \wedge 3212 - x > 3137$.
{ }

Lesson 31

126-A Two

126-B Yes

126-C Examples are (Thanksgiving, June) and (Christmas, December).

126-D It is a member of set A. * It is a member of set B.

126-E Three * Four * Twelve

127-F By finding the product of the number of members in set M and the number of members in set N

127-G No * Yes * No * Since the first component of (June, Thanksgiving) is a member of B and the second component is a member of A, (June, Thanksgiving) is a member of B × A but is not a member of A × B. Since A and B are different sets, the first components of the members of A × B are different from the first

components of the members of B × A. The same is true for the second components.

127-H Yes

127-I G, L

127-J (Labor Day, September), (Christmas, December)

127-K Yes * Yes

128-L Yes * Each member of the solution set of "a is a holiday in b" is also a member of the universe for (a, b).

128-A Two

128-B A universe for (x, y)

128-C 20

128-D {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)}

128-E Yes

128-F F, K, L, P, Q, R

128-G (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)

129-H (Column 1) {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)}

129-I Yes * Each member of the solution set of $x > y$ is also a member of the universe.

129-J By matching each member of S in turn with each member of T * {(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)}

129-K $y = x + 1$. * S × T

129-L Set S * Set T

129-M 2 = 1 + 1. 2 = 3 + 1. 2 = 5 + 1. 4 = 1 + 1. 4 = 3 + 1. 4 = 5 + 1. 6 = 1 + 1. 6 = 3 + 1. 6 = 5 + 1.

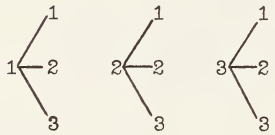
129-N 2 = 1 + 1. 4 = 3 + 1. 6 = 5 + 1.

129-O {(1, 2), (3, 4), (5, 6)}

129-P Yes. Each member of the solution set is also a member of S × T.

129-Q Set S. Since y is the first component of (y, x), you must use the first components of members of S × T to replace y. * Set T. Since x is the second component of (y, x), you must use the second components of members of S × T to replace x.

129-A One possible diagram is shown below.



129-B Set A

129-C $m < 2 + n$. * A × A

129-D Yes * Yes

129-E The universe is given for (m, n). Since m is the first component of (m, n), m should be replaced by the first component of (1, 3). Since n is the second component of (m, n), n should be replaced by the second component of (1, 3).

129-F Yes * Yes

129-G You should obtain 9 statements. * Each of the 9 statements is true except 3 < 2 + 1, which is false. * {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

129-H (Column 2) Yes

On your own

130-1 {(Amazon, Africa), (Amazon, Asia), (Amazon, Europe), (Amazon, Australia), (Danube, Africa), (Danube, Asia), (Danube, Europe), (Danube, Australia), (Nile, Africa), (Nile, Asia), (Nile, Europe), (Nile, Australia)}

130-2 M × P = {(7, 1), (7, 100), (7, 15), (7, 10), (7, 5), (11, 1), (11, 100), (11, 15), (11, 10), (11, 5), (6, 1), (6, 100), (6, 15), (6, 10), (6, 5), (4, 1), (4, 100), (4, 15), (4, 10), (4, 5)}. P × R = {(1, 1), (1, 2), (100, 1), (100, 2), (15, 1), (15, 2), (10, 1), (10, 2), (5, 1), (5, 2)}. M × R = {(7, 1), (7, 2), (11, 1), (11, 2), (6, 1), (6, 2), (4, 1), (4, 2)}.

130-3 No

130-4 P × M = {(1, 7), (1, 11), (1, 6), (1, 4), (100, 7), (100, 11), (100, 6), (100, 4), (15, 7), (15, 11), (15, 6), (15, 4), (10, 7), (10, 11), (10, 6), (10, 4), (5, 7), (5, 11), (5, 6), (5, 4)}. * Yes

130-5 M × M = {(7, 7), (7, 11), (7, 6), (7, 4), (11, 7), (11, 11), (11, 6), (11, 4), (6, 7), (6, 11), (6, 6), (6, 4), (4, 7), (4, 11), (4, 6), (4, 4)}. * 16

130-6 G = {1, 3}. * H = {2, 4, 6, 8, 10}.

130-7 A subset

130-8 Not a subset

130-9 Not a subset

130-10 {(1, 4), (3, 6)}

130-11 {(1, 2), (1, 4), (1, 6), (1, 8), (1, 10), (3, 4), (3, 6), (3, 8), (3, 10)}

130-12 {(3, 2)}

130-13 {(1, 4), (1, 6), (1, 8), (1, 10), (3, 6), (3, 8), (3, 10)}

130-14 {(3, 2)}

130-15 {(3, 2), (3, 4)}

130-16 {(1, 4), (3, 4)}

130-17 {(1, 2), (1, 4), (1, 6), (1, 8), (1, 10), (3, 4), (3, 6), (3, 8), (3, 10)}

130-18 {10, 1} is a set containing two members. (10, 1) is not a set but an ordered pair. {(10, 1)} is a set containing only one member and that member is an ordered pair.

130-19 Whether the universe is for (s, r) or for (r, s)

Lesson 32

130-A x and y

130-B {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)} * $0 + 2 < 0$, $0 + 2 < 1$, $0 + 2 < 2$, are three of the statements obtained. There are 17 other similar statements that are obtained using A × B to make replacements for x and y.

130-C $0 + 2 < 3$, $0 + 2 < 4$, $1 + 2 < 4$.

130-D {(0, 3), (0, 4), (1, 4)} * Yes * Yes

130-E Yes * Yes * Yes

131-F Yes * Yes * (3, 2) * Yes

131-G Yes * Yes * (1, 0)

131-H Yes * Yes * (0, 4) * Yes

131-I (1, 3) is associated with H. (3, 0) is associated with I. (0, 1) is associated with J. (3, 4) is associated with K.

131-J Yes * Yes

131-K 20 * 20

131-L All the dots

131-M Yes * Yes

132-N Yes * Each dot in the graph of the solution set of $x + 2 < y$ is also in the graph of A × B.

132-A {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3)} * The solution set is {(1, 0), (1, 1), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3)}.

132-B Yes

132-C First axis * Second axis

132-D Yes * Yes

132-E Yes. Since each member of the universe is associated with a point represented by a dot and each dot represents a point associated with a member of the universe

132-F Yes * No

132-G Yes. Since each solution of $2x > y$ is associated with a point whose dot is encircled and each encircled dot represents a point associated with a solution of $2x > y$

132-H Yes. Each dot in the graph of the solution set of $2x > y$ is also in the graph of the universe.

132-I The only true statement is $0 + 3 = 3$.

132-J {(3, 0)} * Yes

133-K Yes * Each dot in D8 represents a point associated with a member of A × A and each member of A × A is associated with a point that is represented by a dot in D8.

133-L Yes * The solution set of $y + 3 = x$ contains just one member. There is just one encircled dot in D8 and this dot represents the point that is associated with the one member of the solution set of $y + 3 = x$.

133-M Yes

On your own

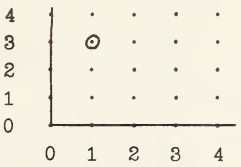
133-1 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (5, 0), (5, 1), (5, 2), (5, 3), (5, 4)} * {(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)}

133-2 {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3)}

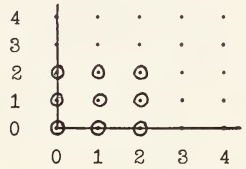
* {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (3, 0)}

133-3 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4)}

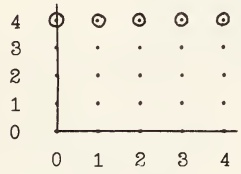
133-4



133-5



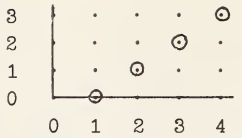
133-6



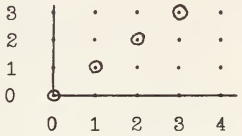
133-7 {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3), (4, 0), (4, 1), (4, 2), (4, 3)}

133-8 {(1, 0), (2, 1), (3, 2), (4, 3)}

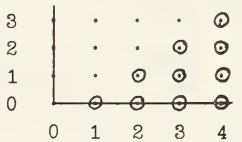
133-9



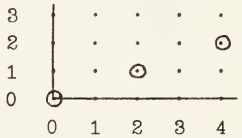
133-10 {(0, 0), (1, 1), (2, 2), (3, 3)}



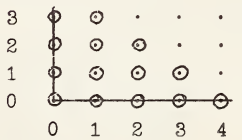
133-11 {(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3)}



133-12 {(0, 0), (2, 1), (4, 2)}



133-13 {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (4, 0)}



Lesson 33

134-A $x = 3$ * {0, 1, 2, ..., 10} * Yes

134-B {3} * One * Yes

134-C Because the universe is a set of numbers, not a set of ordered pairs of numbers

134-D Yes * The graph of the solution set of $x = 3$ contains only the encircled dot in D1 and this dot is also in the graph of the universe.

134-E Members of A × A are ordered pairs. Members of the universe tabulated in D1 are not ordered pairs.

134-F One

134-G The universe, A × A, is given for (x, y), so the first components of the members of A × A may be used as replacements for x.

134-H No

134-I All the dots

134-J Yes * 3 * {(3, 0), (3, 1), (3, 2), (3, 3), (3, 4)}

135-K (Column 1) Yes. When x is replaced by the first component of each of the members of the set tabulated for exercise J, a true statement is obtained from $x = 3$.

135-L (Column 1) No

135-M (Column 1) The encircled dots

135-N (Column 1) Yes * Yes * The solution set has 5 members that are also members of the universe.

135-A A × B * {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 0), (3, 1),

(3, 2), (3, 3), (3, 4), (3, 5), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)}

135–B None

135–C The second component of each member of the solution set must be less than 4.

135–D All the dots

135–E {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3), (4, 0), (4, 1), (4, 2), (4, 3)}

135–F Yes

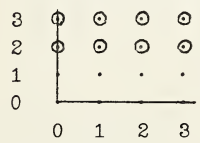
135–G No

135–H Yes * The set tabulated for exercise E contains all the members of the universe that satisfy $y < 4$.

135–I The encircled dots * Yes

135–J $y + 3 > 4$ * R × R

135–K (Column 2) See the graph below. All the dots are in the graph of R × R.



135–L (Column 2) {(0, 2), (1, 2), (2, 2), (3, 2), (0, 3), (1, 3), (2, 3), (3, 3)}

135–M (Column 2) Because the universe is a set of ordered pairs

135–N (Column 2) See the graph for 135–K. The encircled dots are in the graph of the solution set of $y + 3 > 4$.

136–A {(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2), (5, 3), (5, 4)}

136–B No * Yes

136–C No * Yes * Since no member of the set tabulated in exercise A satisfies $x < y$, the set cannot be the solution set of $x < y$. The set tabulated in exercise A contains all the members of the universe that satisfy $x > y$, so this set is the solution set of $x > y$.

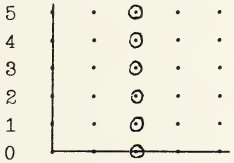
136–D {(0, 2), (1, 3), (2, 4), (3, 5)}

136–E 2

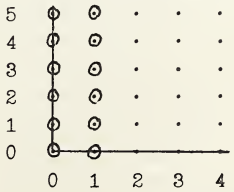
136–F There are many conditions that have this solution set. Some examples are $x + 2 = y$, $x + 3 = y + 1$, $y - 2 = x$.

On your own

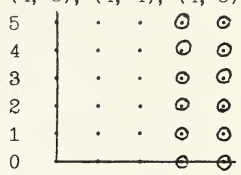
136–1 {(2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5)}



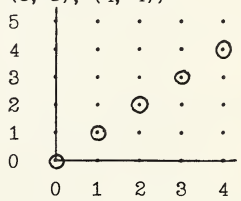
136–2 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5)}



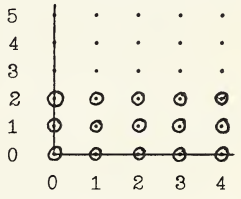
136–3 {(3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5)}



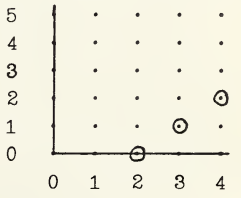
136–4 {(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)}



136–5 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)}



136–6 {(2, 0), (3, 1), (4, 2)}



136–7 {(1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5)}

136–8 {(0, 3), (0, 4), (0, 5), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5)}

136–9 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5)}

136–10 {(0, 2), (1, 2), (2, 2), (3, 2), (4, 2)}

136–11 {(3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5)}

136–12 {(0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}

136–13 {(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} * An example of a condition is $x = y$.

137–14 {(1, 0), (1, 1), (1, 2), (1, 3)} * An example of a condition is $x = 1$.

137–15 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)} * An example of a condition is $y < 3$.

137–16 {(0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)} * An example of a condition is $x < y$.

137–17 {(0, 0), (1, 2), (2, 4), (3, 6)} * An example of a condition is $2x = y$.

Keeping Skilful

137–1 1207

137–2 868

137–3 23,141

137–4 11,948

137–5 13,895

137–6 4069

137–7 27,588

137–8 37,105

137–9 495,600

137–10 90,405

137–11 114,760

137–12 82,655

137–13 755,712

137–14 240, remainder 10

137–15 418, remainder 0

137–16 72, remainder 0

137–17 92, remainder 100

137–18 326, remainder 0

137–19 208, remainder 421

137–20 156, remainder 0

Lesson 34

138–A The set whose members are * that satisfy the condition that

138–B The words "the first component of each ordered pair is greater than the second component" * Yes

138–C $x > y$ * Because $\{(x, y)\ x > y\}$ contains all the members of the universe, and only those members of the universe, that satisfy $x > y$

138–D {(7, 7), (7, 2), (2, 7), (2, 2), (6, 7), (6, 2)} * $x = y$. * $x = 2$. * $x > 8 - y$. * $y > 3$.

138–E A = {(7, 7), (2, 2)}. B = {(2, 7), (2, 2)}. C = {(7, 7), (7, 2), (2, 7), (6, 7)}. D = {(7, 7), (2, 7), (6, 7)}.

138–F Yes * Yes * Sentence A expresses a true statement because (2, 7) and (6, 7) are the only members of the universe that satisfy $x < y$. Sentence B expresses a true statement because there are no members of the universe that satisfy $x = 5$.

139–A (2, 3), (500, 3), (7, 7)

139–B N * N

139–C Yes * Both components of each member of N × N are natural numbers and each ordered pair whose components are natural numbers is a member of N × N.

139–D Yes * The number of members in N × N is not a natural number.

139–E That the points in the first axis that you can associate with natural numbers go on and on * That the points in the second axis that you can associate with natural numbers go on and on

139–F B, point (2, 2). C, point (2, 5). D, point (3, 3). E, point (4, 1). F, point (4, 2). G, point (5, 2). H, point (5, 3).

139–G Yes

139–H No

139–I Yes

139–J Yes

139–K Infinitely many * First axis

139–L Infinitely many * In the second axis

139–M Infinitely many * Point (1, 0) is the same as point 1 in the first axis and all the other points associated with members of N × N having 1 as a first component are located above the point 1 in the first axis.

139-N Infinitely many * Point (0, 9) is the same as point 9 in the second axis and all the other points associated with members of N × N having 9 as a second component are to the right of the point 9 in the second axis.

140-O Point (0, 10) is the same as point 10 in the second axis. Point (1, 17) is located above point 1 in the first axis and to the right of point 17 in the second axis. Point (9, 3) is located above point 9 in the first axis and to the right of point 3 in the second axis. The other points are located similarly. The dots for the points are placed where the points are located.

140-P Yes * Each dot in the graph represents a point associated with a member of N × N. The three small dots at the end of the picture of each axis tell you that each member of N × N is associated with a point whose dot is in the graph even though the dot is not shown.

140-A Yes

140-B Yes * Because you know that since the graph of the solution set is a subset of the universe, the graph of the universe includes a graph of the solution set.

140-C (0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) * Yes

140-D Yes. 98 = 98 is a true statement. * No * Extend the picture of each axis to contain all dots representing points associated with numbers through 98.

140-E Examples are (6, 6), (7, 7), (8, 8), (9, 9), (10, 10).

140-F The entire graph of $\{(x, y) \mid y = x\}$ is not shown. * Each natural number is used for both components of a member of the solution set of $y = x$. Since the set of natural numbers is an infinite set, so is the solution set of $y = x$.

140-G No. None of the ordered pairs associated with points represented by encircled dots satisfy $y > x$.

141-H Examples are the dots that represent point (1, 0) and point (4, 2). * Yes * No * Neither of the points you chose are associated with ordered pairs that satisfy $y > x$.

141-I No

141-J Yes * Each of these dots represents a point associated with an ordered pair that satisfies $y > x$.

141-K Yes * Yes * Because each member of N is used as a component of a member of $\{(x, y) \mid y > x\}$

141-L No * Because not all the points associated with members of $\{(x, y) \mid y > x\}$ are represented by dots

141-M They are encircled. * They are enclosed by squares. * The dots in the graph of $\{(x, y) \mid y < x\}$ are below the dots in the graph of $\{(x, y) \mid y = x\}$.

141-N Yes * Because not all the points associated with members of $\{(x, y) \mid y < x\}$ are represented by dots

141-O Yes * No * Yes

141-P To make a graph of the solution set of $x + y < 4$, use all the dots below those in the solution set of $x + y = 4$. * To make a graph of the solution set of $x + y > 4$, use all the dots above or to the right of the dots in the graph of the solution set of $x + y = 4$.

141-Q Yes * No * The solution set of $x + y < 4$ has 10 members but you cannot determine the number of members in the solution set of $x + y > 4$.

On your own

142-1 $\{(x, y) \mid x = y - 14\}$

142-2 $\{(x, y) \mid y = x - 14\}$

142-3 {(5, 20), (5, 21), (6, 20), (6, 21), (7, 20), (7, 21), (8, 20), (8, 21), (9, 20), (9, 21), (10, 20), (10, 21)}

142-4 $\{(x, y) \mid x = 27 - y\} = \{(7, 20), (6, 21)\}$.

142-5 $\{(x, y) \mid x > 27 - y\} = \{(8, 20), (9, 20), (10, 20), (7, 21), (8, 21), (9, 21), (10, 21)\}$.

142-6 $\{(x, y) \mid x < 27 - y\} = \{(5, 20), (6, 20), (5, 21)\}$.

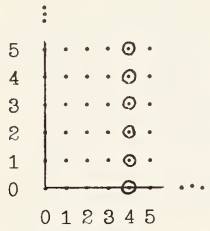
142-7 $\{(x, y) \mid y = 20\} = \{(5, 20), (6, 20), (7, 20), (8, 20), (9, 20), (10, 20)\}$.

142-8 $\{(x, y) \mid y - 12 = x\} = \{(8, 20), (9, 21)\}$.

142-9 $\{(x, y) \mid 3x < y + 9\} = \{(5, 20), (6, 20), (7, 20), (8, 20), (9, 20), (5, 21), (6, 21), (7, 21), (8, 21), (9, 21)\}$.

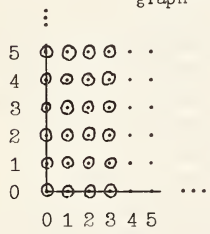
142-10 Infinite set

Incomplete graph



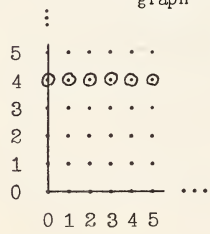
142-11 Infinite set

Incomplete graph



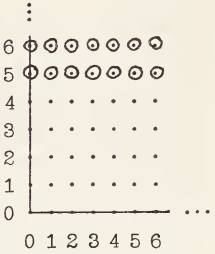
142-12 Infinite set

Incomplete graph

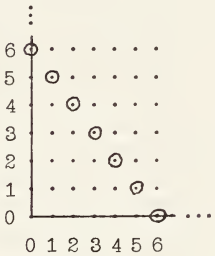


142-13 Infinite set

Incomplete graph

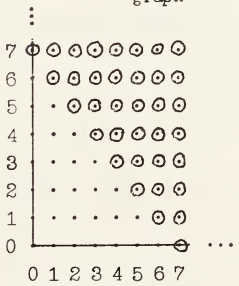


142-14 Finite set



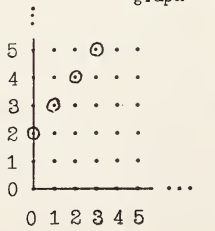
142-15 Infinite set

Incomplete graph



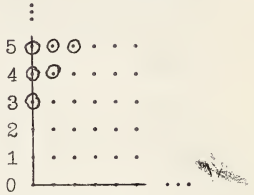
142-16 Infinite set

Incomplete graph



142-17 Infinite set

Incomplete graph



142-18 The dots in the graph of $\{(x, y) \mid x < 4\}$ are to the left of the dots in the graph of $\{(x, y) \mid x = 4\}$.

142-19 The dots in the graph of $\{(x, y) \mid x + y > 6\}$ are above or to the right of the dots in the graph of $\{(x, y) \mid x + y = 6\}$.

Keeping Skilful

142-1 Statement

142-2 Condition

142-3 Condition

142-4 Statement

142-5 Statement

142-6 Condition

142-7 Statement

142-8 Condition

142-9 Statement

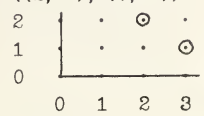
142-10 Condition

142-11 The sentences in exercises 4, 5, and 7 express true statements. The sentences in exercises 1 and 9 express false statements.

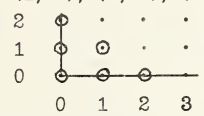
142-12 $\{n \mid n = 861 + 79\} = \{940\}$.
$\{s \mid 34 - s > 18\} = \{0, 1, 2, \ldots, 15\}$.
$\{d \mid d = 19 + 32\} = \{51\}$.
$\{w \mid w > 24 + 9\} = \{34, 35, 36, \ldots\}$.
$\{m \mid m > 3m\} = \{\ \}$.

142–13 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)}

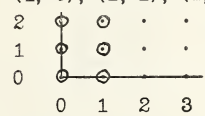
142–14 {(2, 2), (3, 1)}



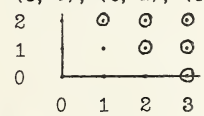
142–15 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 0)}



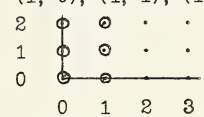
142–16 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)}



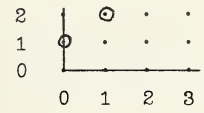
142–17 {(1, 2), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)}



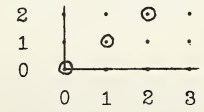
142–18 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)}



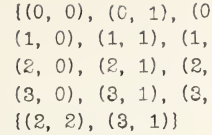
142–19 {(0, 1), (1, 2)}



142–20 {(0, 0), (1, 1), (2, 2)}



142–21 {(1, 2), (2, 1), (3, 0)}



142–22 K = {0, 1, 2, ..., 11}.
L = {0, 1, 2, ..., 8}.
K ∩ L = {0, 1, 2, ..., 8}.
K ∪ L = {0, 1, 2, ..., 11}.

142–23 K = {177}.
L = {0, 1, 2, 3, 4, 5, 6}.
K ∩ L = { }.
K ∪ L = {0, 1, 2, 3, 4, 5, 6, 177}.

142–24 K = {17, 18, 19, ...}.
L = {0, 1, 2, ..., 18}.
K ∩ L = {17, 18}.
K ∪ L = {0, 1, 2, ...}.

142–25 K = {113, 114, 115, ...}.
L = {28, 29, 30, ...}.
K ∩ L = {113, 114, 115, ...}.
K ∪ L = {28, 29, 30, ...}.

142–26 K = {36}.
L = {36}.
K ∩ L = {36}.
K ∪ L = {36}.

Lesson 35

143–A $y = 5 - x$. * A × A * Yes * {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (5, 0), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)}

143–B No * Yes

143–C Yes * Yes * The universe for (x,y) is A × A. Therefore, only members of A can be used as replacements for x.

143–D Yes * 5

143–E Because 5 is the replacement for y that satisfies the condition obtained when x is replaced by 0.

143–F Yes * Because (0, 5) is a member of the universe and a true statement is obtained from $y = 5 - x$ when x is replaced by 0 and y is replaced by 5

143–G

x	0	1	2	3	4	5
y	5	4	3	2	1	0

143–H (0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0) * Yes * No

143–I {(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)}

143–J B × B * No * Yes

144–K (Column 1) Yes * No

144–L When you replaced x by 5, you obtained the condition $5 + y = 4$. There is no member of B that satisfies this condition. When you replace x by 6, you obtain the condition $6 + y = 4$. There is no member of B that satisfies this condition.

144–M They are the only numbers that are first components of solutions of $x + y = 4$.

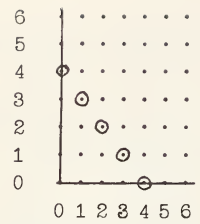
144–N

x	0	1	2	3	4
y	4	3	2	1	0

144–O 5 * Yes * No

144–P {(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)}

144-Q



144-A (Column 1) $x + y < 4$. * B × B

144-B (Column 1) $4 + y < 4$. * No

144-C If a number greater than 3 is used as a replacement for x in $x + y < 4$, no member of B will satisfy the condition obtained.

144-D $0 + y < 4$. * Yes * More than one replacement for y satisfies $0 + y < 4$.

144-E 0, 1, 2, 3 * Below the numeral 0 that names the replacement for x.

144-F No

144-G 0, 1, 2 * Below the numeral 1 that names the replacement for x.

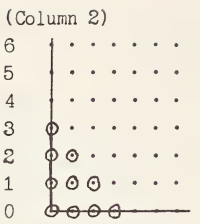
144-H

x	0	1	2	3
y	0	0	0	0
	1	1	1	
	2	2		
	3			

144-I B × B

144-J {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (3, 0)}

144-K (Column 2)



144-A (Column 2) $s = 2r + 1$. * Each of the natural numbers

144-B (Column 2) Yes

145-C 101

145-D

r	0	1	2	3	4
s	1	3	5	7	9

145-E {(0, 1), (1, 3), (2, 5), (3, 7), (4, 9)} * Yes

145-F No * r can be replaced by any natural number and there is a replacement for s that satisfies the condition obtained. Therefore, the solution set of $s = 2r + 1$ is an infinite set.

145-G $s < 2r + 1$.

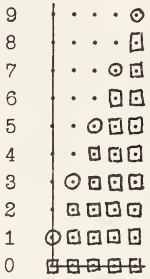
145-H

r	0	1	2	3	4
s	0	0	0	0	0
		1	1	1	1
		2	2	2	2
			3	3	3
			4	4	4
				5	5
				6	6
					7
					8

145-I {(0, 0), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 0), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (4, 7), (4, 8)} * No * It does not contain all the members of N × N that satisfy $s < 2r + 1$.

145-J No * Yes

145-K



145-L $s > 2r + 1$.

r	0	1	2	3	4
s	2	4	6	8	10
	3	5	7	9	
	4	6	8	10	
	5	7	9		
	6	8	10		
	7	9			
	8	10			
	9				
	10				

145-M No. Each of the natural numbers is a first component of infinitely many ordered pairs that satisfy $s > 2r + 1$. The solution set of $s > 2r + 1$ is an infinite set.

On your own

145-I $y = 8 - x$.

x	0	1	2	3	4	5	6	7	8
y	8	7	6	5	4	3	2	1	0

{(0, 8), (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)}

145–2 $3x = y$.

x	0	1	2
y	0	3	6

{(0, 0), (1, 3), (2, 6)}

145–3 $y = x + 1$.

x	0	1	2	3	4	5	6	7
y	1	2	3	4	5	6	7	8

{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8)}

145–4 $y + x < 3$.

x	0	1	2
y	0	0	0
	1	1	
	2		

{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 0)}

145–5 $x + 4 = y$.

x	0	1	2	3	4
y	4	5	6	7	8

{(0, 4), (1, 5), (2, 6), (3, 7), (4, 8)}

145–6 $y = x + 5$.

x	0	1	2	3
y	5	6	7	8

{(0, 5), (1, 6), (2, 7), (3, 8)}

146–7 $s + 17 = r$.

r	17	18	19	42	110	121
s	0	1	2	25	93	104

146–8 $s = 4r + 2$.

r	0	3	4	6	12	17	95	118	213
s	2	14	18	26	50	70	382	474	854

146–9 $4r - 2 > s$.

r	1	2	3	4
s	0	0	0	0
	1	1	1	1
		2	2	2
		3	3	3
		4	4	4
		5	5	5
			6	6
			7	7
			8	8
			9	9
				10
				11
				12
				13

146–10 $3s + r = 35$.

r	32	29	23	17	11	8	5	2
s	1	2	4	6	8	9	10	11

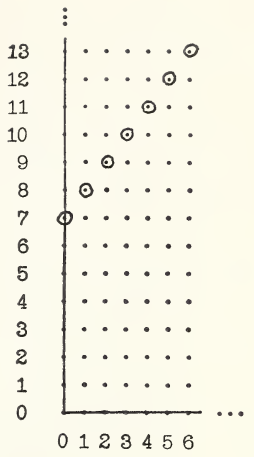
146–11 $r + s < 8$.

r	0	2	4	6
s	0	0	0	0
	1	1	1	1
	2	2	2	
	3	3	3	
	4	4		
	5	5		
	6			
	7			

146–12 $x = y - 7$.

x	0	1	2	3	4	5	6
y	7	8	9	10	11	12	13

{(0, 7), (1, 8), (2, 9), (3, 10), (4, 11), (5, 12), (6, 13)}



146–13 $y = x + 2$.

x	0	1	2	3	4	5	6
y	2	3	4	5	6	7	8

{(0, 2), (1, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8)}

146–14 $2x + 3 = y$.

x	0	1	2	3	4	5	6
y	3	5	7	9	11	13	15

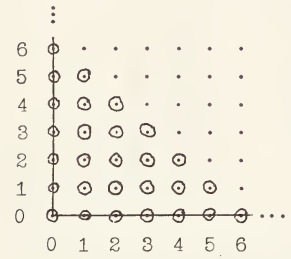
{(0, 3), (1, 5), (2, 7), (3, 9), (4, 11), (5, 13), (6, 15)}

146–15 $7 - x > y$.

x	0	1	2	3	4	5	6
y	0	0	0	0	0	0	0
	1	1	1	1	1	1	
	2	2	2	2	2		
	3	3	3	3			
	4	4	4				
	5	5					
	6						

{(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5),

(0, 6), (1, 0), (1, 1),
(1, 2), (1, 3), (1, 4),
(1, 5), (2, 0), (2, 1),
(2, 2), (2, 3), (2, 4),
(3, 0), (3, 1), (3, 2),
(3, 3), (4, 0), (4, 1),
(4, 2), (5, 0), (5, 1),
(6, 0)}



146-16 $21 - 3x = y$.

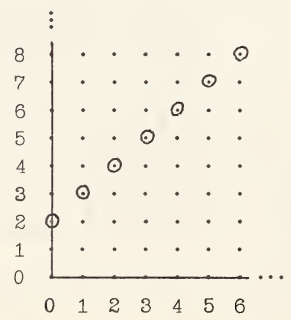
x	0	1	2	3	4	5	6
y	21	18	15	12	9	6	3

{(0, 21), (1, 18), (2, 15),
(3, 12), (4, 9), (5, 6),
(6, 3)}

146-17 $y - 2 = x$.

x	0	1	2	3	4	5	6
y	2	3	4	5	6	7	8

{(0, 2), (1, 3), (2, 4),
(3, 5), (4, 6), (5, 7),
(6, 8)}



Keeping Skilful

146-1 4000, 3200, 3280
146-2 5000, 4700, 4730
146-3 7000, 6800, 6900
146-4 20,000, 19,900, 19,930
146-5 37,000, 36,900, 37,000
146-6 6000, 5500, 5560
146-7 1369
146-8 7165
146-9 19,856
146-10 17,429
146-11 174,337
146-12 90,277
146-13 95,820
146-14 21,060
146-15 75,670
146-16 99,510
146-17 518,280
146-18 259,689
146-19 214,170
146-20 64, remainder 20
146-21 260, remainder 0
146-22 192, remainder 50
146-23 48, remainder 130
146-24 162, remainder 0
146-25 110, remainder 36
146-26 {0, 1, 2, 3, 4}
146-27 {13, 14, 15, 16, 17, 18}
146-28 {476}
146-29 {7, 8}
146-30 {0, 1, 2, ..., 17}
146-31 {5, 6, 7, ...}
146-32 {31, 32}
146-33 {127, 128, 129, ...}
146-34 {(1, 0), (1, 2), (3, 2), (2, 3), (0, 1), (3, 4)}
146-35 { }
146-36 {(3, 4)}
146-37 {(0, 1), (1, 2), (2, 3), (3, 4)}
146-38 {(1, 2), (2, 3)}
146-39 {(3, 4)}

Lesson 36

147-A N × N
147-B Yes * The condition expressed in D1 is made up of two simple conditions and the connective "and." * Two
147-C Because the universe is a set of ordered pairs
147-D Each solution of a compound condition that has two simple conditions and the connective "and" must satisfy both simple conditions.
147-E The dots are encircled. * The dots are enclosed by squares.
147-F The dot that is both encircled and enclosed by a square * (2, 4)
147-G Yes * Yes
147-H {(2, 4)}
148-I {(2, 4)} * Yes
148-J Yes. You know that $\{(x, y) \mid x + y = 6 \wedge y = 2x\}$ consists of all the members of the universe that satisfy both $x + y = 6$ and $y = 2x$. You know that the intersection of $\{(x, y) \mid x + y = 6\}$ and $\{(x, y) \mid y = 2x\}$ also consists of all the members of the universe that satisfy both $x + y = 6$ and $y = 2x$.
148-K N × N
148-L The solution set of $x + y < 7$ is a finite set for which the names of all the members may be listed. The solution set of $y > x$ is an infinite set. * When you replace x in $x + y < 7$ by 7, you obtain $7 + y < 7$. There is no natural number that satisfies $7 + y < 7$. A similar situation arises when any

natural number greater than 6 is used to replace x.

148-M See the chart below.

$x + y < 7$.

x	0	1	2	3	4	5	6
y	0	0	0	0	0	0	0
	1	1	1	1	1	1	
	2	2	2	2	2		
	3	3	3	3			
	4	4	4				
	5	5					
	6						

148-N Yes

148-O The second component of each ordered pair must be greater than the first component. * (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4) * Yes

148-P {(0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4)} * Yes

148-Q $\{(x, y) \mid x + y < 7\} \cap \{(x, y) \mid y > x\}$ contains only those objects that belong to both $\{(x, y) \mid x + y < 7\}$ and $\{(x, y) \mid y > x\}$. These are the same objects that belong to $\{(x, y) \mid x + y < 7 \wedge y > x\}$.

148-A N × N

148-B $x + 1 = 4$. * The solution set of $x + 1 = 4$ contains fewer members than the solution set of $x + y < 8$.

149-C Yes. $x + 1 = 4$ does not place any requirement on the second component. * Yes

149-D 0, 1, 2, 3, 4

149-E (3, 0), (3, 1), (3, 2), (3, 3), (3, 4) * Yes

149-F {(3, 0), (3, 1), (3, 2), (3, 3), (3, 4)} * Yes

149-G The graph of the solution set of $x + y < 8$ * No * The graph of the solution set of $x + 1 = 4$ * Yes

149-H Each dot in the graph of $\{(x, y) \mid x + y < 8 \wedge x + 1 = 4\}$ is both encircled and enclosed by a square. * No

149-I $y + 1 < 7$.

149-J Yes. $y + 1 < 7$ makes no requirement about the first component. * Yes. However, there are only a finite number of replacements for y that satisfy $y + 1 < 7$.

149-K Because $y + 1 < 7$ makes a requirement about the replacements for y but not about the replacements for x * Because the only replacements for y that satisfy $y + 1 < 7$ are 0 through 5

$y + 2 > x$.

y	0	1	2	3	4	5
x	0	0	0	0	0	0
	1	1	1	1	1	1
		2	2	2	2	2
			3	3	3	3
				4	4	4
					5	5
						6

149-L Yes

149-M {(0, 0), (1, 0), (0, 1), (1, 1), (2, 1), (0, 2), (1, 2), (2, 2), (3, 2), (0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (0, 4), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (0, 5), (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)}

149-N That the sum of the components of each ordered pair be less than 6 * That the sum of the components of each ordered pair equals 7

149-O No

149-P { }

149-Q Yes * Yes

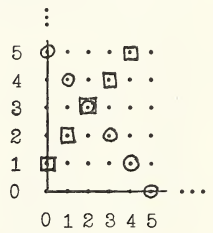
150-R No * Yes

150-S Yes

150-T Yes

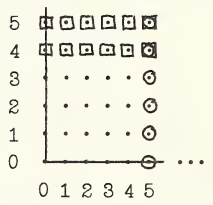
On your own

150-I

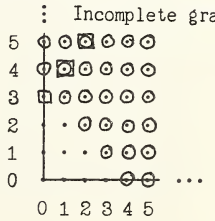


{(2, 3)}

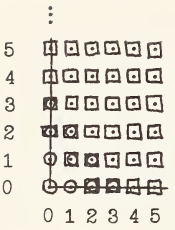
150-2



150-3



150-4



{(0, 2), (0, 3), (1, 1), (1, 2), (2, 0), (2, 1), (3, 0)}

150-5 {(3, 1)}

150-6 {(163, 162)}

150-7 {(48, 77)}

150-8 {(8, 0)}

150-9 {(21, 4)}

150-10 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (2, 0)}

150-11 {(35, 786)}

150-12 {(13, 0), (14, 1), (15, 2), (16, 3), (17, 4), (18, 5), (19, 6)}

150-13 {(36, 31), (36, 32), (36, 33), (36, 34), (36, 35), (36, 36)}

150-14 {(0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (0, 7), (1, 3), (1, 4), (1, 5), (1, 6), (2, 5)}

150-15 { }

150-16 {(18, 122)}

150-17 {(32, 0), (33, 1), (34, 2)}

Checking up

Test 34

150-1 {(2, 3), (2, 4), (4, 3), (4, 4), (7, 3), (7, 4)}

150-2 {(3, 2), (3, 4), (3, 7), (4, 2), (4, 4), (4, 7)}

150-3 {(2, 2), (2, 4), (2, 7), (4, 2), (4, 4), (4, 7), (7, 2), (7, 4), (7, 7)}

150-4 {(3, 3), (3, 4), (4, 3), (4, 4)}

150-5 {(2, 3), (2, 4), (4, 4)}

150-6 {(3, 4), (3, 7), (4, 7)}

Test 35

150-7 {(0, 5), (1, 3), (3, 1), (4, 3), (4, 4), (5, 2), (6, 0)}

150-8 {(0, 1), (1, 4), (2, 5), (3, 3), (4, 0), (6, 5), (7, 2)}

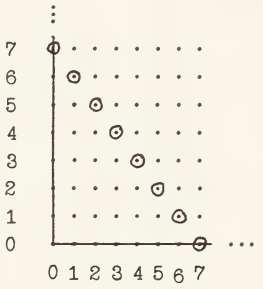
150-9

$7 - x = y \wedge x < 8.$

x	0	1	2	3	4	5	6	7
y	7	6	5	4	3	2	1	0

150-10 {(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)}

150-11



Lesson 37

151-A No

151-B No

151-C Because the action in the problem is additive, not subtractive. Additive action is when two groups are put together. In this case, you should imagine that the fish Steve caught are put with the fish George caught. * The total number of fish George and Steve caught together

151-D Because the total number of fish caught by George and Steve was less than 5 * Two

151-E N × N * Yes. Since the universe is a set of ordered pairs, the solution set must be a set of ordered pairs.

151-F If the two boys together caught fewer than 5 fish, then you know that each boy caught fewer than 5 fish. * 0, 1, 2, 3, 4

151-G When x in $x + y < 5$ is replaced by 5, you obtain the condition $5 + y < 5$. No member of N satisfies $5 + y < 5$.

151-H

x	0	1	2	3	4
y	0	0	0	0	0
	1	1	1	1	
	2	2	2		
	3	3			
	4				

151-I {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (4, 0)}

151-J The number of fish George caught * The number of fish Steve caught

151-K If George caught 2 fish, Steve could have caught 0, 1, or 2 fish. If George caught 3 fish, Steve could have caught 0 or 1 fish. If George caught 4 fish, Steve caught none.

151-L Yes

151-M If George caught 1 fish and Steve caught 3 fish, could George and Steve together have caught fewer than 5 fish? The answer is yes. The other answers can be verified in the same way.

152-A Yes

152-B No

152-C Because the action in the problem is subtractive, not additive. When one group is removed from a larger group, the action is subtractive. In this case, the stamps Judy used are removed from the 8 stamps that Judy bought.

152-D No

152-E Because you know that both $8 - m$ and n refer to the number of stamps that Judy had left

152-F C × C

152-G 7. Since Judy used some, but not all, of the 8 stamps * 1. Since Judy used some of the 8 stamps

152-H 0 is not a member of C. * If m is replaced by 8, you obtain the condition $8 - 8 = n$. There is no member of C that satisfies $8 - 8 = n$.

152-I 1 through 7

152-J

m	1	2	3	4	5	6	7
n	7	6	5	4	3	2	1

152-K {(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1)}

152-L The number of stamps Judy used * The number of stamps Judy had left

152-M If Judy used 1 stamp, she had 7 left. If Judy used 2 stamps, she had 6 left. You can use the other solutions to obtain the five other answers.

On your own

152-1 x, number of boats Dave built.
y, number of boats Ted built.
$x + y = 5$.

x	0	1	2	3	4	5
y	5	4	3	2	1	0

{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (0, 5) is : Dave built 0 boats and Ted built 5 boats. The answer obtained from (3, 2) is: Dave built 3 boats and Ted built 2 boats. For this problem you could have the first component refer to the number of boats Ted built and the second component refer to the number of boats Dave built. The solution set would be the same.

152-2 x, number of fishhooks Ernie gave to Tom.
y, number of fishhooks Ernie kept for himself.
$9 - x = y$.

x	1	2	3	4	5	6	7	8
y	8	7	6	5	4	3	2	1

{(1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 8) is : Ernie gave 1 fishhook to Tom and kept 8 for himself.

152-3 x, number of lemons Martha bought.
y, number of limes Martha bought.
$x + y < 6$.

x	1	2	3	4
y	1	1	1	1
	2	2	2	
	3	3		
	4			

{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 1) is: Martha bought 1 lemon and 1 lime.

152-4 x, number of bushels of apples Mr. Phillips sold.
y, number of bushels of apples Mr. Phillips had left.
$4 - x = y$.

x	1	2	3
y	3	2	1

{(1, 3), (2, 2), (3, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 3) is: Mr. Phillips sold 1 bushel of apples and had 3 bushels left.

153-5 x, number of posters Louise made.
y, number of posters Connie made.

$x + y < 7$.

x	1	2	3	4	5
y	1	1	1	1	1
	2	2	2	2	
	3	3	3		
	4	4			
	5				

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 1) is: Louise made 1 poster and Connie made 1 poster.

153-6 x, number of tickets Jane sold.
y, number of tickets Jane returned.
$6 - x = y$.

x	0	1	2	3	4	5	6
y	6	5	4	3	2	1	0

{(0, 6), (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (0, 6) is: Jane sold no tickets and returned 6.

153-7 x, number of hours Larry works on Friday.
y, number of hours Larry works on Saturday.
$x + y = 8$.

x	1	2	3	4	5	6	7
y	7	6	5	4	3	2	1

{(1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 7) is: Larry works 1 hour on Friday and 7 hours on Saturday.

153-8 x, number of cherry pies Mrs. Ray took to the bake sale.
y, number of lemon pies Mrs. Ray took to the bake sale.
$x + y = 7$.

x	1	2	3	4	5	6
y	6	5	4	3	2	1

{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 6) is: Mrs. Ray took 1 cherry pie and 6 lemon pies to the bake sale.

153-9 x, number of cents Harold spent.
y, number of cents Harold had left.
$10 - x = y$.

x	1	2	3	4	5	6	7	8	9
y	9	8	7	6	5	4	3	2	1

{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 9) is: Harold spent 1¢ and had 9¢ left.

Lesson 38

153-A No

153-B No

153-C Because the action in the problem is additive, not subtractive. Additive action is when two groups are put together. In this case, you should imagine that the records Sue has are put with the records Helen has. * The number of phonograph records Helen and Sue have together

153-D Because you know that both $r + s$ and 22 refer to the number of records Helen and Sue have together * Yes

153-E Because r, not s, is a variable for the number of records Helen has. Therefore, $r + 3$, not $s + 3$, refers to the number of records Helen would have if she had 3 more.

153-F Because you know that both $r + 3$ and 12 refer to the number of records Helen would have if she had 3 more * Yes

153-G Yes. The compound condition is made up of the two simple conditions for the problem and the connective "and."

153-H $N \times N$

153-I 9

153-J Yes * 9 is the only member of N that satisfies $r + 3 = 12$ and r is the first component of (r, s).

154-K (Column 1) 13

154-L (Column 1) {(9, 13)}

154-M The number of records Helen has * The number of records Sue has

154-N Yes * Yes

154-A (Column 1) No

154-B No

154-C Because you know that the number of years that is Karen's age now is greater than the number of years that is Jean's age now * Yes

154-D Because the action is subtractive not additive. To find Karen's age 5 years ago, you must subtract 5, not add 5, to the number of years that is Karen's age now. * To Karen's age 5 years ago

154-E Because you know that both $x - 5$ and 9 refer to the number of years that was Karen's age 5 years ago * Yes

154-F Yes. There are two simple conditions for the problem. These two conditions and the connective "and" make up the compound condition for the problem.

154-G C × C

154-H 14

154-I 14 is the only member of C that satisfies $x - 5 = 9$ and x is the first component of (x, y).

154-J 1 through 13

154-K (Column 2) {(14, 1), (14, 2), (14, 3), (14, 4), (14, 5), (14, 6), (14, 7), (14, 8), (14, 9), (14, 10), (14, 11), (14, 12), (14, 13)}

154-L (Column 2) Number of years that is Karen's age now * Number of years that is Jean's age now

154-A (Column 2) C × C

155-B Yes

155-C Yes

155-D $w + z < 24 \wedge w = 7 + 6$.

155-E 13

155-F 13

155-G 1 through 10

155-H {(13, 1), (13, 2), (13, 3), (13, 4), (13, 5), (13, 6), (13, 7), (13, 8), (13, 9), (13, 10)}

155-I The number of screens that Tom painted * The number of screens that Ed painted

155-J The second components

On your own

155-1 x, number of dollars Mrs. King paid for the stove.
y, number of dollars Mrs. King paid for the freezer.
$x + y = 455 \wedge x - 49 = 139$.
{(188, 267)}.
$188 for the stove and $267 for the freezer.
You may have chosen your variables so that the first component of the solution referred to the number of dollars Mrs. King paid for the freezer and the second component referred to the number of dollars Mrs. King paid for the stove. Then the solution set is {(267, 188)}. However, the answer is the same. When you check your answers, always notice what the variables refer to.

155-2 x, number of fossils Larry has.
y, number of fossils Tom has.
$x + y < 15 \wedge y > 10$.
{(1, 11), (2, 11), (3, 11), (1, 12), (2, 12), (1, 13)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 11) is: Larry has 1 fossil and Tom has 11. If you chose your variables so that the first components of members of the solution set refer to the number of fossils Tom has and the second components refer to the number of fossils Larry has, then the solution set you should obtain is {(11, 1), (11, 2), (11, 3), (12, 1), (12, 2), (13, 1)}.

155-3 x, number of years that is Jane's age now.
y, number of years that is Helen's age now.
$x > y \wedge x + 9 = 21$.
{(12, 1), (12, 2), (12, 3), (12, 4), (12, 5), (12, 6), (12, 7), (12, 8), (12, 9), (12, 10), (12, 11)}.
Jane is 12 years old and Helen is from 1 through 11 years old.

155-4 x, number of points Jay scored.

y, number of points Don scored.
$x > y \wedge x = 6 + 7$.
{(13, 0), (13, 1), (13, 2), (13, 3), (13, 4), (13, 5), (13, 6), (13, 7), (13, 8), (13, 9), (13, 10), (13, 11), (13, 12)}.
From 0 through 12 points. Notice that you are asked to find only the number of points Don could have scored. Therefore, only the second component of each solution is used to obtain an answer.

155-5 x, number of days spent in Alberta.
y, number of days spent in Saskatchewan.
$x + y = 11 \wedge x > 6$.
{(7,4), (8,3), (9,2), (10,1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (7,4) is 7 days in Alberta and 4 days in Saskatchewan.

155-6 x, number of aprons Mrs. Ross made.
y, number of potholders Mrs. Ross made.
$x - y = 6 \wedge x - 8 = 5$.
{(13, 7)}.
13 aprons and 7 potholders

155-7 x, number of boys who received a grade of A.
y, number of girls who received a grade of A.
$x + y < 8 \wedge x > 3$.
{(4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (6, 0), (6, 1), (7, 0)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (4, 0) is 4 boys and 0 girls.

155-8 x, number of sandwiches Sandy made.
y, number of sandwiches Ruth made.
$x + y = 48 \wedge y = 9 + 13$.
{(26, 22)}.
26 sandwiches. Notice that you are asked to find only the number of sandwiches Sandy made. Therefore, only the first component of the solution is used to obtain an answer.

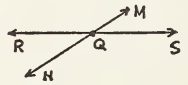
155-9 x, number of pages Beth read the first day.
y, number of pages Beth read the second day.
$x + y = 150 \wedge y < 7$.
{(149, 1), (148, 2), (147, 3), (146, 4), (145, 5), (144, 6)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (149, 1) is: Beth read 149 pages the first day and 1 page the second day.

156-10 x, number of miles that is the length of Lake Huron.
y, number of miles that is the length of Lake Superior.
$307 - x = 101 \wedge y - x = 144$.
{(206, 350)}.
Lake Huron is 206 miles long and Lake Superior is 350 miles long.

156-11 x, number of dollars spent by American tourists in Canada and Mexico.
y, number of dollars spent by American tourists in countries other than Mexico and Canada.
$x + y = 2{,}640{,}000{,}000 \wedge x = 380{,}000{,}000 + 365{,}000{,}000$.
{(745000000, 1895000000)}.
$1,895,000,000. Notice that you are asked to find only the number of dollars spent by American tourists in countries other than Mexico and Canada. Therefore, only the second component of the solution is used to obtain an answer.

Keeping Skilful

156-1 See the drawing below.



156-2 See the drawing above.

156-3 ∠RQM with vertex Q and sides QR and QM. ∠SQM with vertex Q and sides QM and QS. ∠NQS with vertex Q and sides QN and QS. ∠NQR with vertex Q and sides QN and QR.

156-4 Q * QR

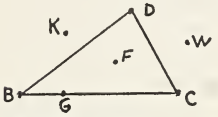
156-5 Line MN

156-6 ∠RQM and ∠MQS, ∠MQS and ∠SQN, ∠SQN and ∠NQR, ∠NQR and ∠RQM

156-7 ∠NQS or ∠RQM * ∠RQN or ∠MQS

156-8 Line RS

156–9 See the drawing below. B, C, and D may be located anywhere just so they are noncollinear.



156–10 $\overline{CB}$, $\overline{BD}$, and $\overline{DC}$

156–11 See the drawing for 156–9. K may be located anywhere in the interior of $\angle BCD$ just so it is in the exterior of $\triangle CBD$.

156–12 See the drawing for 156–9. F may be located anywhere in the interior of $\angle CDB$ just so it is also in the interior of $\angle CBD$.

156–13 See the drawing for 156–9. G may be located anywhere between B and C.

156–14 See the drawing for 156–9. W may be located anywhere in the interior of $\angle CBD$ just so it is also in the exterior of $\angle BCD$.

Lesson 39

156–A No

156–B No

156–C Because the action in the problem is additive, not subtractive. You should imagine that the trees Earl's father planted are put with the trees Earl planted. * The total number of trees Earl and his father planted

156–D Because you know that $x + y$ and 7 both refer to the number of trees Earl and his father planted in all * Yes

156–E Because x, not y, is the variable for the number of trees Earl planted. Therefore, $x + 3$, not $y + 3$, refers to the number of trees Earl would have planted if he had planted 3 more trees.

156–F Yes * The number of trees Earl's father planted

156–G Yes

157–H N × N

157–I Because if you replace x by a number greater than 7, there is no member of N that will satisfy the condition obtained. For example, if you replace x by 8, you obtain the condition $8 + y = 7$. There is no member of N that satisfies $8 + y = 7$.

157–J

x	0	1	2	3	4	5	6	7
y	7	6	5	4	3	2	1	0

157–K {(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)}

157–L (2, 5)

157–M {(2, 5)}

157–N The number of trees Earl planted * The number of trees Earl's father planted

157–O Yes * Yes

157–A No

157–B No

157–C Because the action is additive, not subtractive. To find the total number of boys and girls in the cast, you must add, not subtract. $m + n$ refers to the total number of boys and girls in the cast.

157–D Because the total number of boys and girls in the cast is less than 9 * Yes

157–E Because you know that there are more boys than girls, not more girls than boys, in the cast

157–F Yes

158–G (Column 1) N × N

158–H If a number greater than 8 is used as a replacement for m, you obtain a condition that cannot be satisfied by a member of N.

158–I The numbers from 0 through 8

158–J

m	0	1	2	3	4	5	6	7	8
n	0	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	1	
	2	2	2	2	2	2	2		
	3	3	3	3	3	3			
	4	4	4	4	4				
	5	5	5	5					
	6	6	6						
	7	7							
	8								

158–K (1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2), (5, 3), (6, 0), (6, 1), (6, 2), (7, 0), (7, 1), (8, 0)

158–L {(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2), (5, 3), (6, 0), (6, 1), (6, 2), (7, 0), (7, 1), (8, 0)}

158–M Number of boys in the cast * Number of girls in the cast

158–N If there are 4 boys in the cast, there are 0 through 3 girls in the cast. If

there are 5 boys in the cast, there are 0 through 3 girls in the cast. If there are 6 boys in the cast, there are 0, 1, or 2 girls in the cast. If there are 7 boys in the cast, there is either 0 or 1 girl in the cast. If there are 8 boys in the cast, there are no girls in the cast.

158-A No

158-B No

158-C Yes * Because the action is additive, not subtractive. To find the total number of home runs Bob and Jack hit, you must add, not subtract. * Because you know that the total number of home runs Bob and Jack hit together is less than 8

158-D Since a, not b, is the variable for the number of home runs Bob hit, $a - 5$, not $b - 5$, refers to the number of home runs Bob would have hit if he had hit 5 fewer home runs.

158-E Yes * The number of home runs Jack hit

158-F Yes

158-G (Column 2) N × N

159-H If a number greater than 7 is used to replace a in $a + b < 8$, the condition obtained cannot be satisfied by a member of N.

159-I The numbers from 0 through 7

159-J

a	0	1	2	3	4	5	6	7
b	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	
	2	2	2	2	2	2		
	3	3	3	3	3			
	4	4	4	4				
	5	5	5					
	6	6						
	7							

159-K Each solution of the compound condition must satisfy each of the simple conditions. You know that the replacements for a are the first components of the members of the solution set. If a number less than 5 is used to replace a in $a - 5 = b$, the condition obtained cannot be satisfied by a member of N. For example, if a is replaced by 4, the condition obtained is $4 - 5 = b$. There is no member of N that satisfies $4 - 5 = b$. If a is replaced by a number that is equal to, or greater than, 5, there is a member of N that will satisfy the condition obtained. Therefore, each member of the solution set must have a first component that is equal to, or greater than, 5.

159-L {(5, 0), (6, 1)}

159-M The number of home runs Bob hit * The number of home runs Jack hit

159-N If Bob hit 5 home runs, Jack hit none. If Bob hit 6 home runs, Jack hit 1.

On your own

159-1 x, number of correct answers.
y, number of incorrect answers.
$x + y = 8 \wedge x > y$.
{(5, 3), (6, 2), (7, 1), (8, 0)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (5, 3) is 5 correct answers and 3 incorrect answers.

159-2 x, number of cents spent for the pencil.
y, number of cents spent for the eraser.
$x + y < 5 \wedge x < y$.
{(1, 2), (1, 3)}.
Sarah spent 1¢ for the pencil and either 2¢ or 3¢ for the eraser.

159-3 x, number of senators Nova Scotia has.
y, number of senators Newfoundland has.
$x + y = 16 \wedge x - y = 4$.
{(10, 6)}.
Nova Scotia has 10 senators and Newfoundland has 6 senators.

159-4 x, number of 4-cent stamps.
y, number of 5-cent stamps.
$x + y < 15 \wedge x = y$.
{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (6, 6) is six 4-cent stamps and six 5-cent stamps.

159-5 x, number of votes Dale received.

y, number of votes Harvey received.
$x + y = 21 \wedge x - y = 11$.
$\{(16, 5)\}$.
Dale received 16 votes and Harvey received 5 votes.

159-6 x, number of boxes of candy Jane sold.
y, number of boxes of candy Alice sold.
$x + y < 9 \wedge y - 4 = x$.
$\{(0, 4), (1, 5), (2, 6)\}$.
Each member of the solution set may be used to obtain an answer. The answer obtained from (0, 4) is: Jane sold 0 boxes of candy and Alice sold 4 boxes.

159-7 x, number of home runs Babe Ruth hit during the 1917 season.
y, number of home runs Babe Ruth hit during the 1918 season.
$x + y = 13 \wedge y - x = 9$.
$\{(2, 11)\}$.
2 home runs during the 1917 season and 11 home runs during the 1918 season

159-8 x, number of newspapers Henry sold.
y, number of newspapers Jim sold.
$x + y < 21 \wedge x = y$.
$\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), (10, 10)\}$.
Each member of the solution set may be used to obtain an answer. The answer obtained from (4, 4) is: Henry sold 4 newspapers and Jim sold 4 newspapers.

159-9 x, number of points scored by Rough Riders.
y, number of points scored by Eskimos.
$x + y = 22 \wedge x - y = 10$.
$\{(16, 6)\}$.
The Rough Riders scored 16 points and the Eskimos scored 6 points.

Lesson 40

160-A Yes

160-B $\{(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)\}$

160-C Yes * Yes * (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)

160-D Yes * Yes

160-E Yes

160-F $\{(0, 8), (1, 7), (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)\}$

160-G The second component of each of the ordered pairs you are to find must be less than the product of 2 and the first component. * (3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)

160-H $\{(3, 5), (4, 4), (5, 3), (6, 2), (7, 1), (8, 0)\}$

160-I Yes. When problems concern numbers only, you can use ordered pairs as answers to the problems.

160-J - - -

161-K Yes

161-L 8

161-M Each solution of the compound condition must satisfy both of the simple conditions. This means that the first component of each member of the solution set must satisfy $x - 3 = 5$. The only member of N that satisfies $x - 3 = 5$ is 8.

161-N The first component of each ordered pair you are to find must be less than the second component.

161-O Yes * You know that 8 is the first component of each solution of $x < y \wedge x - 3 = 5$. You also know that the first component of each solution must be less than the second component. Since the first component of each solution is 8, the second component of each solution must be greater than 8. The set of natural numbers greater than 8 is an infinite set. Therefore, there are infinitely many ordered pairs that have a first component of 8 and a second component greater than 8.

161-P Yes * Examples are (8, 10), (8, 25), (8, 1500).

161-Q $\{(x, y) \mid x < y \wedge x - 3 = 5\}$

161-R - - -

161-S Yes

161-T $\{(0, 10), (1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1), (10, 0)\}$

161-U No * No

161-V $\{\ \}$

161-W No

On your own

162-1 $a + b = 15$.
$\{(0, 15), (1, 14), (2, 13), (3, 12), (4, 11), (5, 10), (6, 9), (7, 8), (8, 7),$

(9, 6), (10, 5), (11, 4), (12, 3), (13, 2), (14, 1), (15, 0)}

162–2 $a + b < 20 \wedge a = b$.
{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9)}

162–3 $a + b = 78 \wedge a - 15 = 26$.
{(41, 37)}

162–4 $a + b = 10 \wedge a > b$.
{(6, 4), (7, 3), (8, 2), (9, 1), (10, 0)}

162–5 $b - a = 43 \wedge a + 58 = 321$.
{(263, 306)}

162–6 $a - b = 5 \wedge 8 - a = 2$.
{(6, 1)}

162–7 $a - 7 = 4 \wedge a = 3b$.
{ }

162–8 $a < 4 \wedge b + a < 5$.
{(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1)}

162–9 $a > 9(4) \wedge b + a > 40$.
$\{(a, b) \mid a > 9(4) \wedge b + a > 40\}$

162–10 $a - b = 9 \wedge a + b = 21$.
{(15, 6)}

162–11 $a < b \wedge 13 - b = 8$.
{(0, 5), (1, 5), (2, 5), (3, 5), (4, 5)}

162–12 $a = 645 \div 129 \wedge a - b > 6$.
{ }

162–13 $a + b = 21 \wedge b = 2a$.
{(7, 14)}

Applying mathematics

162–1 x, number of new cars dealer had to begin with.
$x > 17 \wedge x - 8 < 12$.
{18, 19}.
18 or 19 new cars

162–2 x, number of boys.
y, number of girls.
$x + y = 9$.
{(1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 8) is 1 boy and 8 girls.

162–3 x, number of dollars Mr. Foster had in his checking account before he wrote the check for $186.
$x - 186 = 418$.
{604}. $604

162–4 x, number of years that is Carol's age now.
y, number of years that is Ann's age now.
$y - x = 5 \wedge x + 14 = 21$.
{(7, 12)}. Carol is 7 years old and Ann is 12 years old.

162–5 x, number of yards of silk cloth.
y, number of yards of cotton cloth.
$x + y < 5$.
{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 3) is 1 yard of silk cloth and 3 yards of cotton cloth.

162–6 x, number of acres Bill plowed.
y, number of acres Bill's father plowed.
$x + y < 28 \wedge x = 6 + 5$.
{(11, 1), (11, 2), (11, 3), (11, 4), (11, 5), (11, 6), (11, 7), (11, 8), (11, 9), (11, 10), (11, 11), (11, 12), (11, 13), (11, 14), (11, 15, (11, 16)}.
From 1 through 16 acres

162–7 x, number of dolls Rachel has now.
$x > 12 \wedge x - 5 < 11$.
{13, 14, 15}. 13, 14, or 15 dolls

163–8 x, number of boys on the staff.
y, number of girls on the staff.
$x + y < 23 \wedge x = y$.
{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9), (10, 10), (11, 11)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (7, 7) is 7 boys and 7 girls.

163–9 x, number of suits Mr. Baker sold.
y, number of overcoats Mr. Baker sold.
$x + y = 7 \wedge y < x$.
{(4, 3), (5, 2), (6, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (5, 2) is 5 suits and 2 overcoats.

163–10 x, number of magazines Andy sold.
y, number of magazines Peter sold.
$x + y = 22 \wedge x - y = 8$.
{(15, 7)}. Andy sold 15 magazines and Peter sold 7 magazines.

163-11 x, number of stamps Jim sold.
$427 - x > 369$.
$\{1, 2, 3, \ldots, 57\}$. 1 through 57 stamps

163-12 x, number of years that is Beverly's age now.
$x + 9 > 22 \wedge x - 6 < 10$.
$\{14, 15\}$. 14 or 15 years old

163-13 x, number of feet per second faster that sound travels in hard wood than in air.
$12620 - 1088 = x$.
$\{11532\}$. 11,532 feet per second

163-14 x, number of cents Susan spent.
y, number of cents Susan had left.
$91 - x = y \wedge x = 12 + 25$.
$\{(37, 54)\}$. 54¢

163-15 x, number of phonograph records Judy has now.
$x + 15 > 37 \wedge x - 12 < 15$.
$\{23, 24, 25, 26\}$. From 23 through 26 phonograph records

163-16 x, number of persons that was the total population of Tokyo and London.
$9,311,774 + 8,222,340 = x$.
$\{17534114\}$. The total population was 17,534,114.

163-17 x, number of experiments Elizabeth has finished.
y, number of experiments Nancy has finished.
$x > y \wedge x + 8 = 21$.
$\{(13, 0), (13, 1), (13, 2), (13, 3), (13, 4), (13, 5), (13, 6), (13, 7), (13, 8), (13, 9), (13, 10), (13, 11), (13, 12)\}$. Elizabeth has finished 13 experiments. Nancy has finished from 0 through 12 experiments.

163-18 x, number of persons who entered Canada as immigrants in 1956.
$x - 71,719 = 93,138$.
$\{164857\}$.
164,857 persons

163-19 x, number of stamps John had to begin with.
$x + 13 < 145 \wedge x + 35 > 161$.
$\{127, 128, 129, 130, 131\}$. From 127 through 131 stamps

163-20 x, number of sq. mi. that is the area of Kootenay National Park.
y, number of sq. mi. that is the area of Elk Island National Park.
$x - 203 = 340 \wedge x - y = 468$.
$\{(543, 75)\}$. The area of Kootenay Park is 543 sq. mi., and that of Elk Island Park is 75 sq. mi.

163-21 $x + y = 9$.
$\{(0, 9), (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1), (9, 0)\}$

163-22 $x > 466(238)$.
$\{110909, 110910, 110911, \ldots\}$

163-23 $x > 78 \wedge 75 + x < 160$.
$\{79, 80, 81, \ldots, 84\}$

164-24 $x + y < 17 \wedge x = 2y$.
$\{(0, 0), (2, 1), (4, 2), (6, 3), (8, 4), (10, 5)\}$

164-25 $y - x = 5 \wedge x < 16$.
$\{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10), (6, 11), (7, 12), (8, 13), (9, 14), (10, 15), (11, 16), (12, 17), (13, 18), (14, 19), (15, 20)\}$

Checking up

Test 36

164-1 F

164-2 T

164-3 T

164-4 F

164-5 T

164-6 F

164-7 $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)\}$

164-8 $\{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4), (7, 2), (7, 4)\}$

164-9 $X = \{0, 1, 2, 3, 4, 5\}$.

164-10 $Y = \{0, 1, 2, 3, 4\}$.

164-11 (0, 1)

164-12 E

164-13 C

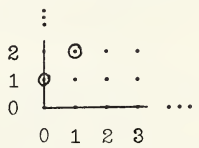
164-14 $\{(1, 0), (2, 1), (3, 2), (4, 3), (5, 4)\}$

Test 37

164-15 $\{(a, b) \mid b = a + 1\}$

164-16 $\{(0, 1), (1, 2)\}$

164-17



164-18 See the graph above. The encircled dots represent points associated with solutions of $b = a + 1$.

164-19

$x + y < 6$.

x	0	1	2	3	4	5
y	0	0	0	0	0	0
	1	1	1	1	1	
	2	2	2	2		
	3	3	3			
	4	4				
	5					

{(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (5, 0)}

164-20 {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9)}

164-21 {(3, 6)}

164-22 {(0, 0), (0, 1), (1, 1), (0, 2), (1, 2), (2, 2), (0, 3), (1, 3), (2, 3)}

164-23 { }

164-24 {(0, 2), (1, 6), (2, 10), (3, 14), (4, 18), (5, 22), (6, 26)}

Test 38

165-25 x, number of sandwiches Mary ate.
y, number of sandwiches Donna ate.
$x + y = 4$.
{(1, 3), (2, 2), (3, 1)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (1, 3) is: Mary ate 1 sandwich and Donna ate 3 sandwiches.

165-26 x, number of cupcakes Barbara took to the sale.
y, number of cupcakes Kathy took to the sale.
$x + y = 18 \wedge x - y = 4$.
{(11, 7)}. Barbara took 11 cupcakes and Kathy took 7 cupcakes.

165-27 x, number of inches of snow that fell in January.
y, number of inches of snow that fell in February.
$x + y = 8 \wedge y < 5$.
{(4, 4), (5, 3), (6, 2), (7, 1)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (5, 3) is: 5 inches of snow fell during January and 3 inches of snow fell during February.

165-28 x, number of hits Henry had.
y, number of hits Steve had.
$x + y = 6 \wedge x > y$.
{(6, 0), (5, 1), (4, 2)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (4, 2) is: Henry had 4 hits and Steve had 2 hits.

165-29 x, number of steers the farmer has.
y, number of hogs the farmer has.
$x - y = 32 \wedge x + 47 = 172$.
{(125, 93)}.
125 steers and 93 hogs

165-30 $x + y < 12 \wedge y = 2x$.
{(0, 0), (1, 2), (2, 4), (3, 6)}

165-31 $y - x = 7 \wedge x + 2 = 11$.
{(9, 16)}

165-32 $x + y < 5$.
{(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (4, 0)}

165-33 $y - x = 9 \wedge x < 7$.
{(0, 9), (1, 10), (2, 11), (3, 12), (4, 13), (5, 14), (6, 15)}

Checking up

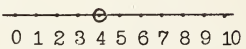
Test 39

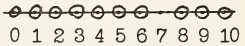
165-1 F
165-2 T
165-3 F
165-4 F
165-5 F
165-6 F
165-7 T
165-8 intersection
165-9 the empty set, or { }
165-10 $\overleftrightarrow{BC}$
165-11 intersection, or $\cap$
165-12 $\overline{JL}$
165-13 $\overleftrightarrow{RS}$

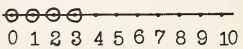
Test 40

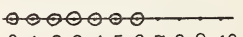
166-14 Diagram C
166-15 Diagram A
166-16 Diagram C
166-17 Diagram A
166-18 Diagram C
166-19 Diagram B
166-20 Diagram A
166-21 Diagram C
166-22 Diagram C

Test 41

166-23 

166-24 

166-25 

166-26 

166-27 {(50, 8)}
166-28 { }

166-29 {(1, 0), (1, 1), (2, 0), (2, 1), (2, 2), (2, 3), (3, 0), (3, 1), (3, 2), (3, 3), (4, 0), (4, 1), (4, 2), (5, 0), (5, 1), (6, 0)}

166-30 {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5)}

Test 42

166-31 x, number of words in the old constitution.
$2670 - x = 875$.
{1795}. 1795 words

166-32 x, number of boys.
y, number of girls.
$x + y = 5$.
{(0, 5), (1, 4), (2, 3), (3, 2), (4, 1), (5, 0)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (2, 3) is 2 boys and 3 girls.

166-33 x, number of pennants the Pep Club had to begin with.
$x < 32 \wedge x + 18 > 40$.
{23, 24, 25, ..., 31}.
From 23 through 31 pennants

166-34 x, number of points scored by Bill's opponent.
$21 + x < 38$.
{0, 1, 2, ..., 16}.
From 0 through 16 points

166-35 x, number of times Chuck caddied.
y, number of times Will caddied.
$x + y = 15 \wedge y > 9$.
{(5, 10), (4, 11), (3, 12), (2, 13), (1, 14)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (3, 12) is: Chuck caddied 3 times and Will caddied 12 times.

166-36 x, number of boys.
y, number of girls.
$x + y < 20 \wedge x - y = 8$.
{(9, 1), (10, 2), (11, 3), (12, 4), (13, 5)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (10, 2) is 10 boys and 2 girls.

Test 43

166-37 1216

166-38 49,686

166-39 36,585

166-40 408,240

166-41 185,136

166-42 66,688

166-43 437,579

166-44 9

166-45 74

166-46 43

166-47 34

166-48 3008

166-49 140

166-50 305

Lesson 41

167-A The first component

167-B The second component

167-C Yes

167-D Yes

167-E Yes

168-F Yes

168-G Yes

168-H Yes

168-I There are many ordered pairs that you can use. Some of these are (8, 32), (10, 40), (11, 44), (12, 48), and (13, 52).

168-J No * For any number of pencils, there is an ordered pair in which the second component is the number of cents paid for the pencils. Therefore, there are infinitely many ordered pairs that represent the rate at which the pencils are sold.

168-K $1 \times 8 * 2 \times 4$

168-L Yes * Yes

168-M Yes * The first component of (4, 16) and the second component of (3, 12)

168-N Yes * Yes

168-O The first component of one ordered pair and the second component of the other ordered pair * Yes

169-P Because $1 \times 8 = 2 \times 4$ is a true statement * Because $3 \times 16 = 4 \times 12$ is a true statement

169-Q Yes * $2 \times 12 = 3 \times 8$ is a true statement.

169-R Equivalent ordered pairs because $55 \times 2 = 5 \times 22$ is a true statement

169-S Equivalent ordered pairs because $9 \times 72 = 54 \times 12$

is a true statement

169-T Not equivalent ordered pairs because $5 \times 54 = 35 \times 9$ is a false statement

169-U Equivalent ordered pairs because $2 \times 30 = 20 \times 3$ is a true statement

169-V Not equivalent ordered pairs because $1 \times 42 = 7 \times 7$ is a false statement

169-W Equivalent ordered pairs because $63 \times 5 = 7 \times 45$ is a true statement

169-X Not equivalent ordered pairs because $27 \times 1 = 9 \times 9$ is a false statement

169-Y Equivalent ordered pairs because $4 \times 78 = 24 \times 13$ is a true statement

169-A No * To decide if (a, b) and (c, d) are equivalent ordered pairs, you must decide whether or not $a \times d = c \times b$ is a true statement. But $a \times d = c \times b$ is a condition, not a statement, and a condition is neither true nor false.

169-B The first component of (a, b) and the second component of (c, d) * By multiplying the first component of (c, d) and the second component of (a, b)

169-C C * a has been replaced by 2 and b by 6. * c has been replaced by 3 and d by 9.

169-D $2 \times 9 = 3 \times 6$ is obtained from $a \times d = c \times b$ by making replacements for the variables. * Yes * Because $2 \times 9 = 3 \times 6$ is a true statement

169-E a must be replaced by 1, b by 4, c by 4, and d by 16. * $1 \times 16 = 4 \times 4$. * Yes * Yes. Since (1, 4) and (4, 16) are equivalent, you can say that (1, 4) is equivalent to (4, 16) and that (4, 16) is equivalent to (1, 4).

169-F No * (2, 5) and (4, 7) are not equivalent ordered pairs because $2 \times 7 = 4 \times 5$ is a false statement.

169-G Yes. Since $2 \times 25 = 10 \times 7$ is a true statement, (2, 7) and (10, 35) are equivalent ordered pairs. * Yes. Since (2, 7) and (10, 35) are equivalent ordered pairs, you know that each of them is equivalent to the other.

170-H False statement because $3 \times 10 = 4 \times 9$ is a false statement

170-I True statement because $1 \times 63 = 9 \times 7$

170-J True statement because $3 \times 3 = 1 \times 9$

170-K False statement because $1 \times 1 = 7 \times 7$ is a false statement

170-L Four

170-M Yes. $1 \times 4 = 2 \times 2$. * Yes. $1 \times 6 = 3 \times 2$. * Yes. $1 \times 8 = 4 \times 2$.

170-N You already know that (1, 2) and (2, 4) are equivalent ordered pairs. This means that each of two ordered pairs is equivalent to the other. * Since you already know that (1, 2) and (2, 4) are equivalent ordered pairs as well as that (1, 2) and (3, 6) are equivalent ordered pairs, you know that (2, 4) and (3, 6) are equivalent ordered pairs. Since you already know that (1, 2) and (2, 4) are equivalent ordered pairs as well as that (1, 2) and (4, 8) are equivalent ordered pairs, you know that (2, 4) and (4, 8) are equivalent ordered pairs.

170-O You know that (1, 2) is equivalent to each of the other ordered pairs in set B including (3, 6). This means that (3, 6) must be equivalent to each of the other ordered pairs in set B. The same is true for (4, 8).

170-P Yes

170-Q Yes * Any ordered pair is equivalent, as well as equal, to itself. For example: you know that (2, 4) is equivalent to (2, 4) because $2 \times 4 = 2 \times 4$.

170-R Yes * Yes * Yes

170-S Yes * Since you know that one member of M is equivalent to each of the other members of M, you know that each member of M is equivalent to each of the other members. A set of ordered pairs in which each member is equivalent to each of the other mem-

bers is a proportional relation.

170-T No * If you can find two members of a set of ordered pairs that are not equivalent, then the set is not a proportional relation. In set G, you know that (1, 1) is not equivalent to either (1, 2) or (2, 1).

170-A Yes. $12 \times 8 = 6 \times 16$. * Yes. $3 \times 8 = 6 \times 4$.

170-B 2

170-C Multiply the second component of (6, 8) by 2.

170-D 2

170-E 2

170-F Divide the second component of (6, 8) by 2.

171-G (Column 1) 2

171-H (Column 1) Yes * $3 \times 16 = 12 \times 4$ is a true statement.

171-I (Column 1) Multiply each component of (6, 8) by 3.

171-J (Column 1) Multiply each component of (6, 8) by 7.

171-K (Column 1) Multiply each component of (6, 8) by 11.

171-L (Column 1) Multiply each component of (6, 8) by 8.

171-M (Column 1) Multiply each component of (6, 8) by 2.

171-N (Column 1) Multiply each component of (6, 8) by 12.

171-O (Column 1) Yes. You know that (6, 8) is equivalent to (6, 8). For each replacement of x by a member of C, true statements are obtained from both $(6x, 8x) \sim (6, 8)$ and $6x \times 8 = 6 \times 8x$.

171-P Divide each component of (120, 160) by 10.

171-Q Divide each component of (120, 160) by 20.

171-R Divide each component of (120, 160) by 40.

171-S Divide each component of (120, 160) by 8.

171-T Divide each component of (120, 160) by 4.

171-U Divide each component of (120, 160) by 2.

171-A C

171-B Multiply the first component of (5, 8) by 6.

171-C Multiply the second component of (5, 8) by 6. * 48

171-D Yes. $5 \times 48 = 30 \times 8$ is a true statement.

171-E - - -

171-F Divide the second component of (10, 24) by 2.

171-G (Column 2) Divide the first component of (10, 24) by 2. * 5

171-H (Column 2) Yes. $10 \times 12 = 5 \times 24$ is a true statement.

171-I (Column 2) 4

171-J (Column 2) 48

171-K (Column 2) 72

171-L (Column 2) 64

171-M (Column 2) 12

171-N (Column 2) 2

171-O (Column 2) For exercise I, you divide the first component of (16, 24) by 4. For exercise J, you multiply the second component of (16, 24) by 2. For exercise K, you multiply the second component of (16, 24) by 8. For exercise L, you multiply the first component of (16, 24) by 4. For exercise M, you divide the second component of (16, 24) by 2. For exercise N, you divide the first component of (16, 24) by 8.

On your own

171-1a No

171-1b Yes

171-1c Yes

171-1d No

171-1e Yes

171-1f Yes

171-2 $5 \times 16 = 10 \times 8$ is a true statement.

172-3 $10 \times 72 = 45 \times 16$ is a true statement.

172-4 $5 \times 8 = 10 \times 8$ is a false statement.

172-5 $(2, 3) \sim (4, 6)$.

172-6 $(3, 2) \sim (6, 4)$.

172-7 $(2, 1) \sim (6, 3)$.

172-8 $(1, 2) \sim (3, 6)$.

172-9 $(5, 4) \sim (10, 8)$.

172-10 $(4, 5) \sim (8, 10)$.

172-11 $(4, 3) \sim (12, 9)$.

172-12 $(3, 2) \sim (9, 6)$.

172-13 $(3, 4) \sim (6, 8)$.

172-14 Not a proportional relation. (60, 3) and (3, 60) are not equivalent ordered pairs.

172-15 A proportional relation. Each member is equivalent to each of the other members.

172-16 A proportional relation. Each member is equivalent to each of the other members.

172-17 A proportional relation. Each member is equivalent to each of the other members.

172-18 Not a proportional relation. (100, 7) and (200, 7) are not equivalent ordered pairs.

172-19 Not a proportional relation. Each member is not equivalent to each of the other members.

172-20 Multiply each component of (5, 3) by 5. * $25 \times 3 = 5 \times 15$ is a true statement.

172-21 Divide each component of (50, 30) by 2. * $25 \times 30 = 50 \times 15$ is a true statement.

172-22 12

172-23 150

172-24 10

172-25 21

172-26 26

172-27 84

172-28 12

172-29 5

172-30 10

172-31 24

172-32 20

172-33 6

Keeping Skilful

172-1 Closed sentence

172-2 Open sentence

172-3 Open sentence

172-4 Closed sentence

172-5 Open sentence

172-6 Open sentence

172-7 Open sentence

172-8 Closed sentence

172-9 Open sentence

172-10 Closed sentence

172-11 The sentence in exercises 4 and 10 express equations. The sentences in exercises 1 and 8 express inequalities.

172-12 The sentences in exercises 2, 5, and 7 express conditions for equality. The sentences in exercises 3, 6, and 9 express conditions for inequality.

172-13 {(0, 1), (0, 3), (0, 5), (0, 7), (2, 1), (2, 3), (2, 5), (2, 7), (4, 1), (4, 3), (4, 5), (4, 7)}

172-14 {(0, 1), (0, 3), (0, 5), (0, 7), (2, 1), (2, 3), (2, 5), (2, 7)}

172-15 {(0, 7), (2, 5), (4, 3)}

172-16 {(0, 7), (2, 5), (2, 7), (4, 3), (4, 5), (4, 7)}

172-17 {(0, 1), (0, 3), (0, 5), (0, 7), (2, 3), (2, 5), (2, 7), (4, 5), (4, 7)}

172-18 {(0, 5), (0, 7), (2, 7)}

172-19 {(0, 1), (2, 3), (4, 5)}

172-20 {406}

172-21 {0, 1, 2, ..., 30}

172-22 {39}

172-23 {0, 1, 2, ..., 16}

172-24 {99, 100, 101, ...}

172-25 {0, 1, 2, ...}

172-26 {27}

172-27 {0, 1, 2, ..., 12}

172-28 {(141, 262)}

172-29 {(0, 191), (1, 191), (2, 191), (3, 191), (4, 191), (5, 191), (6, 191), (7, 191), (8, 191)}

172-30 {(0, 2), (1, 3), (2, 4), (3, 5)}

172-31 {(304, 221)}

172-32 {(1, 0), (2, 0), (2, 1)}

Lesson 42

173-A Yes * It is a set of ordered pairs in which each member is equivalent to each of the other members.

173-B Yes * A member of a proportional relation is a rate pair.

173-C Six to five

173-D Four to fifteen

173-E Six to nine

173-F One hundred to one hundred six

173-G Seven to twenty-one

173-H The slant mark separates the names of the first and second components. The first component is named to the left of the slant mark. * The second component is named to the right of the slant mark.

173-I {1/15, 2/30, 3/45, 4/60, 5/75}

174-J Yes

174-K What numbers must be multiplied before you can tell if sentence A expresses a true statement

174-L Yes

174-M Yes

174-N Sentence A contains the ratios 6/27 and 2/9. B contains 5/9 and 15/21. C contains 3/5 and 12/20. D contains 10/1 and 50/5. E contains 2/13 and 3/18. F contains 7/9 and 28/36.

174-O A, C, D, F * Because of the definition of equivalent ordered pairs. For example, $6/27 \sim 2/9$ is a true statement because $6 \times 9 = 2 \times 27$.

174-A Yes * Because of the definition of equivalent ordered pairs

174-B No

174-C No * There are infinitely many rate pairs that are equivalent to 4/7.

174-D – – –

174-E Yes * Yes * Yes * Examples are 100/175, 32/56, 40/70, 800/1400, 4000/7000.

174-F No * Yes

174-G With the use of the definition of equivalent ordered pairs and with this one member, you can tell which objects are members of the proportional relation and which objects are not members.

174-H Yes * {8/14, ...} is the set whose members are the rate pair 8/14 and all rate pairs equivalent to 8/14. Since 4/7 is equivalent to 8/14, 4/7 is a member of {8/14, ...}

175-I Yes. You know that 4/7 is a member of the proportional relation {8/14, ...}, and you also know that each member of a proportional relation is equivalent to each of the other members. * Yes. Since 8/14 and 4/7 are equivalent rate pairs, {8/14, ...} and {4/7,...} are the same set. * Yes. The rate pairs equivalent to 100/175 are the same as the rate pairs equivalent to 4/7.

175-J C × C

175-K Yes * Yes * Yes * Yes

175-L Because there are infinitely many rate pairs equivalent to 4/7 * Yes

175-M Yes. Each member of $\{(x,y) \mid x/y \sim 4/7\}$ is equivalent to 4/7. Each member of $\{(x,y) \mid x/y \sim 8/14\}$ is equivalent to 8/14. Each ordered pair equivalent to 4/7 is also equivalent to 8/14, and each ordered pair equivalent to 8/14 is also equivalent to 4/7.

175-N The set whose members are the ordered pairs x,y that satisfy the condition that x to y is equivalent to seven to fifteen

175-O The set whose members are the ordered pairs x,y that satisfy the condition that x to y is equivalent to ten to one

175-P The set whose members are the ordered pairs r,s that satisfy the condition that r to s is equivalent to one to one hundred

175-Q The set whose members are the ordered pairs m,n that satisfy the condition that m to n is equivalent to n to m

175-R (1, 3), (2, 6), (3, 9) * Yes

175-S Yes * Examples are (4, 12), (5, 15), (6, 18).

On your own

176-1a One to six

176-1b Two to twelve

176-1c Three to one

176-1d Eight to five

176-1e One hundred to one

176-2 {1/1, 2/2, 3/3, 4/4, 5/5}

176-3 {9/1, 18/2, 27/3, 36/4}

176-4 {10/3, 20/6}

176-5 False statement

176-6 True statement

176-7 True statement

176-8 True statement

176-9 True statement

176-10 True statement

176-11 False statement

176-12 False statement

For each of the next five exercises, you could have used any letters you wished as names for the variables. You could also have used any rate pair that is equivalent to the rate pair named in each exercise when you tabulated and described a set. For example, you could have written, for exercise 14, "{1/5, ...} = $\{(a, b) \mid a/b \sim 1/5\}$."

176-13 {3/8, ...} = $\{(x,y) \mid x/y \sim 3/8\}$.

176-14 {5/25, ...} = $\{(x,y) \mid x/y \sim 5/25\}$.

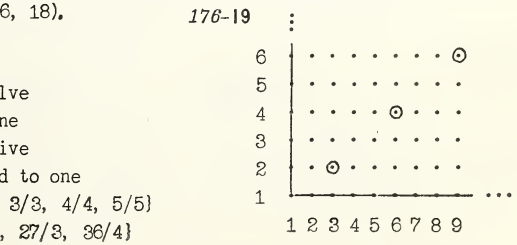
176-15 {1/4, ...} = $\{(x,y) \mid x/y \sim 1/4\}$.

176-16 {2/2, ...} = $\{(x,y) \mid x/y \sim 2/2\}$.

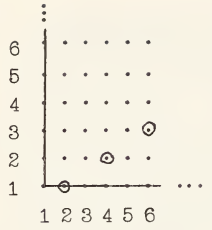
176-17 {4/17, ...} = $\{(x,y) \mid x/y \sim 4/17\}$.

176-18 Each of these proportional relations have infinitely many members. Examples of five others for exercise 13 are: 6/16, 8/24, 12/32, 15/40, 18/48.

176-19



176-20



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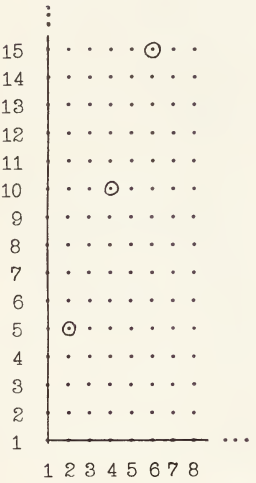
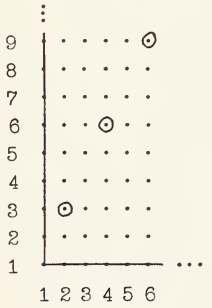


176-22



Lesson 43

176-A 8 dollars * Yes * Number of plants * Number of dollars

176-B You know that the first component of 16/8 refers to the number of plants, and since 4 is the number of plants for which you want to find the cost, 4 must be the first component of the rate pair you are to find.

176-C Yes * Number of dollars that is the cost of 4 plants

177-D What components to multiply to help find the replacement for x that will make a true statement from $16/8 \sim 4/x$. * By using the definition of equivalent ordered pairs

177-E Yes

177-F 16

177-G Yes * Yes

177-H Yes

177-I Yes

177-J Number of dollars * Number of plants

177-K Because now the second component of 8/16 refers to the number of plants

177-L Yes * Because x is the variable for the number of dollars that is the cost of 4 plants, and the first component of 8/16 refers to a number of dollars * For the number of dollars that is the cost of 4 plants

178-M Yes

178-N Divide 32 by 16. * {2}

178-O {2} * {2} * Yes. The two sets contain the same member.

178-P A number of plants

178-Q 2 dollars

178-R Yes

178-A By using the definition of equivalent ordered pairs

178-B 480 * Divide 480 by 15.

178-C 32

178-D {32}

178-E {32} * Because $15x = 480$ was obtained from $48/15 \sim x/10$

178-F 32/10

178-G $10/32 * 10/32 \sim 15/48$.

178-H By using the definition of equivalent ordered pairs * Divide 4284 by 17.

178-I {252}

178-J 252/63

178-K 63/252

On your own

179-1 {6}

179-2 {6}

179-3 {1}

179-4 {30}

179-5 {50}

179-6 {10}

179-7 {135}

179-8 {5}

179-9 {250}

179-10 {30}

179-11 {24}

179-12 {28}

179-13 {26}

179-14 {10}

179-15 {60}

179-16 {44}

179-17 {4}

179-18 {32}

179-19 {5}

179-20 {40}

179-21 {84}

179-22 {102}

179-23 {90}

179-24 {12}

Keeping Skilful

179-1 6642

179-2 12,051

179-3 159,025

179-4 198,584

179-5 76,426

179-6 3287

179-7 1129
179-8 1203
179-9 64,656
179-10 6554
179-11 51,273
179-12 229,215
179-13 3332
179-14 2641
179-15 165,517
179-16 240,786
179-17 449,580
179-18 96, remainder 36
179-19 173, remainder 20
179-20 710, remainder 12
179-21 42, remainder 110
179-22 178, remainder 135
179-23 260, remainder 0

Checking up

Test 44

179-1 6/1
179-2 2/7
179-3 3/2
179-4 3/7 ~ 12/24.
179-5 2/9 ~ 4/18.
179-6 5/8 ~ 10/80.
179-7 4/6 ~ 16/24.
179-8 Sentences in exercises 5 and 7
179-9 3(24) = 12(7). 2(18) = 4(9). 5(80) = 10(8) 4(24) = 16(6).
179-10 The sentences that express the statements obtained using the replacements named in exercises 5 and 7

Test 45

179-11 6/12
179-12 30/60
179-13 12/28
179-14 15/9
179-15 6/15
179-16 35/21
179-17 {72}
179-18 {105}
179-19 {6}

Lesson 44

180-A Number of blocks Tim walks
180-B Since the first component of 6/8 refers to the number of blocks Jerry walks, the first component of the rate pair you are to find must also refer to the number of blocks Jerry walks.
180-C Number of blocks Tim walks
180-D The rate pair 6/8 represents the comparison of the number of blocks Jerry walks with the number of blocks Tim walks. Since each rate pair you are to find must represent this same comparison, you use x/y, and not y/x.
180-E Yes * Each rate pair you are to find must be equivalent to 6/8.
180-F Yes * Examples are 3/4, 6/8, 9/12, 12/16, 15/20, 18/24. * Yes. Since 6/8 is equivalent to 3/4, each rate pair equivalent to 3/4 is also equivalent to 6/8.
180-G The number of blocks Jerry walks with the number of blocks Tim walks
180-H The number of blocks Tim has walked when Jerry has walked 24 blocks
180-I There are two simple conditions for the problem. $x = 24$ is one of these conditions. The rate pair you are to find must satisfy both simple conditions.
180-J Yes
180-K 24 * The only replacement for x that satisfies $x = 24$ is 24.
181-L 24/32
181-M {24/32}
181-N One * Number of blocks Jerry walks * Number of blocks Tim walks
181-O Yes. Since 6/8 ~ 24/32, 6/8 and 24/32 represent the same comparison.
181-P Yes
181-Q Yes
181-R By using the definition of equivalent ordered pairs
181-S 32
181-T Yes * By using the definition of equivalent ordered pairs you know that if 6(32) = 24(8), then 6/8 ~ 24/32. This means that 32 satisfies 6/8 ~ 24/y.
181-A Yes * Number of cars * Number of minutes
181-B The number of cars that can be washed in 15 minutes * Because 15 refers to minutes and the second component of 20/60 also refers to minutes * The number of cars that can be washed in 15 minutes
181-C Yes * w/15 is equivalent to 20/60.
181-D By using the definition of equivalent ordered pairs
181-E 5
181-F Yes
182-G (Column 1) Yes
182-H It represents the rate of 60 minutes per 20 cars. * The number of minutes * The number of cars

182-I Yes * Since 15 refers to minutes and the first component of 60/20 refers to minutes, 15 must be the first component of the rate pair you are to find. * The number of cars that can be washed in 15 minutes

182-J Yes * $15/w$ is equivalent to 60/20.

182-K {5} * The number of cars that can be washed in 15 minutes

182-L Both of the rate pairs 20/60 and 60/20 represent the rate at which cars can be washed. If you think of the rate pair 20/60, then $20/60 \sim w/15$ is the condition for the problem. If you think of the rate pair 60/20, than $60/20 \sim 15/w$ is the condition for the problem.

182-A Yes * Number of cents * Number of packages of notebook paper

182-B The number of cents in all Jim paid for the 3 packages of paper * 3 is a number of packages of paper. Since the second component of 18/1 refers to the number of packages of paper, 3 must be a second component of the rate pair you are to find. * Number of cents

182-C Yes

182-D {54}

182-E Yes

182-F Yes * Number of votes Janet received * Number of votes Sue received

182-G (Column 2) Since the first component of 39/13 refers to the number of votes Janet received, the first component of the rate pair you are to find must also refer to the number of votes Janet received. Therefore, x must be the first component of the second rate pair. Since the second component of 39/13 refers to the number of votes Sue received, the second component of the second rate pair must refer to the number of votes Sue received. Therefore, 1 must be the second component of the second rate pair.

183-H Yes * $x/1$ is equivalent to 39/13.

183-I {3}

183-J Number of inches * Number of miles * Number of miles represented by 4 inches

183-K {70}

183-L 70 miles are represented by 4 inches on the map.

183-M Both of the rate pairs, 35/2 and 2/35 represent the rate. If you think of the rate pair 35/2, then you will use either the condition $s/4 \sim 35/2$ or the condition $35/2 \sim s/4$. If you think of the rate pair 2/35, then you will use either the condition $2/35 \sim 4/s$ or the condition $4/s \sim 2/35$.

On your own

183-1 x, number of satellites Mars has.
$6/1 \sim 12/x$.
{2}. 2 satellites. You could also use one of the following conditions: $12/x \sim 6/1$, $1/6 \sim x/12$, and $x/12 \sim 1/6$. In the answers that follow only, one condition will be expressed.

183-2 x, number of cents 5 cans of soup cost.
$3/54 \sim 5/x$.
{90}. 90¢

183-3 x, number of votes Roger received.
$2/3 \sim 98/x$.
{147}. 147 votes

183-4 x, number of new cars sold.
$8/6 \sim 72/x$.
{54}. 54 new cars

183-5 x, number of English coins for each French coin Lois has in her collection.
$95/19 \sim x/1$.
{5}. 5 times as many

183-6 x, number of dollars that Jerry would be paid for delivering 16 grocery orders.
$6/24 \sim x/16$.
{4}. $4

183-7 x, number of calories in a piece of mince pie.
$4/1 \sim x/170$.
{680}. 680 calories

183-8 x, number of cents Mrs. Adams paid for 18 ears of sweet corn.
$6/29 \sim 18/x$.
{87}. 87¢

183-9 x, number of senators Prince Edward Island had in 1958.
$12/2 \sim 24/x$.
{4}. 4 senators

184-10 x, number of points John scored.
$8/7 \sim 192/x$.
{168}. 168 points

184-11 x, number of miles that is the actual distance between the two cities.
$2/28 \sim 9/x$.
{126}. 126 miles

184-12 x, number of persons who live in Canada.
$4/1 \sim x/3,560,238$.
{14240952}. Approximately 14,240,952 persons

184-13 x, number of square miles that is the area of Australia.
$3/1 \sim 8,925,000/x$.
{2975000}. About 2,975,000 square miles

184-14 x, number of feet that was the height of the new building for each 1 foot of height of the old building.
$76/228 \sim 1/x$.
{3}. 3 times the height of the old building

184-15 x, number of dollars Alice will have saved when she has earned $216.
$4/9 \sim x/216$.
{96}. $96

184-16 x, number of motor vehicles in Canada in 1935.
$16/1 \sim x/74,246$.
{1187936}. 1,187,936 motor vehicles

184-17 x, number of papers the press can print in 3 hours.
$237,500/5 \sim x/3$.
{142500}. 142,500 newspapers

184-18 x, number of square miles the land area of Saskatchewan is.
$4/1 \sim 880,000/x$.
{220000}. About 220,000 square miles

Applying mathematics

184-1 x, number of cents the service charge is for 12 checks.
$8/1 \sim x/12$.
{96}. 96¢

184-2 x, number of cents the service charge was per check.
$91/13 \sim x/1$.
{7}. 7¢

184-3 x, number of dollars interest Mr. Peters will pay at the end of one year.
$6/100 \sim x/700$.
{42}. $42

184-4 x, number of dollars the interest per year is for each $100 borrowed.
$49/980 \sim x/100$.
{5}. $5

184-5 x, number of dollars the discount was.
$3/75 \sim x/375$.
{15}. $15

184-6 x, number of dollars discount for each $100 of the regular price.
$20/250 \sim x/100$.
{8}. $8

184-7 x, number of dollars commission Mr. Baker would be paid.
$16/80 \sim x/560$.
{112}. $112

185-8 x, number of dollars worth of merchandise Mr. Thompson has to sell to earn a commission of $144.
$24/240 \sim 144/x$.
{1440}. $1440

185-9 x, number of dollars Mr. Hill's yearly take-home pay is.
$436/1 \sim x/12$.
{5232}. $5232

185-10 x, number of dollars Mrs. Smith's yearly salary is.
$4/5 \sim 5276/x$.
{6595}. $6595

185-11 x, number of dollars worth of merchandise Tom had to sell to receive a commission of one dollar.
$16/288 \sim 1/x$.
{18}. $18

185-12 x, number of dollars the regular price of the camera was.
$2/10 \sim 8/x$.
{40}. $40

185-13 x, number of dollars Mr. Harper borrowed.
$5/100 \sim 33/x$.
{660}. $660

185-14 x, number of dollars Mr. Martin's monthly salary is.
$12/8472 \sim 1/x$.
{706}. $706

185-15 x, number of dollars worth of merchandise Mr. Johnson sold.
$18/200 \sim 261/x$.
{2900}. $2900

185-16 x, number of dollars interest per year for each $100 he borrowed.
$64/1280 \sim x/100$.
{5}. $5

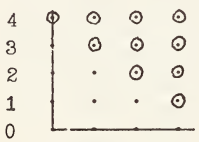
185-17 x, number of dollars take-home pay for each $10 of salary.
$288/360 \sim x/10$.
{8}. $8

185-18 x, number of dollars discount for each $10 of regular price.
$20/200 \sim x/10$.
{1}. $1

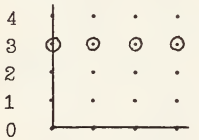
185-19 x, number of dollars commission for selling $405 worth of merchandise.
$3/45 \sim x/405$.
{27}. $27

Keeping Skilful

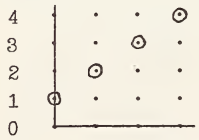
185-1 {(0, 4), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}



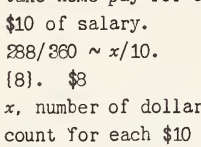
185-2 {(0, 3), (1, 3), (2, 3), (3, 3)}



185-3 {(0, 1), (1, 2), (2, 3), (3, 4)}



185-4 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (1, 3), (2, 4)}



185-5 {36}

185-6 {3280}

185-7 {0, 1, 2, ..., 12}

185-8 {254}

185-9 {118854}

185-10 {29072}

185-11 {386}

185-12 {6993, 6994, 6995, ...}

185-13 {(6, 0), (6, 1), (6, 2)}

185-14 {(7, 11), (8, 10), (9, 9), (10, 8), (11, 7), (12, 6), (13, 5), (14, 4), (15, 3), (16, 2), (17, 1), (18, 0)}

185-15 {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10), (6, 11), (7, 12), (8, 13)}

185-16 {(23, 0), (23, 1), (23, 2), (23, 3), (23, 4)}

185-17 {(5, 7), (6, 6), (7, 5), (8, 4), (9, 3), (10, 2), (11, 1), (12, 0)}

Lesson 45

186-A The chart below shows the number of dollars raised by the students in each room and the goal for that room.

Room	Amount raised	Goal
Room 101	$22	$25
Room 102	$20	$20
Room 103	$24	$30
Room 104	$22	$20
Room 105	$21	$25

186-B The number of dollars raised * The number of dollars that was the goal

186-C Room 104 * Yes * The goal is how much each room was expected to raise. Room 104 was the only one of the five rooms that did better than expected.

186-D Both rooms had the same goal but Room 101 raised more money.

186-E Because these two rooms had different goals and neither room reached its goal.

186-F Yes * The number of dollars that would have been raised by the students in Room 101 if their goal had been $100 and if they had done just as well.

187-G Yes. For each room you know one rate pair that represents the comparison of the amount raised with the goal, and you want to find an equivalent rate pair that has a second component of 100. * The number of dollars that would have been raised by the students in each room if their goal had been $100 and if they had done just as well

187-H By using the definition of equivalent ordered pairs. For example, since $22/25$ and $n/100$ are equivalent ordered pairs, you know that the replacement for n must satisfy $22(100) = 25n$.

187-I Yes

187-J Yes * All you need do is compare the first components. Since the greatest first component is 110, you know that Room 104 did best. The next great-

est first component is 100, so Room 102 did second best. 88 is the third greatest number, so Room 101 did third best. Next is Room 105 and then Room 103.

187-K Room 104, Room 102, Room 101, Room 105, Room 103

187-L No. The rate pair 22/25 represents a comparison but does not have a second component of 100. * Yes. The rate pair 88/100 represents a comparison and has a second component of 100.

187-M Yes

187-N Eighty-eight per cent, one hundred per cent, eighty per cent, one hundred ten per cent

187-O Yes

187-P Yes. For each room, the symbol in column B and the symbol in column C are different names of the same rate pair.

188-Q 110%

188-R 80 * 81 * Yes * Yes

188-S 110%

188-T The first component of the rate pair is 100 and the second component is 100.

188-U Less than 100%

188-V Greater than 100%

188-W Equal to 100%

188-X Less than 100%

188-Y Greater than 100%

188-Z Less than 100%

188-A Test 1 18/29
Test 2 12/15
Test 3 36/40
Test 4 28/28
Test 5 47/50

188-B 18/20 ~ 90/100. 12/15 ~ 80/100. 36/40 ~ 90/100. 28/28 ~ 100/100. 47/50 ~ 94/100.

188-C 90/100 = 90%. 80/100 = 80%. 90/100 = 90%. 100/100 = 100%. 94/100= 94%.

188-D No * No * To receive a grade greater than 100%, the first component of the rate pair representing the comparison would have to be greater than the second component. This means that Susan would have to answer correctly more questions than are in the test, which is impossible.

188-E Test 4 * Test 2 * The rate pairs whose second components are 100. In other words, the per cents * The first components

On your own

189-1 78%

189-2 125%

189-3 1%

189-4 10%

189-5 200%

189-6 9%

189-7 99%

189-8 100%

189-9 66%

189-10 120/100

189-11 2/100

189-12 500/100

189-13 71/100

189-14 5/100

189-15 90/100

189-16 1000/100

189-17 98/100

189-18 43/100

189-19 4/5 ~ 80/100. 80%

189-20 9/50 ~ 18/100. 18%

189-21 32/64 ~ 50/100. 50%

189-22 90/30 ~ 300/100. 300%

189-23 75/150 ~ 50/100. 50%

189-24 96/32 ~ 300/100. 300%

189-25 10/1000 ~ 1/100. 1%

189-26 1000/10 ~ 10000/100. 10,000%

189-27 5/4 ~ 125/100. 125%

189-28 Camera Club 35/35
Drama Club 23/25
Glee Club 32/40
Mathematics Club 33/30
Pep Club 63/50
Space Club 18/15

189-29 Camera Club 35/35 ~ 100/100. 100%
Drama Club 23/25 ~ 92/100. 92%
Glee Club 32/40 ~ 80/100. 80%
Mathematics Club 33/30 ~ 110/100. 110%
Pep Club 63/50 ~ 126/100. 126%
Space Club 18/15 ~ 120/100. 120%

189-30 110%, 126%, 120% * 92%, 80% * 100%

189-31 Pep Club * Glee Club

189-32 Pep Club 126%
Space Club 120%
Mathematics Club 110%
Camera Club 100%
Drama Club 92%
Glee Club 80%

189-33 Yes * If the Space Club had obtained 3 more members, they would have had 21 new members. The rate pair that would represent the comparison of the actual number of new members with the goal would be 21/15. 21/15 is equivalent to 140/100, or 140%. 140% is greater than 126%.

Lesson 46

190-A The number of students enrolled at Lee School who live more than five blocks from the school * The number of students enrolled at Lee School

190-B You are to find a per cent, and you know that a per cent is a rate pair that represents a comparison and has a second component of 100. * The number of students out of each 100 students enrolled who live more than five blocks from school

190-C It is equivalent to 540/720.

190-D By using the definition of equivalent ordered pairs. You know that the product of the first component of 540/720 and the second component of $x/100$ is equal to the product of the first component of $x/100$ and the second component of 540/720

190-E 75

190-F {75}

190-G Yes * 75% is a rate pair that represents a comparison and has a first component of 75 and a second component of 100. 75/100 is also a rate pair that represents a comparison and has a first component of 75 and a second component of 100.

190-H The number of questions Jack answered correctly on the test

190-I - - -

191-J Yes * The number of questions Jack would have answered correctly if the test had 100 questions and if Jack had done as well * 100 questions in a test

191-K Since the second component of 85/100 refers to a number of questions in a test, the second component of the rate pair whose first component you are to find must also refer to the number of questions in a test. * The number of questions Jack answered correctly

191-L It is equivalent to 85/100.

191-M {34}

191-N The number of dollars Doris earned last summer

191-O Doris saved 54% of her earnings which means that she saved 54 dollars out of each 100 dollars she earned.

191-P Yes * A number of dollars saved * A number of dollars earned * The comparison of the number of dollars Doris saved with the number of dollars she earned

191-Q Since the first component of 54/100 refers to the number of dollars Doris saved, the first component of the rate pair whose second component you are to find must also refer to the number of dollars Doris saved. * The number of dollars Doris earned

191-R It is equivalent to 54/100.

191-S {150}

191-T Doris earned $150 last summer.

191-U No * You cannot save more of your earnings than you earn.

191-A - - -

191-B Yes * Yes

191-C You know that $x\%$ is the same as $x/100$. Therefore, both 6 is $x\%$ of 10 and $6/10 \sim x/100$ require you to find the first component of a rate pair that has a second component of 100 and is equivalent to 6/10.

191-D 60 * Yes * Since 6 is $x\%$ of 10 is the same condition as $6/10 \sim x/100$, a number that satisfies $6/10 \sim x/100$ also satisfies 6 is $x\%$ of 10.

191-E {60}

191-F 60 per cent * 60 per cent

192-G - - -

192-H Yes * Yes

192-I Yes * Yes * 30% of 320 is s is the same condition as $30/100 \sim s/320$ since both conditions require you to find the first component of a rate pair that has a second component of 320 and is equivalent to 30/100. You know that $30/100 \sim s/320$ is the condition for both problem C and problem D. Therefore, 30% of 320 is s is also the condition for both problem C and problem D.

192-J 96 * Since 30% of 320 is s is the same condition as $30/100 \sim s/320$, a number that satisfies $30/100 \sim s/320$ also satisfies 30% of 320 is s.

192-K 96 pages * 96

192-L - - -

192-M Yes * Yes

192-N Yes. Both 270 is 5% of n and $5/100 \sim 270/n$ require you to find the second component of a rate pair that has a first component of 270 and is equivalent to 5/100.

192-O 5400

192-P $5400 * 5400

192-Q Yes

192-R You want to compare the number of tickets Martin sold with the number of tickets June sold. 12/8, not 8/12, represents this comparison. 8/12 represents the comparison of the number of tickets June sold with the number of tickets Martin sold.

192-S {150}

192-T Martin sold 150% of the number of tickets June sold.

192-U The first component

192-V The second component. The problem means that you are to find the number that compares with 32 as 125 compares with 100.

192-W $125/100 \sim x/32$, or 125% of 32 is x.

192-X {40}

192-Y 125% of 32 is 40.

On your own

193-1 x, number of cloudy days in April.
$40/100 \sim x/30$.
{12}. 12 days

193-2 x, number of baskets he would have made if he had made 100 tries and had done just as well.
$7/20 \sim x/100$.
{35}. 35%

193-3 x, number of pitchers.
$40/100 \sim x/25$.
{10}. 10 pitchers

193-4 x, number of students enrolled.
$20/100 \sim 92/x$.
{460}. 460 students

193-5 x, number of correct answers.
$100/100 \sim x/15$.
{15}. 15 correct answers

193-6 x, number of dollars raised by students in Cook School for each $100 raised by the students in Fisher School.
$52/65 \sim x/100$.
{80}. 80%

193-7 x, number of dollars raised by students in Fisher School for each $100 raised by the students in Cook School.
$65/52 \sim x/100$.
{125}. 125%

193-8 x, number of dollars paid for bicycle.
$65/100 \sim 39/x$.
{60}. $60

193-9 x, number of dollars that was sale price for each $100 of regular price.
$30/40 \sim x/100$.
{75}. 75%

193-10 x, number of persons that was the estimated population in 1960.
$175/100 \sim x/800000$.
{1400000}. 1,400,000 persons

193-11 x, number of dollars take-home pay for each $100 of salary.
$340/425 \sim x/100$.
{80}. 80%

193-12 x, number of members in Science Club.
$5/100 \sim 4/x$.
{80}. 80 members

193-13 x, number of square miles suitable for agriculture.
$16/100 \sim x/3,560,000$.
{569600}. 569,600 square miles

193-14 x, number of dollars Mr. White paid for house.
$138/100 \sim 17,250/x$.
{12500}. $12,500

193-15 x, number of persons that was the urban population in 1956.
$65/100 \sim x/16,080,791$.
{10452514}. Approximately 10,452,514 persons

193-16 x, number of persons that was the total population in 1956.
$60/100 \sim 510,000/x$.
{850000}. Approximately 850,000 persons

193-17 $20/100 \sim x/185$.
{37}

193-18 $150/30 \sim x/100$.
{500}

193-19 $16/100 \sim x/425$.
{68}

193-20 $84/100 \sim 42/x$.
{50}

193-21 $174/100 \sim 348/x$.
{200}

193-22 $47/94 \sim x/100$.
{50}

193-23 $94/47 \sim x/100$.
{200}

193-24 $150/100 \sim x/242$.
{363}

193-25 $32/100 \sim 1536/x$.
{4800}

Applying mathematics

194-1 x, number of dollars commission was for each $100 worth of merchandise he sold.
$200/2500 \sim x/100$.
{8}. 8%

194-2 x, number of dollars commission for each $100 worth of merchandise he sold.
$450/5000 \sim x/100$.
{9}. 9%

194-3 x, number of dollars commission.
$4/100 \sim x/9650$.
{386}. $386

194-4 x, number of dollars worth of merchandise sold.
$5/100 \sim 142/x$.
{2840}. $2840

194-5 x, number of dollars discount per $100 of regular price.
$5/25 \sim x/100$.
{20}. 20%

194-6 x, number of dollars discount per $100 of regular price.
$45/180 \sim x/100$.
{25}. 25%

194-7 x, number of dollars discount.
$20/100 \sim x/375$.
{75}. $75

194-8 x, number of dollars worth of merchandise.
$2/100 \sim 60/x$.
{3000}. $3000

194-9 x, number of dollars interest paid on each $100 borrowed.
$30/500 \sim x/100$.
{6}. 6%

194-10 x, number of dollars interest per $100 borrowed.
$10/200 \sim x/100$.
{5}. 5%

194-11 x, number of dollars interest.
$4/100 \sim x/8500$.
{340}. $340

194-12 x, number of dollars borrowed.
$5/100 \sim 69/x$.
{1380}. $1380

194-13 x, number of dollars weekly salary.
$80/100 \sim 112/x$.
{140}. $140

194-14 x, number of dollars discount per $100 of regular price.
$9/75 \sim x/100$.
{12}. 12%

194-15 x, number of dollars that was the selling price of the house.
$5/100 \sim 545/x$.
{10900}. $10,900

194-16 x, number of dollars interest.
$4/100 \sim x/600$.
{24}. $24

194-17 x, number of dollars take-home pay per $100 of salary.
$64/80 \sim x/100$.
{80}. 80%

195-18 x, number of dollars interest for each $100 borrowed.
$105/1750 \sim x/100$.
{6}. 6%

195-19 x, number of dollars that was the price of the rug before the sale.
$15/100 \sim 21/x$.
{140}. $140

195-20 x, number of dollars of monthly take-home pay.
$78/100 \sim x/450$.
{351}. $351

195-21 x, number of dollars of commission per $100 of selling price.
$852/28,400 \sim x/100$.
{3}. 3%

195-22 x, number of dollars discount.
$32/100 \sim x/175$.
{56}. $56

195-23 x, number of dollars yearly salary.
$83/100 \sim 5312/x$.
{6400}. $6400

195-24 x, number of dollars paid auctioneer.
$5/100 \sim x/16,700$.
{835}. $835

Keeping Skilful

195-1 {60}

195-2 {80}

195-3 {8905}

195-4 {6}

195-5 {2750}

195-6 {416}

195-7 {72}

195-8 {18}

195-9 {64}

195-10 {120900}

195-11 {1, 2, 3, ..., 146}

195-12 {150}

195-13 {2879}

195-14 {100}

195-15 {380}

195-16 {1, 2, 3, ..., 205}

195-17 {150}

195-18 {227}

195-19 {225}

195-20 {100}

195-21 {935}

195-22 {3067, 3068, 3069, ...}

195-23 {500}

195-24 {8}

Lesson 47

195-A One

195-B Because the first component, not the second component, of 4/1 refers to the number of records Jane has

195-C To the number of records Sally can have

196-D Yes

196-E You know that Jane has fewer than 25 records. This means that the number of records Jane has is less than 25. * Yes * The number of records Jane can have

196-F Yes

196-G Yes. Each member of {4/1, ...} satisfies $4/1 \sim x/y$. * Yes

196-H Examples are 8/2, 12/3, 16/4, 20/5, 24/6.

196-I 4/1, 8/2, 12/3, 16/4, 20/5, 24/6 * Yes

196-J {4/1, 8/2, 12/3, 16/4, 20/5, 24/6}

196-K If Jane has 16 records, Sally has 4 records. If Jane has 20 records, Sally has 5 records. If Jane has 24 records, Sally has 6 records.

196-L Yes

197-M No

197-N Since the first component of $m/100$ refers to the number of dollars interest, the first component of the second rate pair must refer to the number of dollars interest. Since the second component of $m/100$ refers to the number of dollars on which the interest is paid, the second component of the second rate pair must refer to the number of dollars on which the interest is paid. * Interest is paid on the number of dollars that is borrowed. In this case, $260, not $273, was borrowed.

197-O Yes

197-P n, not m, refers to the number of dollars interest Mr. Taylor paid. In the problem another requirement is given about the number of dollars interest Mr. Taylor paid. * Yes

197-Q Yes

197-R 13

197-S Yes

197-T 5

197-A No * No * You should imagine that the cents spent for erasers are put with the cents spent for pencils. This is additive action and is represented by $a + b$, not $a - b$.

197-B No

197-C Yes

197-D Yes * The number of pencils * The number of cents

197-E Since the first component of 3/15 refers to the number of pencils, the first component of the second rate pair must refer to the number of pencils. Since the second component of 3/15 refers to the number of cents, the second component of the second rate pair must refer to the number of cents.

197-F Yes

197-G Yes * The number of erasers * The number of cents

197-H Since the first component of 1/4 refers to the number of erasers, the first component of the second rate pair must refer to the number of erasers. Since the second component of 1/4 refers to the number of cents, the second component of the second rate pair must refer to the number of cents.

197-I Yes

197-J Yes. The compound condition for the problem consists of three simple conditions and two of the connectives "and."

198-K 10 * 12

198-L Yes

198-M 22

198-N 22¢

On your own

198-I x, number of points Mike scored. y, number of points Paul scored. $4/3 \sim x/y \wedge y < 14$. {4/3, 8/6, 12/9, 16/12}. Each member of the solution set may be used to obtain an answer. The answer obtained from 8/6 is: Mike scored 8 points and Paul scored 6 points.

198-2 x, number of votes Carl received. y, number of votes George received.
$2/5 \sim x/y \wedge x + 18 = y$.
{12/30}.
Carl received 12 votes and George received 30 votes.

198-3 x, number of dollars discount. y, number of dollars net price.
$10/100 \sim x/20 \wedge 20 - x = y$.
{(2, 18)}. $18

198-4 x, number of dollars discount. y, number of dollars net price.
$15/100 \sim x/220 \wedge 220 - x = y$.
{(33,187)}. $187

198-5 x, number of new stamps Larry bought. y, number of stamps Larry had after he bought the 5 packages.
$274 + x = y \wedge 1/24 \sim 5/x$.
{(120, 394)}. 394 stamps

198-6 x, number of cents spent for oranges. y, number of cents spent in all.
$27 + x = y \wedge 12/48 \sim 6/x$.
{(24, 51)}. 51¢

198-7 x, number of stamps Jay has. y, number of stamps Ed has.
$125/100 \sim x/y \wedge y + 128 = 364$.
{295/236}. Jay has 295 stamps and Ed has 236 stamps.

198-8 x, number of dollars interest Mr. Thomas paid. y, number of dollars interest the credit union charged on each $100.
$303 - 300 = x \wedge x/300 \sim y/100$.
{(3, 1)}. 1%

198-9 x, number of dollars interest Mr. Henderson paid. y, number of dollars interest bank charged on each $100.
$1325 - 1250 = x \wedge x/1250 \sim y/100$.
{(75, 6)}. 6%

198-10 x, number of ounces of mints. y, number of ounces of candy.
$12 + x = y \wedge 1/8 \sim 3/x$.
{(24, 36)}. 36 ounces

198-11 x, number of dollars discount on chair. y, number of dollars discount for each $100 of the regular price.
$140 - 91 = x \wedge x/140 \sim y/100$.
{(49, 35)}. 35%

198-12 x, number of lawns Ed mowed. y, number of lawns Frank mowed.
$4/3 \sim x/y \wedge x + y < 15$.
{4/3, 8/6}. Either Ed mowed 4 lawns and Frank mowed 3 lawns or Ed mowed 8 lawns and Frank mowed 6 lawns.

198-13 x, number of cents spent on tickets for adults. y, number of cents spent on tickets for children. z, number of cents spent in all.
$x + y = z \wedge 1/25 \sim 3/x \wedge 1/10 \sim 2/y$.
Mr. Brown spent 95 cents in all for the tickets.

Lesson 48

199-A Yes * Yes

199-B $m/n \sim 5/3 \wedge n < 12$.

199-C Yes * Yes

199-D 20/12 is a member of {5/3, ...} because 20/12 is equivalent to 5/3. You know 20/12 is equivalent to 5/3 because $20 \times 3 = 5 \times 12$. 25/15 is a member of {5/3,...} because 25/15 is equivalent to 5/3. You know 25/15 is equivalent to 5/3 because $25 \times 3 = 5 \times 15$.

199-E Because neither 12 nor 15 satisfy $n < 12$

199-F 5/3, 10/6, 15/9 * Yes. These rate pairs satisfy both $m/n \sim 5/3 \wedge n < 12$.

199-G {5/3, 10/6, 15/9} * No

199-H Yes

199-I Yes * Yes

199-J $x/y \sim 2/8 \wedge x + y = 20$.

199-K Yes * Yes

199-L Yes * Yes * Yes * The set tabulated in D4 is the set of all rate pairs equivalent to 1/4. 5/20, 6/24, and 8/32 are contained in this set because each of them is equivalent to 1/4.

199-M The sum of the components of each of these rate pairs is greater than 20. * No. To satisfy the compound condition, they must satisfy $x + y = 20$ as well as $x/y \sim 2/8$.

200-N 4/16 * Yes. 4/16 satisfies both $x/y \sim 2/8$ and $x + y = 20$.

200-O {4/16}

200-P Yes

200-Q - - -

200-R Yes

200-S {6/8, ...}. This could also be tabulated as

{3/4, ...} since 3/4 is equivalent to 6/8.

200-T All the members

200-U {6/8, ...} * Yes

200-V Yes. Each rate pair equivalent to 6/8 has a first component less than the second component.

On your own

200-1 $x/y \sim 4/6 \wedge x + y < 20$.
{2/3, 4/6, 6/9}

200-2 $x/y \sim 3/7 \wedge x < 25$.
{3/7, 6/14, 9/21, 12/28, 15/35, 18/42, 21/49, 24/56}

200-3 $x/y \sim 4/9 \wedge x = 2(4)$.
{8/18}

200-4 $x/y \sim 15/18 \wedge y - x = 1$.
{5/6}

200-5 $x/y \sim 2/10 \wedge y < 10x$.
{1/5, ...}

200-6 $x/y \sim 2/2 \wedge x + y < 15$.
{1/1, 2/2, 3/3, 4/4, 5/5, 6/6, 7/7}

200-7 $x/y \sim 7/8 \wedge y - x = 3$.
{21/24}

200-8 $x/y \sim 9/6 \wedge x < y + 4$.
{3/2, 6/4, 9/6}

201-9 $x/y \sim 2/3 \wedge x + y < 5$.
{ }

201-10 $x/y \sim 8/5 \wedge x + y = 52$.
{32/20}

Applying mathematics

201-1 x, number of dollars that the Canadian railways earned.
$x - 1,232,387,115 = 37,220,149$.
{1269607264}.
$1,269,607,264

201-2 x, number of miles traveled in 15 minutes.
$337/1 \sim x/15$.
{5055}. Approximately 5055 miles

201-3 x, number of tickets Harriet had to begin with.
$x > 20 \wedge x - 14 < 9$.
{21, 22}. 21 or 22 tickets

201-4 x, number of strokes Joe had on the second nine holes.
$54 + x > 113$.
{60, 61, 62, ...}. A score of 60

201-5 x, number of dollars that was the regular price of the jacket.
$15/100 \sim 72/x$.
{480}. $480

201-6 x, number of moths Donna mounted. y, number of butterflies Donna mounted.
$x + y = 6$.
{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (2, 4) is 2 moths and 4 butterflies.

201-7 x, number of shrubs planted. y, number of trees planted.
$x + y < 12 \wedge x = y$.
{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (5, 5) is 5 shrubs and 5 trees.

201-8 x, number of dollars that was the discount. y, number of dollars that was the net price of the dress.
$20/100 \sim x/30 \wedge 30 - x = y$.
{(6, 24)}. $24

201-9 x, number of degrees the melting point of iron is higher than the melting point of copper.
$1533 - 1083 = x$.
{450}. 450 degree centigrade

201-10 x, number of dollars that was Mr. Lee's commission during October.
$4/100 \sim x/7475$.
{299}. $299

201-11 x, number of hits the outfielder had.
$9/47 \sim 36/x$.
{188}. 188 hits

201-12 x, number of cents Carol had before her mother gave her the 15 cents.
$x < 57 \wedge x + 15 > 62$.
{48, 49, 50, ..., 56}.
From 48 through 56 cents

201-13 x, number of dollars discount.
$348 - x < 252$.
{97, 98, 99, ..., 348}.
97 dollars

201-14 x, number of dollars interest for each $100 borrowed.
$45/750 \sim x/100$.
{6}. 6%

201-15 x, number of years that is Helen's age now.
y, number of years that is Anita's age now.
$x - y = 9 \wedge y + 4 = 17$.
{(22, 13)}. Helen is 22 years old and Anita is 13 years old.

202-16 x, number of dollars raised for each $100 of the goal.
$8625/7500 \sim x/100$.
{115}. 115%

202-17 x, number of arrowheads Bill has. y, number of arrowheads Ken has.
$x - y = 15 \wedge y + 23 = 50$.
{(42, 27)}. Bill has 42 arrowheads and Ken has 27 arrowheads.

202-18 x, number of cents that was the total cost of the track. y, number of cents that was Tom's change.
$x/4 \sim 12/1 \wedge 75 - x = y$.
{(52, 23)}. 23 cents

202-19 x, number of nickels Janet had. y, number of dimes Janet had.
$x + y = 11 \wedge y - x = 5$.
{(3, 8)}. 3 nickels and 8 dimes

202-20 $36/225 \sim x/100$.
{16}

202-21 $x + y < 15 \wedge y - x = 9$.
{(0, 9), (1, 10), (2, 11)}

202-22 $x > 386 \wedge 134 + x > 525$
{392, 393, 394, ...}

202-23 $x/y \sim 8/12 \wedge y - x < 8$.
{2/3, 4/6, 6/9, 8/12, 10/15, 12/18, 14/21}

202-24 $68/100 \sim x/9350$.
{6358}

202-25 $x + y = 15 \wedge y < 78 \div 13$.
{(15, 0), (14, 1), (13, 2), (12, 3), (11, 4), (10, 5)}

202-26 $x/y \sim 6/5 \wedge x + y < 35$.
{6/5, 12/10, 18/15}

202-27 $125/100 \sim 15/x$.
{12}

202-28 $28/112 \sim x/100$. {25}

202-29 $11/100 \sim x/200$. {22}

Checking up

Test 46

202-1 T

202-2 F

202-3 T

202-4 F

202-5 T

202-6 T

202-7 T

202-8 F

202-9 T

202-10 F

Test 47

202-11 (a) 12/15

202-12 (c) $8(12) = 32(3)$

202-13 (d) discount

203-14 (b) 7

203-15 (c) interest

203-16 (c) {2, 3}

203-17 (c) 10%

203-18 (b) 8/12

203-19 (b) 3/10

Test 48

203-20 18

203-21 48

203-22 6

203-23 6

203-24 108

203-25 2

203-26 {12}

203-27 {32}

203-28 {256}

203-29 {8}

203-30 {9/15}

203-31 {4/5, 8/10, 12/15, 16/20}

203-32 {2/3, ...}

Test 49

203-33 x, number of cents that 18 pencils cost.
$6/25 \sim 18/x$.
{75}. 75¢

203-34 x, number of correct answers Barbara had.
$5/4 \sim 45/x$.
{36}. 36 correct answers

203-35 x, number of students enrolled at Westview School.
$15/100 \sim 54/x$.
{360}. 360 students

203-36 x, number of dollars commission that Mr. Hanson receives for each $100 worth of merchandise he sells.
$70/1000 \sim x/100$.
{7}. 7%

203-37 x, number of dollars discount on suit.
$20/100 \sim x/45$.
{9}. $9

203-38 x, number of dollars discount on set of tools. y, number of dollars Mr. Bender paid for set of tools.
$15/100 \sim x/140 \wedge 140 - x = y$.
{(21, 119)}. $119

203-39 x, number of hits Bob had. y, number of hits Larry had.
$3/2 \sim x/y \wedge x - y = 8$.
{(24, 16)}. Bob had 24 hits and Larry had 16 hits.

203-40 $60/100 \sim x/25$.
{15}

203-41 $200/100 \sim 48/x$.
{24}

203-42 $13/25 \sim x/100$.
{52}

203-43 $15/100 \sim x/120$.
{18}

203-44 $105/75 \sim x/100$.
{140}

203-45 $x/y \sim 5/6 \wedge y - x = 4$.
{20/24}

203-46 $x/y \sim 2/3 \wedge y < 25$.
{2/3, 4/6, 6/9, 8/12, 10/15, 12/18, 14/21, 16/24}

Test 50

204-1 T

204-2 T

204-3 F

204-4 T

204-5 F

204-6 T

204-7 T

Test 51

204-8 (d) $x > 1 \wedge x < 6$.

204-9 (c) { }

204-10 (d) {0, 2, 4}

204-11 (b) is the empty set

204-12 (a) $x - 2 > 2$.

204-13 (b) A and B meet

204-14 (b) ray

204-15 (d) 6/30

Test 52

204-16 30

205-17 {0, 1, 2, 3, 4, 5}

205-18 {0, 1, 2, 3, 4}

205-19 They are encircled.

205-20 They are enclosed by triangles.

205-21 They are enclosed by squares.

205-22 They are both encircled and enclosed by triangles.

205-23 {(3, 3), (4, 4)}

205-24 { }

Test 53

205-25 x, number of aprons Martha made.
$13 - x = 5$.
{8}. 8 aprons

205-26 x, number of tickets Ruth sold.
$23 + x < 39$.
{1, 2, 3, ..., 15}. From 1 through 15 tickets

205-27 x, number of points scored by winning team. y, number of points scored by losing team.
$x + y = 87 \wedge x > 45$.
{(46, 41), (47, 40), (48, 39), (49, 38), (50, 37), (51, 36), (52, 35), (53, 34), (54, 33), (55, 32), (56, 31), (57, 30), (58, 29), (59, 28), (60, 27), (61, 26), (62, 25), (63, 24), (64, 23), (65, 22), (66, 21), (67, 20), (68, 19), (69, 18), (70, 17), (71, 16), (72, 15), (73, 14), (74, 13), (75, 12), (76, 11), (77, 10), (78, 9), (79, 8), (80, 7), (81, 6), (82, 5), (83, 4), (84, 3), (85, 2), (86, 1), (87, 0)}.
41 points

205-28 x, number of days Carol was absent.
$5/100 \sim x/180$.
{9}. 9 days

205-29 x, number of points Pete's team scored. y, number of points Harold's team scored.
$3/2 \sim x/y \wedge x < 16$.
{3/2, 6/4, 9/6, 12/8, 15/10}.
10 points

Lesson 49

206-A Yes

206-B One. The stroke

206-C 17

206-D Yes

206-E Set B has one more member than set A. * The tally numeral for set B has one more stroke than the tally numeral for set A.

206-F Set C has one more member than set B. * The tally numeral for set C has one more stroke than the tally numeral for set B.

206-G Set D has one more member than set C. * The tally numeral for set D has one more stroke than the tally numeral for set C.

207-H (Column 1) Set A has 1 member. Set B has 2 members. Set C has 3 members. Set D has 4 members.

207-I (Column 1) 1

207-J (Column 1) 2

207-K 3

207-L 4

207-M 5

207-N 6

207-O 7

207-P 8

207-Q 9

207-R 10

207-A The number that each symbol expresses

207-B ⋔ names 1. # names 2. ∾ names 3. ∋ names 4.

207-C ∾

207-D #

207-E ∋

207-F ⋔

207-G You may use any symbols you wish as code numerals. For example, you

could use λ to name the number of members in set E, Δ to name the number of members in set F, θ to name the number of members in set G, and Λ to name the number of members in set H.

207-H (Column 2) No * No * Since the same symbol cannot be used more than once in a code numeral, the code system does not have the property of repetition. Since each numeral in a code system consists of just one symbol, addition is not used to find the number expressed. Therefore, the code system does not have the property of addition.

207-I (Column 2) Yes. In a code system, a different symbol is used to name each member of {1, 2, 3, ...}. Since {1, 2, 3, ...} is an infinite set, an infinite set of symbols must be used to name the members of {1, 2, 3, ...}.

207-J (Column 2) Yes * Yes * A tally system is easier to learn because it has only one symbol which is used again and again. In a code system, each number is expressed with just one symbol.

208-A One

208-B How many strokes there are * 30

208-C 2 * 4

208-D 4

208-E There are seven groups of four objects.

208-F There were 2 objects left over after the grouping was completed.

208-G 7 * 2

208-H No. There are only 9 stars and strokes but 30 objects.

208-I 30

208-J Yes * 14. Since each numeral contains 3 stars and 2 strokes, they name the same number. The arrangement of the stars and strokes in the numeral does not matter.

208-K 16

208-L Yes

208-M No. There are only 6 diamonds, stars, and strokes. * Find the sum of the numbers expressed by the symbols in the numeral. This sum is 16 + 4 + 4 + 4 + 1 + 1. * 30

209-N Yes

209-O There are 6 groups of 4 × 4, or 16. * Yes. 16 + 16 + 16 + 16 + 16 + 16 + 4 + 4 + 1 + 1 = 106.

209-P Since you are using a base of four, you must group by 4's.

209-Q There is just one group of 4 × 4 × 4 objects.

209-R 106

209-S Yes * Four of the diamonds in numeral H express the same number as the arrowhead in numeral I.

210-T 1 * 10

210-U Yes * 10 × 10, or 100

210-V You must know what number is expressed by each symbol.

210-W Yes * 111

210-X Yes * The arrangement of the symbols in numeral K is different from the arrangement of the symbols in numeral M.

On your own

210-1 ☆ ☆

210-2 / /

210-3 ☆ //

210-4 ☆ ☆ / /

210-5 ◈ ☆ ☆

210-6 ◈ ☆

210-7 ☆ ///

210-8 ☆ ☆

210-9 ///////

210-10 ☆ ☆ ☆ ☆ ☆ /////

211-11 (base three) ◈
(base eight) ☆/

211-12 (base three) ◈☆ ☆ //
(base eight) ☆ ☆ /

211-13 9 + 9 + 3 + 1, or 22

211-14 8 + 4 + 2 + 1, or 15

211-15 16 + 16 + 4 + 4 + 4, or 44

211-16 216 + 36 + 36 + 6 + 1 + 1 + 1 + 1, or 298

211-17 121 + 11 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1, or 141

211-18 64 + 64 + 1 + 1 + 1, or 131

211-19 100 + 100 + 100 + 10 + 100 + 100, or 510

211-20 12 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1, or 21

Keeping Skilful

211-1 10,596

211-2 19,507

211-3 98,824

211-4 242,429

211-5 10,783

211-6 1165

211-7 704

211-8 2472

211-9 9778

211-10 6382

211-I1 17,167
211-I2 72,044
211-I3 5526
211-I4 43,758
211-I5 1786
211-I6 6510
211-I7 211,509
211-I8 448,000
211-I9 259,415
211-I20 609,120
211-I21 98, remainder 56
211-I22 39, remainder 0
211-I23 713, remainder 0
211-I24 413, remainder 94
211-I25 207, remainder 0
211-I26 108, remainder 0
211-I27 727, remainder 38

Lesson 50

211-A 3
211-B 53
211-C Some of the symbols are used more than once in the numeral.
211-D When you found the number expressed by the numeral, you found the sum of the numbers expressed by the symbols in the numeral.
212-E So you can tell which strokes express the number 1 and which strokes express the number 3
212-F Since each of these strokes expresses 3, the two strokes express 2(3), or 6.
212-G Yes
212-H Yes. 1(3 × 3 × 3) = 27.
212-I 53
212-J Yes * Yes * The stroke is used more than once in numeral C and when you found the number expressed by numeral C, you found the sum of the numbers expressed by the symbols in the numeral.
212-K 2(3 × 3 × 3) + 1(3 × 3) + 2(3) + 1(1), or 70 * 1(3 × 3 × 3) + 2(3 × 3) + 1(3) + 2(1), or 50
212-L Yes * By distributing the 6 strokes in the spaces in two different ways
212-M You could use an open space to separate the strokes in one position from those in another position. Numeral E in D3 could be written as / // / //.
213-N (Column 1) No * No matter where the stroke is located in the numeral, it will express the number 1. No matter where the star is located in the numeral, it will express the number 3. Likewise, the diamond will always express 3 × 3, and the arrowhead will always express 3 × 3 × 3.
213-A 10 × 10, or 100
213-B 10 × 10 × 10, or 1000
213-C Yes * 4754
213-D The numbers expressed by the symbols in the numeral are different. For example, a stroke in the second space from the right in a base-ten numeral expresses the number 10, while a stroke in the same position in a base-three numeral expresses the number 3. A stroke in the third space from the right in a base-ten numeral expresses the number 10 × 10, or 100, while a stroke in the same position in a base-three numeral expresses the number 3 × 3, or 9.
213-E Numerals I and J
213-F Numeral J * It is easier to determine what number is named. It is also easier to write the numeral.
213-G Numerals K and L * Numeral L
213-H Three strokes in any space may be replaced by one stroke in the next space to the left.
213-I 9 * Ten strokes in any space may be replaced by one stroke in the next space to the left.
213-J 30
213-K 63
213-L The number of strokes in each space in numeral P is the sum of the number of strokes in the corresponding spaces in numerals M and N. For example, since there are 3 strokes in the second space from the right in numeral M and 3 strokes in the second space from the right in numeral N, there are 3 + 3, or 6 strokes in the second space from the right in numeral P.
213-M 4(16) + 6(4) + 5(1), or 93
213-N (Column 2) 3
213-O You know that you can regroup the strokes in numeral P because it is a base-four numeral and has more than 3 strokes in each space. * You

should replace four of the strokes in the space farthest to the right by one stroke in the second space from the right because 1 four is the same as 4 ones.

213-P Yes

214-Q Because the one stroke in the third space from the right expresses the same number as the four strokes in the second space from the right * Yes

214-R Four of the strokes in the space third from the right can be replaced by one stroke in a new space fourth from the right.

214-S Yes

214-T 93 * Yes * Yes

214-U The greatest number of strokes needed in a space is one less than the base. When the number of strokes in any position is equal to the number which is the base, these strokes can be replaced by one stroke in the next space to the left.

On your own

214-1 1(7 × 7) + 4(7) + 3(1), or 80

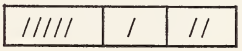
214-2 3(7 × 7) + 6(7) + 4(1), or 193

214-3 5(7 × 7) + 3(7) + 6(1), or 272

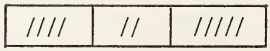
214-4 2(7 × 7 × 7) + 4(7 × 7) + 5(7) + 3(1), or 920

214-5 5(7 × 7 × 7) + 6(7 × 7) + 4(7) + 2(1), or 2039

214-6

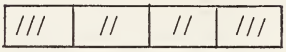


214-7



214-8

214-9



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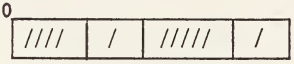
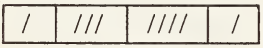


214-11 188, 161, 176, 735, 931

214-12



Lesson 51

215-A Yes * Because each of the other strokes expresses a number other than 1

215-B 64 + 12 + 3, or 79

215-C 3 * Yes * Yes

215-D Yes * Yes

215-E Yes * Yes

215-F 64

215-G Yes

215-H No strokes

215-I 2 strokes

215-J 1 is the fourth digit from the right, not the third digit from the right.

215-K Yes. In numeral B the digit farthest to the right expresses 3(1) while the same digit second from the right expresses 3(8).

215-L Yes. The position of a symbol in the numeral determines the number expressed by the symbol.

215-M 79

215-N Yes * Yes

216-O Yes * When you found the number expressed by numeral B, you found the sum of the numbers expressed by the digits.

216-P Four * Numeral D

216-Q Numeral E is read "two three zero one, base four." Numeral F is read "two three one zero, base four." Numeral G is read "three zero two, base four." Numeral H is read "two zero, base four." Numeral I is read "three zero one, base four." Numeral J is read "one zero zero zero, base four."

216-R To find the number named by each numeral in D4, you must find the sum of the numbers expressed by the digits in each numeral. * Numeral D names the number 141. Numeral E names the number 2(64) + 3(16) + 0(4) + 1(1), or 177. Numeral F names the number 2(64) + 3(16) + 1(4) + 0(1), or 180. Numeral G names the number 3(16) + 0(4) + 2(1), or 50. Numeral H names the number 2(4) + 0(1), or 8. Numeral I names the number 3(16) + 0(4) + 1(1), or 49. Numeral J names the number 1(64) + 0(16) + 0(4) + 0(1), or 64.

216-A 5(1), or 5

216-B Yes * Each stroke in the second space from the right expresses the number 10. So 9 strokes in this space express 9(10), or 90.

216-C 4(10 × 10), or 400 * 5(10 × 10 × 10), or 5000

216-D Yes * 2(10,000), or 20,000

216-E 25,495

216-F Yes * Yes

216-G The digit 5 that is farthest to the right expresses 5(1), or 5. The digit 9 expresses 9(10), or 90. The digit 4 expresses 4(10 × 10), or 400. The digit 5 that is the fourth digit from the right expresses 5(10 × 10 × 10), or 5000. The digit 2 expresses 2(10 × 10 × 10 × 10), or 20,000.

216-H Yes

217-I Think of a base-ten numeration system that uses strokes. The most strokes you need in any space is 9. This means that you need 10 different digits to represent from 0 through 9 strokes. * In a base-four system, you need 4 different digits.

217-J The number associated with the fourth place from the right is (10 × 10 × 10), or 1000. Therefore, the digit 5 in the fourth place from the right expresses the number 5(10 × 10 × 10), or 5000.

217-K Yes

217-L The position of a digit in the numeral determines the number expressed by that digit. * The symbol 5 is used twice in numeral B. * The number expressed by numeral B is the sum of the numbers expressed by the digits in numeral B.

217-M Since there is exactly one digit in each position in the numeral, you can decide what position each digit is in without a chart. * Numeral B is easier to write than numeral A.

217-A Yes * Each stroke in the space farthest to the right expresses 1. Therefore, 10 strokes in this space express 10(1), or 10.

217-B Yes * Since numeral C is a base-twelve numeral, each stroke in the space second from the right expresses 12. Four strokes in this space express 4(12), or 48.

217-C Yes * 4(12 × 12 × 12), or 6912

217-D Find the sum of the numbers expressed by the digits in numeral C.

217-E No * The familiar digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. No one of these digits expresses the number 10.

217-F 10(1), or 10

217-G 4(12), or 48

217-H There is no familiar digit in the decimal system that expresses the number 11.

218-I Yes

218-J 4(12 × 12 × 12), or 4(1728), or 6912

218-K Yes

218-L Since there is exactly one digit in each position in numeral D, you can determine what position a digit is in without using a chart.

218-M "Four el four dek, base twelve."

On your own

218-1 415

218-2 1785

218-3 3160

218-4 17,460

218-5 13

218-6 21

218-7 147

218-8 313

218-9 287

218-10 590

218-11 9

218-12 108

218-13 22

218-14 121

218-15 1587

218-16 2147

218-17 15

218-18 11

218-19 23

218-20 28

218-21 25

218-22 17

218-23 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30

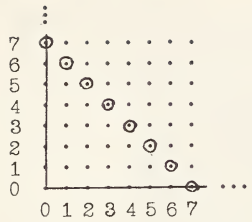
218-24 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23

218-25 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16

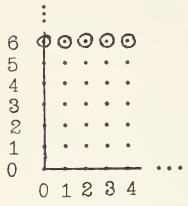
218-26 1, 2, 3, 4, 5, 6, 7, 8, 9, ↄ, 10, 11, 12, 13, 14

Keeping Skilful

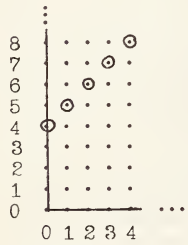
219-1 {(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (7, 0)}



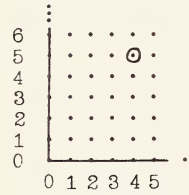
219-2 {(0, 6), (1, 6), (2, 6), (3, 6), (4, 6)}



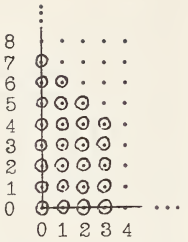
219-3 {(0, 4), (1, 5), (2, 6), (3, 7), (4, 8)}



219-4 {(4, 5)}



219-5 {(0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6), (0, 7), (1, 0), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 0), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4)}



219-6 {3}

219-7 {1, 2, 3, 4, 5, 6, 7}

219-8 {51}

219-9 {120}

219-10 {1, 2, 3, ..., 24}

219-11 {161}

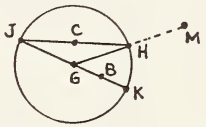
219-12 {1, 2, 3, ..., 16}

219-13 {216}

219-14 {456}

219-15 {195}

219-16 See the drawing below. Point H can be any point in circle G.



219-17 See the drawing for 219-16. J can be any point in circle G just so that H, J, and G are noncollinear.

219-18 See drawing for 219-16 Points J, G, and K must be collinear and K must be in circle G.

219-19 See drawing for 219-16. M can be anywhere in the exterior of circle G such that points G, H, and M are collinear, and H is between G and M.

219-20 See drawing for 219-16. Point B can be anywhere in the interior of circle G just so it is between G and K. Point C can be anywhere in the interior of circle G just so it is between H and J.

219-21 $\{X \mid \overline{GX} \cong \overline{GH}\}$, or $\{X \mid \overline{GX} \cong \overline{GJ}\}$, or $\{X \mid \overline{GX} \cong \overline{GK}\}$

219-22 $\{X \mid \overline{GX} < \overline{GH}\}$, or $\{X \mid \overline{GX} < \overline{GJ}\}$, or $\{X \mid \overline{GX} < \overline{GK}\} * \{X \mid \overline{GX} > \overline{GH}\}$, or $\{X \mid \overline{GX} > \overline{GJ}\}$, or $\{X \mid \overline{GX} > \overline{GK}\}$

219-23 $\overline{GK} \cong \overline{JG}$. Since segments GK and JG are both radii of circle G, they are congruent.

219-24 $\overline{GC} < \overline{GM}$. Since point M is located in the exterior of the circle, you know that $\overline{GH} < \overline{GM}$. Since point C is located in the interior of circle G, you know that $\overline{GC} < \overline{GH}$. You also know that if $\overline{GC} < \overline{GH}$ and $\overline{GH} < \overline{GM}$, then $\overline{GC} < \overline{GM}$. This is an application of the property of "less than" for line segments. See lesson 11, page 43.

219-25 $\overline{HG} < \overline{MG}$. Since H is between G and M, $\overline{HG}$ must be less than $\overline{MG}$.

219-26 $\overline{GH} < \overline{JK}$. You know that $\overline{GH}$ is a radius of circle G and that $\overline{JK}$ is a diameter of circle G. Since $\overline{JK}$ is the union of two radii, $\overline{GJ}$ and $\overline{GK}$, you know that $\overline{GH}$ is less than $\overline{JK}$.

Lesson 52

219-A 10 * 10

219-B 10

220-C (Column 1) No * Numeral A has only 4 symbols but

numeral A names the number 220.

220-D 220 * Finding the sum of the numbers expressed by the symbols in numeral B.

220-E Yes

220-F 27 * 20,016

220-G Yes * Yes * The Egyptian numeration system had the property of addition because to find the number named by a numeral, you found the sum of the numbers expressed by the symbols in the numeral. The Egyptian numeration system had the property of repetition because the same symbol could be used more than once in a numeral.

220-H Numeral E expresses the number 240. Numeral F expresses the number 1324. Numeral G expresses the number 43.

220-I Yes

220-J 1607 * By finding the sum of the numbers expressed by the symbols in the numeral

220-K The 10 heelbones were replaced by the symbol for 100. * Yes

220-L The decimal numeration system requires fewer symbols to name most numbers than the Egyptian numeration system requires. For example, the decimal numeral 1607 has four symbols, while numeral I in D4 has 14 symbols. There are some exceptions. The Egyptian numeral for 1,000,000, for example, has only one symbol.

220-A X * C * M

220-B V

220-C (Column 2) L * D

221-D 6

221-E VII * VIII

221-F Numeral B consists of the same symbols as numeral A. * The symbols are in a different order.

221-G Add 1 to 10. * 11

221-H Subtract 1 from 10. * 9

221-I Yes * Yes

221-J Numeral A expresses 6; B expresses 4; E expresses 11; F expresses 9; G expresses 60; H expresses 40; J expresses 110, and K expresses 90.

221-K 6, 11, 60, and 110 * 4, 9, 40, and 90

221-L X expresses 10; C expresses 100, and I expresses 1. * C expresses 100; X expresses 10, and I expresses 1.

221-M 93 * 113

221-N No. The X in both XC and CX expresses the number 10 even though you subtract the 10 in one case and add the 10 in the other case. * No. For example, in the numeral XXX, each X expresses the number 10.

221-O If a symbol is written at the left of another symbol that expresses a greater number, you subtract the lesser number from the greater. If a symbol is written at the right of another symbol that expresses a greater number, you add the lesser number to the greater. Therefore, to find the number expressed by XCIII, you subtract 10 from 100 and then add 3 to the difference. To find the number expressed by CXIII, you add 100, 10, and 3.

221-P Yes * The same symbol may be used more than once in a numeral.

221-Q Numeral Q expresses 20; R expresses 53, and S expresses 137.

221-R 5 * 5

222-S Yes * V * L

222-T Yes

222-U LL was replaced by C and VV was replaced by X. * 210

222-V Yes * Numeral W

222-W The arrangement of the symbols is important in XCVII and XLV. The symbol X must remain on the left of C to show that 10 is subtracted from 100. Likewise, the symbol X must remain on the left of L to show that 10 is subtracted from 50.

222-X The decimal system is more convenient than the Roman system. * Fewer symbols are necessary to express most numbers in the decimal system than in the Roman system. The use of subtraction and a mixed base makes Roman numerals more difficult to interpret than decimal numerals.

222-A The symbol on the left expresses 1 and the symbol on the right expresses 10.

222-B 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1, or 54

222-C Yes * Yes

222-D 22

222-E 10(60) + 10(60), or 1200

222-F 1(60 × 60) + 10(60) + 1(60) + 10 + 1, or 4271

222-G 10(60) + 10(60) + 10 + 10 + 1 + 1, or 1222

222-H 1(60 × 60) + 1(60 × 60), or 7200 * 1(60 × 60) + 1(60 × 60) + 10(60) + 10(60) + 10(60) + 1 + 1 + 1, or 9003

223-I Yes * In the Babylonian numeration system, the position of each symbol in a numeral determined the number expressed by the symbol.

223-J Numeral F expresses 10 + 10 + 1 + 1 + 1, or 23; G expresses 10(60) + 10 + 10 + 10 + 1 + 1, or 632, and H expresses 1(60 × 60) + 10(60) + 10(60) + 10 + 1 + 1 + 1 + 1 + 1, or 4815.

223-K Yes

223-L 1(60 × 60) + 10(60) + 10(60) + 10(60) + 10 + 10 + 10 + 10 + 10 + 10 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1, or 5470

223-M 10 * The symbol for 10

223-N 6 * In the second group from the right, the symbol for 1 expresses 1(60). This is the same number expressed by the 6 symbols for 10 in the group farthest to the right.

223-O Yes * Numeral J because fewer symbols are used than in numeral I

223-P The decimal numeration system is more convenient because it has only one base and more code symbols. For example, to express each of the numbers from 2 through 9 in the decimal system, you use only one symbol. In the Babylonian system, you would use more than one symbol to express each of the numbers from 2 through 9.

On your own

223-1 31,300

223-2 54

223-3 1,120,000

223-4 1776

223-5 644

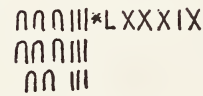
223-6 99

223-7 21

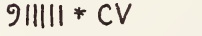
223-8 1812

223-9 73,203

223-10



223-11

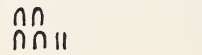


223-12

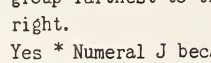
223-13

223-14

223-15



223-16



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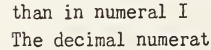

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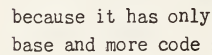

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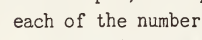
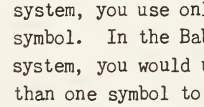


223-20



Checking up

Test 54

224-1 599

224-2 1814

224-3 4073

224-4 8

224-5 14

224-6 19

224-7 46

224-8 91

224-9 109

224-10 33

224-11 1963

224-12 520

Test 55

224-13 (e) Code system

224-14 (d) Grouping system with place value and the stroke

224-15 (c) Grouping system with place value and digits

224-16 (a) Tally system

Applying mathematics

224-1 x, number of tons of iron ore mined in both Newfoundland and Quebec. $8{,}174{,}779 + 8{,}872{,}948 = x$. {17047727}. 17,047,727 tons

224-2 x, number of tons fewer mined in Newfoundland than in Quebec.

$8,872,948 - 8,174,779 = x$.
{698169}. 698,169 tons

224-3 x, number of lawns Ken mowed during July and August.
y, number of lawns Jerry mowed during July and August.
$x + y = 42 \wedge 14 + 9 = x$.
{(23, 19)}. 19 lawns

224-4 x, number of factories in Montreal.
$4/1 \sim x/1200$.
{4800}. About 4800 factories

224-5 x, number of cupcakes Mrs. Brown made.
$x > 35 \wedge x - 24 < 17$.
{36, 37, 38, 39, 40}. From 36 through 40 cupcakes

224-6 x, number of years that is Ann's age now.
y, number of years that is Sue's age now.
$y < x \wedge x + 5 = 19$.
{(14, 1), (14, 2), (14, 3), (14, 4), (14, 5), (14, 6), (14, 7), (14, 8), (14, 9), (14, 10), (14, 11), (14, 12), (14, 13)}.
Ann is 14 years old and Sue is from 1 through 13 years old.

224-7 x, number of raincoats sold.
$115 - x < 59$.
{57, 58, 59, ..., 115}. From 57 through 115 raincoats

225-8 x, number of chocolate cookies Kay bought.
y, number of ginger cookies Kay bought.
$x < y \wedge x + y = 7$.
{(1, 6), (2, 5), (3, 4)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (2, 5) is: Kay bought 2 chocolate cookies and 5 ginger cookies.

225-9 x, number of tons of raw sugar produced in North America.
$34/100 \sim x/39,138,000$.
{13306920}. About 13,306,920 tons

225-10 x, number of telephones in Canada.
y, number of telephones in the United States.
$x + y = 71,768,000 \wedge x + 877,000 = 6,000,000$.
{(5123000, 66645000)}. There are 5,123,000 telephones in Canada and 66,645,000 telephones in the United States.

225-11 x, number of feet that is the width of the Chubb Crater.
$64/10 \sim x/1800$.
{11520}. 11,520 feet

225-12 x, number of dollars interest.
y, number of dollars Mr. Horton paid the bank at the end of the year.
$y = 1350 + x \wedge 6/100 \sim x/1350$.
{(81,1431)}. $1431

225-13 x, number of points Ted scored.
y, number of points Bill scored.
$x + y = 21 \wedge x - y = 7$.
{(14, 7)}. Ted scored 14 points and Bill scored 7 points

225-14 x, number of dollars discount on freezer.
y, number of dollars discount for each $100 of the regular price.
$215 - 172 = x \wedge x/215 \sim y/100$.
{(43, 20)}. 20%

225-15 x, number of degrees that the temperature increases from the surface of the earth to 18,120 feet beneath the surface.
y, number of degrees that is the temperature 18,120 feet beneath the surface of the earth.
$60/1 \sim 18,120/x \wedge 49 + x = y$.
{(302, 351)}. 351 degrees Fahrenheit

Keeping skilful

225-1 1000, 2000, 1920
225-2 47,000, 47,500, 47,490
225-3 6000, 6300, 6200
225-4 1000, 2000, 1990
225-5 35,000, 35,200, 35,160
225-6 9000, 9100, 9000
225-7 2000, 2800, 2710
225-8 95,000, 95,700, 95,650
225-9 34,207
225-10 81,526
225-11 196,700
225-12 201,181
225-13 740,955
225-14 4284
225-15 1981
225-16 836
225-17 562,324
225-18 7232
225-19 12,098
225-20 516,026
225-21 5365

225-22 30,160
225-23 151,844
225-24 55,096
225-25 441,100
225-26 234, remainder 0
225-27 176, remainder 80
225-28 49, remainder 98
225-29 195, remainder 113
225-30 156, remainder 312
225-31 304, remainder 0
225-32 {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}
225-33 {12/16}
225-34 {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9), (7, 10), (8, 11)}
225-35 {15/10, ...}
225-36 {(99, 51)}
225-37 {3/5, 6/10, 9/15, 12/20}
225-38 {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5)}
225-39 {2/3, 4/6, 6/9, 8/12, 10/15}
225-40 {(625, 575)}
225-41 {(1, 27), (2, 27), (3, 27), (4, 27), (5, 27), (6, 27), (7, 27), (8, 27)}
225-42 {15/18}
225-43 {(72, 1), (72, 2), (72, 3), (72, 4), (72, 5), (72, 6), (72, 7)}
225-44 {(1, 3), (2, 6), (3, 9), (4, 12), (5, 15), (6, 18)}
225-45 {(3, 150)}
225-46 { }

Lesson 53

226-A Yes * Yes
226-B The product of 16 and 1. The product of 4 and 4. The product of four 2's.
226-C Yes * Yes * Yes * No
226-D Four times
226-E Yes * Yes
226-F Yes * Yes * Yes
226-G Yes * 2
226-H Yes
226-I *a* was replaced by 12, *b* by 1, and *c* by 12. * Yes * Since 12 and 1 can be used to obtain the product 12, you know that 12 and 1 are factors of 12.
226-J Yes * Yes
226-K 1, 3, 5, 9, 15, 45
226-L Yes * Yes
226-M Yes * Yes
226-N Yes. You know that if you choose any number in {1, 2, 3, ...} and multiply this number by 1, the product will be the number you chose.
226-O Yes * Yes. Since the product of 1 and the number you chose is the number, both 1 and the number are factors of the number.
227-A - - -
227-B Yes * Because $10^2 = 10 \times 10$ and $100 = 10 \times 10$
227-C Yes * Yes. $10^3 = 10 \times 10 \times 10$ and $1000 = 10 \times 10 \times 10$.
227-D 10^4 * $10 \times 10 \times 10 \times 10$, or 10,000
227-E 5
227-F 10^4 is the fourth power of 10.
227-G 2 * 3 * 7 * Twice * Three times * Seven times
227-H *a* was replaced by 3 and *n* by 6. * *a* was replaced by 7 and *n* by 1. * *a* was replaced by 5 and *n* by 4.
227-I Yes
227-J 3 * 6 * 6 times
227-K The base of 7^1 is 7, of 5^4 is 5, of 11^2 is 11, and of 10^7 is 10. * The exponent of 7^1 is 1, of 5^4 is 4, of 11^2 is 2, and of 10^7 is 7.
227-L $3^6 = 729$. $7^1 = 7$. $5^4 = 625$. $11^2 = 121$. $10^7 = 10,000,000$.
227-M Yes * Yes
227-N Yes * Yes. The first power of any number is the number.
227-O 1
227-P 1
227-Q 1
227-R 1
227-S 1
227-T Yes * If you replace *n* by 1, a true statement is obtained because the first power of any number is the number. If you replace *n* by any other counting number, a true statement is obtained because regardless of how many times 1 is used as a factor, the product is always 1.
228-A Yes * Yes * Yes
228-B 10^2
228-C 10^4
228-D 10^3
228-E 10^5
228-F Yes
228-G By using exponential notation * Yes
228-H 2 * 4
228-I $3(10^4) + 5(10^3) + 9(10^2) + 2(10^1) + 7(1)$
228-J $1(10^2) + 8(10^1) + 5(1)$

228-K $7(10^3) + 3(10^2) + 3(10^1) + 0(1)$

228-L $2(10^4) + 9(10^3) + 5(10^2) + 3(10^1) + 7(1)$

228-M $6(10^5) + 6(10^4) + 3(10^3) + 5(10^2) + 0(10^1) + 1(1)$

On your own

228-1 2, 12, 18, 30, 36, 54

228-2 3, 12, 18, 30, 36, 45, 54, 99

228-3 5, 30, 45, 85

228-4 There are none.

228-5 {1, 2, 4, 8}

228-6 {1, 19}

228-7 {1, 2, 7, 14}

228-8 {1, 11}

228-9 {1, 5, 7, 35}

228-10 {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

228-11 Yes * Yes

228-12 The base is 10 and the exponent is 4.

228-13 The base is 15 and the exponent is 3.

228-14 The base is 7 and the exponent is 7.

228-15 The base is 9 and the exponent is 10.

228-16 2

228-17 4

228-18 16

228-19 32

228-20 9

228-21 81

228-22 1

228-23 125

228-24 243

228-25 100,000

228-26 4

228-27 144

228-28 3^4

229-29 2^6

229-30 11^2

229-31 10^6

229-32 True statement

229-33 False statement

229-34 True statement

229-35 $1(10^1) + 6(1)$

229-36 $5(10^1) + 8(1)$

229-37 $7(10^2) + 4(10^1) + 9(1)$

229-38 $1(10^3) + 0(10^2) + 0(10^1) + 7(1)$

229-39 $2(10^5) + 6(10^4) + 2(10^3) + 8(10^2) + 4(10^1) + 4(1)$

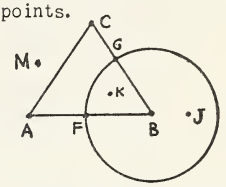
Keeping Skilful

229-1 $S = \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\}$.
$T = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)\}$.
$S \cup T = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3), (3, 4), (4, 5)\}$.
$S \cap T = \{(0, 1), (1, 2), (2, 3)\}$.

229-2 $S = \{(0, 7), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2)\}$.
$T = \{(0, 6), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)\}$.
$S \cup T = \{(0, 6), (0, 7), (1, 6), (2, 5), (2, 6), (3, 4), (3, 6), (4, 3), (4, 6), (5, 2), (5, 6)\}$.
$S \cap T = \{(1, 6)\}$.

229-3 $S = \{(0, 7), (1, 7), (2, 7), (3, 7), (4, 7), (5, 7)\}$.
$T = \{(9, 0), (8, 1), (7, 2), (6, 3), (5, 4), (4, 5), (3, 6)\}$.
$S \cup T = \{(0, 7), (1, 7), (2, 7), (3, 7), (4, 7), (5, 7), (9, 0), (8, 1), (7, 2), (6, 3), (5, 4), (4, 5), (3, 6)\}$.
$S \cap T = \{\ \}$.

229-4 See the sketch below. Points A, B, and C can be any three noncollinear points.



229-5 See the sketch for 229-4. Circle B can be any circle that has its center at B and intersects $\overline{AB}$ in F and $\overline{BC}$ in G.

229-6 See the sketch for 229-4. J can be any point in the interior of circle B just so it is in the exterior of ΔABC.

229-7 See the sketch for 229-4. K can be any point that is in the interior of both circle B and ΔABC.

229-8 See the sketch for 229-4. M can be any point that is in both the exterior of $\angle CAB$ and the interior of $\angle CBA$.

229-9 B

229-10 G, F, J, M

229-11 A, C

Lesson 54

229-A 3

229-B 2 * 6 * 5

229-C $5(10^3) + 6(10^2) + 2(10^1) + 3(1)$

229-D Seven * Yes

230-E By using exponential notation

230-F Yes

230-G Yes

230-H 6 * 5

230-I Nine

230-J 9

230-K 7 * The digit 1 is associated with the first power of the base. The digit 8 is associated with the second power of the base. The digit 4 is associated with the third power of the base.

230-L $1(4^3) + 3(4^2) + 0(4^1) + 2(1)$

230-M $3(6^3) + 5(6^2) + 4(6^1) + 2(1)$

230-N $3(8^3) + 2(8^2) + 7(8^1) + 7(1)$

230-A The base of numeral A is four. The base of numeral B is two. The base of numeral C is six. The base of numeral D is twelve.

230-B Zero ones * First * Second

231-C (Top of column 1) First * Second * First

231-D (Column 1) Yes

231-E (Column 1) Numeral A expresses the number 52. Numeral B expresses the number 5. Numeral C expresses the number 176. Numeral D expresses the number 861.

231-F (Column 1) $3(7^1) + 0(1)$

231-G (Column 1) $2(8^1) + 5(1)$

231-H (Column 1) $2(3^2) + 1(3^1) + 0(1)$

231-I (Column 1) $4(6^2) + 3(6^1) + 5(1)$

231-J (Column 1) $1(5^3) + 3(5^2) + 1(5^1) + 4(1)$

231-K (Column 1) $1(2^4) + 0(2^3) + 1(2^2) + 0(2^1) + 1(1)$

231-L In exercise F, the numeral expresses 21. In exercise G, the numeral expresses 21. In exercise H, the numeral expresses 21. In exercise I, the numeral expresses 167. In exercise J, the numeral expresses 209. In exercise K, the numeral expresses 21.

231-A 25 * Yes

231-B No * 97 is less than 5^3.

231-C (Bottom of column 1) Yes * Three

231-D (Column 2) Yes

231-E (Column 2) The computation shows that there are 3 twenty-fives in 97. * The space third from the right

231-F (Column 2) 4 * The space second from the right

231-G (Column 2) 2 * The space farthest to the right

231-H (Column 2) The digit 2 * The digit 4 is associated with the first power of 5. The digit 3 is associated with the second power of 5.

231-I (Column 2) $3(5^2) + 4(5^1) + 2(1)$, or 97 * Yes

231-J (Column 2) 8

231-K (Column 2) 64 * Yes

232-L 132 is less than 8^3. * Three

232-M To find the digit that belongs in the space associated with 8^2 * 2 * 4

232-N There are no groups of 8 in 4.

232-O 4

232-P 4 * The digit 0 is associated with the first power of 8. The digit 2 is associated with the second power of 8.

232-Q $2(8^2) + 0(8^1) + 4(1)$, or 132 * Yes

232-R 221_{six}

232-S $\mathcal{X}3_{twelve}$

232-T 11111_{two}

232-U 285

232-V 285 ÷ 144 = 1, remainder 141.
141 ÷ 12 = 11, remainder 9.
$285 = 1\mathcal{E}9_{twelve}$

232-W Yes. You know that both the numerals 346_{nine} and $1\mathcal{E}9_{twelve}$ express the number 285.

On your own

232-1 3123_{five} * 413

232-2 645_{seven} * 327

232-3 5700_{eight} * 3008

232-4 $2\mathcal{E}8_{twelve}$ * 428

232-5 320

232-6 9

232-7 1382

232-8 797

232-9 147

232-10 842

232-11 1330

232-12 588

232-13 1279

232-14 404_{five}

232-15 54_{twelve}

232-16 1010111_{two}

232-17 10043_{five}

232-18 12332_{seven}

232-19 1121_{three}

232-20 216401_{eight}

232-21 256_{nine}

232-22 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

232-23 Base-two numerals are 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110

232-24 Base-twelve numerals are 1, 2, 3, 4, 5, 6, 7, 8, 9, $\mathcal{X}$, $\mathcal{E}$, 10, 11, 12

232-25 Base-eight numerals are 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16

232-26 131_{twelve}

232-27 $\mathcal{E}_{twelve}$

232-28 $1\mathcal{E}\mathcal{E}_{twelve}$

232-29 $8\mathcal{X}_{twelve}$

232-30 55_{twelve}

232-31 $121\mathcal{X}_{twelve}$

232-32 110_{twelve}

232-33 1828_{twelve}

Lesson 55

233-A 10, 11, 12, 13, 14 * 22, 23, 30, 31, 32

233-B 10 * 4

233-C Only 4 different digits are needed to name any natural number using a base-four system.

233-D The numeral 11 * Since only the digits 0, 1, 2, and 3 are used in the base-four system, you use the numeral 11, not the numeral 5, to express the sum of 2 and 3.

233-E 12 * 10

233-F

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

233-G Yes * In the place farthest to the right, or the one's place.

234-H 6 * In the second place from the right, or the ten's place * 64

234-I 25 * 39

234-J Yes * To show that there are zero ones

234-K 2 * 1 * 4 * The digit 4 is not used in the base-four system so you must regroup the 4 fours into 1 group of sixteen. * To show that there are 0 fours * 1 sixteen

234-L $2(4^2)$, or 2(16) * Yes * To show that there are 0 sixteens * To show that there is 1 sixty-four

234-M Yes. $1(4^3) + 0(4^2) + 0(4^1) + 0(1) = 64$.

234-N $233_{four} = 47$. $212_{four} = 38$. $47 + 38 = 85$.

234-O Yes * Yes. $1(4^3) + 1(4^2) + 1(4^1) + 1(1) = 85$. * Yes

234-P

$$\begin{array}{r} {}^{1} \\ 203 \\ \underline{123} \\ 332 \end{array}$$

$203_{four} = 35$. $123_{four} = 27$. $35 + 27 = 62$.

$332_{four} = 62$.

234-Q See the table below. * $\mathcal{X}$ and $\mathcal{E}$

*	0	1	2	3	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$
0	0	1	2	3	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$
1	1	2	3	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10
2	2	3	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11
3	3	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12
4	4	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13
5	5	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14
6	6	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14	15
7	7	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14	15	16
8	8	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14	15	16	17
9	9	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14	15	16	17	18
$\mathcal{X}$	$\mathcal{X}$	$\mathcal{E}$	10	11	12	13	14	15	16	17	18	19
$\mathcal{E}$	$\mathcal{E}$	10	11	12	13	14	15	16	17	18	19	$1\mathcal{X}$

234-R Yes

235-S $19\mathcal{E}9_{twelve} = 3165$.

$\mathcal{X}0\mathcal{E}\mathcal{X}_{twelve} = 17,422$.

$3165 + 17,422 = 20,587$.

$\mathcal{E}\mathcal{X}\mathcal{E}7_{twelve} = 20,587$.

235-T

$$\begin{array}{r} 2\mathcal{E}\mathcal{X} \\ \underline{504} \\ 802 \end{array}$$

$2\mathcal{E}\mathcal{X}_{twelve} = 430$.

$504_{twelve} = 724$.

$430 + 724 = 1154$.

$802_{twelve} = 1154$.

235-U

$$\begin{array}{r} 137 \\ \underline{\mathcal{X}89} \\ 1004 \end{array}$$

$137_{twelve} = 187$.

$\mathcal{X}89_{twelve} = 1545$.

$187 + 1545 = 1732$.

$1004_{twelve} = 1732$.

235-A 10 * The digit 6 is not used in the base-four numeration system. You should use the numeral 12 to express the product of 2 and 3. * Since the digit 9 is not used in the base-four numeration system, you should use the numeral 21 to express the product of 3 and 3.

235-B

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

235-C To show that there are 3 ones

235-D Because the 2 is in the fours' place * Because the 1 is in the sixteens' place and the 2 is in the fours' place

235-E 300_{four} names 3 sixteens, 0 fours, and 0 ones. * 2100_{four} names 2 sixty-fours, 1 sixteen, 0 fours, and 0 ones. * You have 2 sixty-fours and 1 sixteen from multiplying the sixteens and 1 sixteen from multiplying the fours.

235-F From multiplying the sixty-fours, there are 3. * The sixty-fours and the 2 sixty-fours obtained from multiplying the sixteens are five, or 11_{four} sixty-fours in all * Since 5(64) is 1(256) + 1(64), you write 1 in the sixty-fours' place. * 1

235-G $3_{four} = 3$. $1321_{four} = 121$. $3 \times 121 = 363$. * Yes * $11223_{four} = 363$. * Yes

236-H $3232_{four} = 238$. $3_{four} = 3$. $3 \times 238 = 714$. $23022_{four} = 714$. * Yes

236-I

×	0	1	2	3	4	5	6	7	8	9	ↄ	Ɛ
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	ↄ	Ɛ
2	0	2	4	6	8	ↄ	10	12	14	16	18	1ↄ
3	0	3	6	9	10	13	16	19	20	23	26	29
4	0	4	8	10	14	18	20	24	28	30	34	38
5	0	5	ↄ	13	18	21	26	2Ɛ	34	39	42	47
6	0	6	10	16	20	26	30	36	40	46	50	56
7	0	7	12	19	24	2Ɛ	36	41	48	53	5Ɛ	65
8	0	8	14	20	28	34	40	48	54	60	68	74
9	0	9	16	23	30	39	46	53	60	69	76	83
ↄ	0	ↄ	18	26	34	42	50	5Ɛ	68	76	84	92
Ɛ	0	Ɛ	1Ɛ	29	38	47	56	65	74	83	92	ↄ1

236-J From the table you know that $4 \times$ Ɛ $= 38$. 38_{twelve} means 3 twelves and 8 ones. You write 8 in ones' place. You must remember to add the 3 twelves to the other twelves. * You know that $4 \times 1 = 4$. Since 4 twelves + 3 twelves = 7 twelves, you write 7 in twelves' place. * In one hundred forty-fours' place * In one thousand seven hundred twenty-eights' place

236-K $31Ɛ_{twelve} = 455$. $4_{twelve} = 4$. $455 \times 4 = 1820$. $1078_{twelve} = 1820$.

236-L

$$\begin{array}{r} 603 \\ \times\quad 8 \\ \hline 4020 \end{array}$$

$603_{twelve} = 867$. $8_{twelve} = 8$. $8 \times 867 = 6936$. $4020_{twelve} = 6936$.

236-M 0 and 1 are the only digits in the base-two numeration system.

236-N The digit 2 is not used in the base-two numeration system. You should use the numeral 10 to express the sum of 1 and 1.

236-O Yes

236-P Yes

236-Q Example A. $101_{two} = 5$. $111_{two} = 7$. $5 + 7 = 12$. $1100_{two} = 12$.

Example B. $101_{two} = 5$. $1_{two} = 1$. $5 \times 1 = 5$. $101_{two} = 5$.

236-R Yes * $111_{two} = 7$. $11_{two} = 3$. $3 \times 7 = 21$. $10101_{two} = 21$.

On your own

237-1 20_{four}

237-2 111_{four}

237-3 12001_{four}

237-4 12023_{four}

237-5 191_{twelve}

237-6 27_{twelve}

237-7 31_{twelve}

237-8 $16Ɛ_{twelve}$

237-9 120_{four}

237-10 132_{four}

237-11 32_{four}

237-12 3300_{four}

237-13 $Ɛ64_{twelve}$

237-14 42008_{twelve}

237-15 $13Ɛ70_{twelve}$

237-16 $ↄ068_{twelve}$

237-17

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	10
2	2	3	4	5	6	10	11
3	3	4	5	6	10	11	12
4	4	5	6	10	11	12	13
5	5	6	10	11	12	13	14
6	6	10	11	12	13	14	15

237-18

x	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	11	13	15
3	0	3	6	12	15	21	24
4	0	4	11	15	22	26	33
5	0	5	13	21	26	34	42
6	0	6	15	24	33	42	51

237-19 33_{seven}

237-20 264_{seven}

237-21 155_{seven}

237-22 1445_{seven}

237-23 112_{seven}

237-24 161_{seven}

237-25 63_{seven}

237-26 1515_{seven}

237-27 1212_{seven}

237-28 2304_{seven}

237-29 11_{two}

237-30 1000_{two}

237-31 1000_{two}

237-32 10110_{two}

237-33 110_{two}

237-34 100011_{two}

237-35 11110_{two}

237-36 11011_{two}

Lesson 56

238-A The digits 4, 6, and 2 * The digits 1, 9, and 3 * The digits 2, 5, and 6

238-B 256 billion

238-C 193 million

238-D 462 thousand

238-E 109

238-F 69 billion 840 million 20 thousand 2

238-G They are separated by commas.

238-H 721 billion 635 million 599 thousand 483

238-I 20 billion 840 million 500 thousand 80

238-J 87 million 692 thousand 110

238-K 660 billion 30 million 900 thousand 2

238-L 1 million 5 thousand

238-M 2 billion 470 million 2

238-N See answers for exercises H through M.

238-O 721,635,599,483 * 1,005,000

238-P 4 trillion 672 billion 792 million 844 thousand 631

238-Q 25 trillion 985 billion 234 million 96 thousand 741

238-R 61 trillion 234 billion 492 million 766 thousand 900

238-S 500 billion 370 million 4 thousand 19

239-T Yes * Place value

239-U The name of the periods

239-A The digits 3, 0, and 1 * One thousand

239-B The digits 4, 5, and 7 * Seven million * Fifty million * Four hundred million

239-C 0 * 9 * 6

239-D Yes * 1000 * 10,000 * Yes

239-E Yes * Yes * Yes

239-F 500,000 * 4000 * Each of numerals E and F expresses a number as the product of a power of 10 and a number from 1 to 10.

240-G 3000 * 3000 * 3×10^3 * 3 is a number from 1 to 10, and 10^3 is a power of 10. 30×10^2 is not in scientific notation because 30 is not a number from 1 to 10.

240-H 6×10^2

240-I 7×10^4

240-J 3×10^6

240-K 4×10^8

240-L 1×10^5

240-M 8×10^9

On your own

240-1 Eighty-five million ninety thousand eighty

240-2 Fifty-nine thousand seventy-three

240-3 Three hundred fifty-six thousand five hundred twenty-one

240-4 Six hundred seventy-four million nine hundred eighty-two thousand three hundred fifteen

240-5 Seven hundred sixty-five million one hundred thousand two hundred

240-6 Two billion nine hundred sixty-five million four hundred sixty-one thousand three hundred thirty-two

240-7 70,004

240-8 900,006

240-9 400,730

240-10 623,040

240-11 7,007,070

240-12 45,600,000

240-13 52,089,014

240-14 1,000,024,319

240-15 580,700,016

240-16 3,000,400,000,000

240-17 4000

240-18 200,000

240-19 900

240-20 80,000

240-21 5,000,000

240-22 70,000,000,000

240-23 8×10^3

240-24 2×10^2

240-25 7×10^5

240-26 4×10^4

240-27 3×10^3

240-28 5×10^7

240-29 4×10^8

240-30 6×10^8

240-31 8×10^7

240-32 2×10^8

Applying mathematics

240-1 x, number of newspapers Terry sold.
$110 - x = 25$.
{85}. 85 newspapers

240-2 x, number of months to build a road 115 miles long.
$5/3 \sim 115/x$.
{69}. 69 months

240-3 x, number of square miles that is the total area of the Atlantic and Pacific oceans.
$63,801,668 + 31,839,306 = x$.
{95640974}. 95,640,974 square miles

240-4 x, number of yards of material Mrs. Smith had before she bought 13 more yards.
$x + 13 > 41$.
{29, 30, 31, ...}. 29 yards

240-5 x, number of dollars that was the goal.
$9213 - x = 758$.
{8455}. $8455

241-6 x, number of posters Carl has.
$x < 18 + 13 \wedge x > 13$.
{14, 15, 16, ..., 30}.
From 14 through 30 posters

241-7 x, number of dollars interest that Mr. Fisher paid per $100 he borrowed.
$57/950 \sim x/100$.
{6}. 6%

241-8 x, number of provincial parks for every one national park.
$300/30 \sim x/1$.
{10}. 10 times as many

241-9 x, number of cars Mr. Roberts had to sell.
$x > 63 \wedge x - 17 < 54$.
{64, 65, 66, ..., 70}.
From 64 through 70 cars

241-10 x, number of baseballs used during the game.
$48 - x < 19$.
{30, 31, 32, ..., 48}.
From 30 through 48 baseballs

241-11 x, number of dollars Mr. Lang saved last year.
$2/17 \sim x/7395$.
{870}. $870

241-12 x, number of dollars interest.
y, number of dollars Miss Wilson owed the bank at end of year.
$6/100 \sim x/750 \wedge 750 + x = y$.
{(45, 795)}. $795

241-13 x, number of square miles that is the area of France.
$17/1 \sim 3,851,809/x$.
{226577}. About 226,577 square miles

241-14 x, number of electric fans sold.
y, number of steam irons sold.
$x + y < 6 \wedge x > y$.
{(2, 1), (3, 1), (3, 2), (4, 1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (3, 2) is 3 electric fans and 2 steam irons.

241-15 x, number of dollars commission Mr. Davis received in March.
$125/100 \sim x/260$.
{325}. $325

241-16 x, number of dollars discount on dresser.
y, number of dollars discount per $100 of the regular price.
$350 - 273 = x \wedge x/350 \sim y/100$.
{(77, 22)}. 22%

241-17 $2/3 \sim x/y \wedge x(y) < 100$.
{2/3, 4/6, 6/9, 8/12}

241-18 $34/100 \sim 51/x$.
{150}

241-19 $96/100 \sim x/7625$.
{7320}

241-20 $660/528 \sim x/100$.
{125}

Checking up

Test 56

242-1 F
242-2 T
242-3 T
242-4 F
242-5 F
242-6 T
242-7 F
242-8 F
242-9 T
242-10 T

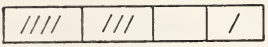
Test 57

242-11 (c) powers
242-12 (b) expanded form
242-13 (c) four hundred seven billion
242-14 (a) 19
242-15 (b) 7
242-16 (c) 12
242-17 (c) base-nine
242-18 (a) base-four
242-19 (c) base

Test 58

242-20 3
242-21 1
242-22 10100_{two}
242-23 1001101_{two}
242-24 2258_{eight}
242-25 $\mathcal{E}37_{twelve}$
242-26 177_{twelve}
243-27 118
243-28 714
243-29 1312

243-30



Test 59

243-31 11
243-32 95
243-33 830
243-34 455
243-35 9231
243-36 53
243-37 2011_{four}
243-38 11101_{two}
243-39 $11X8_{twelve}$
243-40 1234_{five}

Test 60

243-41 70,000
243-42 10,000,000
243-43 8,000,000
243-44 9,000,000,000
243-45 2×10^3
243-46 6×10^4
243-47 5×10^6
243-48 4×10^7

Test 61

243-1 F
243-2 T
243-3 T
243-4 F
243-5 T
243-6 T
243-7 F
243-8 T
243-9 T
243-10 F

Test 62

243-11 principal
243-12 net price
243-13 solution
243-14 plane
244-15 set A, or A
244-16 the empty set
244-17 {0}
244-18 subset
244-19 rate pair

Test 63

244-20 (b) perpendicular
244-21 (b) half-planes
244-22 (b) one member
244-23 (d) {1, 2, 3, ...}
244-24 (c) three hundred forty-three
244-25 (c) four members
244-26 (d) infinitely many planes
244-27 (a) exactly one plane
244-28 (d) the empty set

Test 64

244-29 x, number of dollars Fred paid per \$100 of the regular price.
$3/4 \sim x/100$.
{75}. 75%

244-30 x, number of dollars that is the basic price of the cheaper car.
y, number of dollars that is the price of the cheaper car with a heater.
$90/100 \sim x/2450 \wedge x + 69 = y$.
{(2205, 2274)}. \$2274

244-31 x, number of students enrolled in Scott School.
y, number of students enrolled in Brown School.
$x - y = 231 \wedge x + 78 = 550$.
{(472, 241)}.
472 students in Scott School and 241 students in Brown School

244-32 x, number of quarts of antifreeze.
y, number of quarts of water.
$1/2 \sim x/y \wedge x + y = 21$.
{7/14}. 7 quarts of antifreeze and 14 quarts of water

244-33 $x/100 \sim 42/70$.
{60}

244-34 $65/100 \sim x/180$.
{117}

244-35 $30/100 \sim 24/x$.
{80}

244-36 $x/100 \sim 391/230$.
{170}

244-37 $150/100 \sim 57/x$.
{38}

Lesson 57

245-A Yes * Yes
245-B One
246-C The set of straws
246-D One * Yes * Yes * The mapping in D2 is a one-to-one mapping because each member of set B is mapped onto just one member of set A, and each member of A is used only once.
246-E Each member of set L is mapped onto just one member of set K, and each member of K is used only once. * Yes
246-F Yes
246-G 6 is mapped onto 17, 5 onto 14, 9 onto 21, 3 onto 0, and 1 onto 6. * 17 is mapped onto 6, 14 onto 5, 21 onto 9, 0 onto 3, and 6 onto 1. * Each member of set X is mapped onto exactly one member of set Y, and each member of set Y is mapped onto exactly one member of set X.
246-H 9 is mapped onto 17, 3 onto 14, 6 onto 21, 5 onto 0, and 1 onto 6. * 17 is mapped onto 9, 14 onto 3, 21 onto 6, 0 onto 5, and 6 onto 1. * Yes
246-I Yes * The diagram below represents a third way that set X can be put in

one-to-one correspondence with set Y. There are 120 different ways that sets X and Y can be put in one-to-one correspondence.

X = {3, 6, 1, 9, 5}.
↕ ↕ ↕ ↕ ↕
Y = {21, 14, 17, 0, 6}.

247-J Yes * The number of members in each set is a natural number.

247-K Set T. Set P and set T can be put in one-to-one correspondence. * Set S. Set Q and set S can be put in one-to-one correspondence.

247-L No * There are infinitely many subsets of the set of natural numbers that are equivalent to set R. One example is {11, 22, 33}.

247-M A finite set cannot be put in one-to-one correspondence with an infinite set.

247-N Yes. If two sets are equal, then they are the same set. Every set can be put in one-to-one correspondence with itself. Therefore, every set is equivalent to itself. * No. From your responses to exercises K and L, you know that two different sets can be equivalent.

247-A The three dots tell you that there is no end to the members of each set. * The set of natural numbers

247-B Yes * Yes * Yes * Yes * Yes * Yes

247-C 2 × 17, or 34 * 2 × 28, or 56 * Yes

247-D 10 * 50 * 1000 * 20,002

247-E Yes * Yes

247-F Yes

247-G Yes * Yes

247-H Yes * Yes * 18

248-I Divide each member of E by 2 * 4 * 9 * 25 * 408

248-J Yes * Yes

248-K Yes * Yes

248-L To find the member of E onto which you map each member of N, multiply each member of N by 2. To find the member of N onto which you map each member of E, divide each member of E by 2.

248-M
N = {0, 1, 2, 3, 4, ...}.
↕ ↕ ↕ ↕ ↕
E = {0, 2, 4, 6, 8, ...}.

248-N Yes * The set of even numbers and the set of natural numbers can be put in one-to-one correspondence.

248-O
N = {0, 1, 2, 3, 4, ...}.
↕ ↕ ↕ ↕ ↕
O = {1, 3, 5, 7, 9, ...}.

248-P Yes * The set of natural numbers and the set of odd numbers can be put in one-to-one correspondence.

On your own

248-I {Mary, Jill, Susan}
↕ ↕ ↕
{June, July, August}
There are other correct subsets and diagrams for this exercise. For this and the following exercises, only one diagram is given.

248-2
{ 2, 4, 6 }
↕ ↕ ↕
{Sunday, Monday, Tuesday}

248-3
{Mexico}
↕
{Canada}

248-4
{1, 2, 3, 4}
↕ ↕ ↕ ↕
{0, 1, 2, 3}

248-5
{ 3, 1, 5 }
↕ ↕ ↕
{(0,1), (1,0), (1,1)}

248-6
{0, 1, 2, 3, 4}
↕ ↕ ↕ ↕ ↕
{0, 1, 2, 3, 4}

248-7
{12}
↕
{48}

248-8
{3/4, 6/8, 9/12}
↕ ↕ ↕
{4/3, 8/6, 12/9}

249-9
{red, blue}
↕ ↕
{ 0, 1 }

249-10
{0, 1, 2}
↕ ↕ ↕
{0, 1, 2}

249-11
{0}
↕
{0}

249-12
{(*a*, *b*), (*c*, *d*)}
↕ ↕
{ 0, 1 }

249-13
{*a*, *b*, *c*, *d*}
↕ ↕ ↕ ↕
{0, 1, 2, 3}

249-14
{5/6, 10/12, 15/18, 20/24, 25/30, 30/36}
↕ ↕ ↕ ↕ ↕ ↕
{ 0, 1, 2, 3, 4, 5 }

249-15
{10, 100, 1000, 10000, ...}
↕ ↕ ↕ ↕
{1, 2, 3, 4, ...}

Keeping Skilful

249-1 25,256

249-2 2473

249-3 23,091

249-4 138,131

249-5 305,588

249-6 112,461

249-7 7500

249-8 1747

249-9 829

249-10 15,853

249-11 23,674

249-12 76,561

249-13 51,064

249-14 72,294

249-15 {138}

249-16 {0, 1, 2, ..., 431}

249-17 {38, 39, 40, ...}

249-18 {78192}

249-19 {554}

249-20 {0, 1, 2, ..., 15}

249-21 {9120}

249-22 {60, 61, 62, ...}

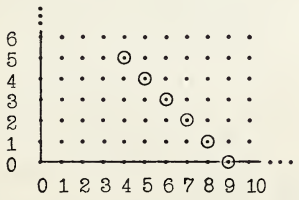
249-23 {104}

249-24 {0, 1, 2, ..., 57}

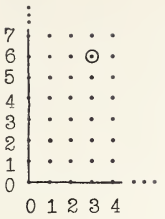
249-25 {1, 2, 3, ...}

249-26 {79}

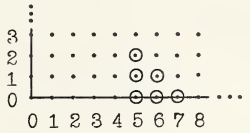
249-27 {(4,5), (5,4), (6,3), (7,2), (8,1), (9,0)}



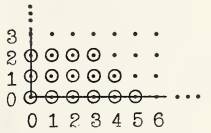
249-28 {(3,6)}



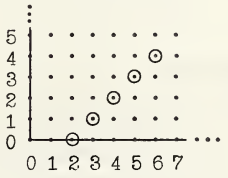
249-29 {(5,0), (5,1), (5,2), (6,0), (6,1), (7,0)}



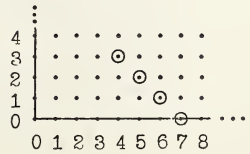
249-30 {(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (5,0)}



249-31 {(2,0), (3,1), (4,2), (5,3), (6,4)}



249-32 {(4,3), (5,2), (6,1), (7,0)}



Lesson 58

249-A None * The empty set

249-B { } is a set that has no members, and n{ } is not a set but is the number of members in the empty set.

250-C The set whose only member is 0 * One

250-D {0} is a set with one member, and { } is a set with no members. * {0} is a set whose only member is zero, and n{ } is the number of members in the empty set, or the number 0.

250-E Yes * Yes

250-F Yes * {0} is a set whose member is zero, and n{0} is not a set, but is the number of members in the set whose member is zero.

250-G Yes

250-H Set G is equivalent to {0, 1}. * Set H is equivalent to {0, 1}.

250-I Yes * The natural number 2

250-J The natural number 0 is associated with set A, the natural number 1 with set B, and the natural number 2 with set C.

250-K Yes

251-L The natural number 3 * {tuba, drum, clarinet} is equivalent to {0, 1, 2}. Since 3 is the natural number associated with {0, 1, 2}, you know that 3 is also associated with {tuba, drum, clarinet}.

251-M The natural number 3

251-N The natural number 0 is associated with set A, 1 with set B, 2 with set C, 3 with set D, 4 with

set E, 5 with set F, 6 with set G, and 7 with set H.

251-O {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday} * Set H * 7 * You know that the natural number 7 is associated with set H and each set equivalent to set H.

251-P {0, 1, 2, 3, 4, 5, 6, 7, 8}

251-Q {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

251-R {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}

251-S {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

251-T {0, 1, 2, ..., $m - 1$} * Form the set that contains the natural numbers from zero through the number that is one less than the given natural number.

251-U Yes. The number of members in any given standard set is a natural number; therefore, the standard set of any natural number is a finite set. * No. Since the number of members in the set of natural numbers cannot be counted, the set of natural numbers is an infinite set.

On your own

251-1 An example is: {beet, potato, carrot, onion, radish}

251-2 An example is: {chair, table, lamp, desk, footstool}

251-3 An example is: {Mississippi, Columbia, Hudson, Colorado, Potomac}

251-4 The natural number 5

251-5 10 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.

251-6 13 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.

251-7 11 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

251-8 15 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}.

251-9 0 = n{ }.

251-10 21 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}.

251-11 {0, 1, 2, 3, 4, 5, 6, 7}

251-12 {0}

251-13 { }

251-14 {0, 1, 2, 3, 4, 5, 6}

251-15 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

251-16 9 = n{0, 1, 2, 3, 4, 5, 6, 7, 8}.

251-17 12 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.

251-18 1 = n{0}.

251-19 m = {0, 1, 2, ..., $m - 1$}.

Lesson 59

252-A Yes * The member of the standard set of 1 is also a member of the standard set of 2.

252-B No

252-C Yes * The standard set of 2 is a subset of the standard set of 3, and the standard set of 2 does not contain all the members of the standard set of 3.

252-D No * The standard set of 3 contains all the members of the standard set of 3.

253-E Yes. The standard set of 1 is a proper subset of the standard set of 2. * Yes. The standard set of 2 is a proper subset of the standard set of 3.

253-F {0, 1, 2, 3} * Yes * The standard set of 3 is a subset of the standard set of 4, and the standard set of 3 does not contain all the members of the standard set of 4.

253-G Yes * From your answer to exercise F, you know that the standard set of 3 is a proper subset of the standard set of 4. From the definition for "less than" for natural numbers, you know that this also means $3 < 4$.

253-H { }

253-I Yes * Yes * Yes * Yes * The standard set of 1 is a proper subset of the standard set of each of the numbers 2, 3, 4, and 5.

253-J Yes

253-K {3, 4, 5, ...} * Yes

253-L Yes. Each natural number is less than infinitely many other natural numbers. For example, if a is replaced by 5 in $a < b$, you obtain $5 < b$. The solution set of $5 < b$ is {6, 7, 8, ...}.

253-M Point 0

253-N Each of the points associated with a number less

than 5 is to the left of point 5 in the natural-number line.

253-O Point a is to the left of point b.

253-P Yes

253-Q $15 < 7$.

254-R Yes

254-S Point b is to the right of point a.

254-A Yes * 6

254-B Yes * The number that immediately follows any given natural number is the sum of the given number and 1.

254-C 8 * 20 * 511 * Yes

254-D $x + 1$

254-E Yes. Since the set contains 0 and 1 is the successor of 0, the set also contains 1. Since the set contains 1 and 2 is the successor of 1, the set also contains 2. Since 2 is a member of the set, so is the successor of 2 which is 3. Since 3 is contained in the set, so is 4 because 4 is the successor of 3. You can continue in this way and show that any given natural number is contained in the set.

254-F No * Zero is not the successor of a natural number because it is not the sum of a natural number and 1.

254-G Point 3, point 4, and point 5 * Yes

254-H {9, 10, 11, 12, 13, 14, 15} * No. This set contains 7 members.

254-I $8 < x \wedge x < 16$.

254-J No * { }

254-K {5, 6, 7, 8, 9, 10} * { } * {473, 474, 475, 476, 477, 478, 479}

254-L There are 6 members in the first set, 0 members in the second set, and 7 members in the third set.

254-M Yes * The set of natural numbers between any two given natural numbers has a greatest member. Any set of natural numbers that has a greatest member is a finite set.

On your own

255-1 Examples are: { }, {0}, {0, 1}, {1, 2}.

255-2 Examples are: {20, 25}, {5}, {10, 15, 20}, {5, 10, 15, 20}.

255-3 Examples are: {0, 1, 2, 3, 4, 5, 6, 7, 8}, {1, 2, 3}, {4}, {4, 5, 6, 7}.

255-4 Examples are: {0, 1, 2}, {0, 1}, {1}, {0, 2}.

255-5 Proper subset

255-6 Proper subset

255-7 Not a proper subset

255-8 Not a proper subset

255-9 Proper subset

255-10 Not a proper subset

255-11 (6,9), (1,7), (2,11)

255-12 Yes * The condition for inequality, $y > x$ makes the same requirement on the ordered pairs that satisfy the condition as the condition $x < y$.

255-13 Yes * They contain the same members.

255-14 2044

255-15 102

255-16 600

255-17 10010

255-18 9. The successor of 7 is 8, so the successor of the successor of 7 is 9.

255-19 {10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

255-20 4 * None

255-21 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

255-22 { }

255-23 {12}

255-24 {16, 17, 18, 19, 20}

Checking up

Test 65

255-1 {0, 1} * Not equivalent

255-2 {3, 4, 5, 6} * Equivalent

255-3 {(0,0), (0,1), (0,2), (1,0), (1,1), (2,0)} * Not equivalent

255-4 {3/4, 6/8, 9/12, 12/16, 15/20} * Not equivalent

255-5 {(0,15), (1,14), (2,13), (3,12)} * Equivalent

255-6 {0}

255-7 {0, 1, 2, 3, 4, 5, 6, 7}

255-8 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

255-9 {0, 1}

255-10 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

255-11 {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}

255-12 Each of the sets

255-13 No. It contains all the members of the standard set of 17.

Test 66

255-14 0 * 1

255-15 6 * 7

255-16 12 * 13

255-17 10 * 11

255-18 7 * 8

255-19 {1, 2, 3, 4, 5}

255-20 {4, 5, 6, 7, 8, 9}

255-21 {2, 3, 4, 5, 6}

255-22 One example is:

{5, 10, 15, 20, 25}
↓ ↓ ↓ ↓ ↓
{3/7, 6/14, 9/21, 12/28, 15/35}

Applying mathematics

256-1 x, number of shades the manager needs before he can fill the order.
$176 + x = 320$.
{144}. 144 shades

256-2 x, number of miles Mr. Robinson drove in 15 min.
$60/75 \sim x/15$.
{12}. 12 miles

256-3 x, number of inches of ribbon Kay had before she used the 13 in.
$x > 27 \wedge x - 13 < 19$.
{28, 29, 30, 31}. From 28 through 31 inches

256-4 x, number of boys who entered the contest.
y, number of girls who entered the contest.
$x + y = 12 \wedge x > y$.
{(11,1), (10,2), (9,3), (8,4), (7,5)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (9,3) is 9 boys and 3 girls.

256-5 x, number of copies the company printed.
$x > 1245 \wedge x - 1195 < 66$.
{1246, 1247, 1248, ..., 1260}. From 1246 through 1260 copies

256-6 x, number of perch Bill caught.
y, number of sunfish Bill caught.
$x + y < 15 \wedge x - y = 5$.
{(6, 1), (7,2), (8,3), (9,4)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (6,1) is 6 perch and 1 sunfish.

256-7 x, number of dollars discount for each $100 of the regular price.
y, number of dollars that was the amount of discount.
$x/100 \sim y/210 \wedge 210 - 147 = y$.
{(30,63)}. 30%

256-8 x, number of pints of raspberries Sally picked.
y, number of pints of raspberries Linda picked.
$x + y < 40 \wedge 7/6 \sim x/y$.
{7/6, 14/12, 21/18}. Each member of the solution set may be used to obtain an answer. The answer obtained from (14/12) is Sally picked 14 pt. and Linda picked 12 pt.

256-9 $58/100 \sim x/2300$.
{1334}

256-10 $75/100 \sim 69/x$.
{92}

256-11 $x > 52 \wedge 689 + x < 756$.
{53, 54, 55, ..., 66}

256-12 $x + y = 13 \wedge x - y = 4$.
{ }

256-13 $x/y \sim 6/9 \wedge x + y < 32$.
{2/3, 4/6, 6/9, 8/12, 10/15, 12/18}

Lesson 60

256-A Yes * 2 is associated with set A. 3 is associated with set B.

257-B Yes * Set A is the same set as {1, 3}. Therefore, the number associated with set A is the same as the number associated with {1, 3}.

257-C Yes * Set B is the same set as {0, 2, 4}.

257-D Yes

257-E Yes

257-F {1, 3, 0, 2, 4}

257-G Yes

257-H Set E and set F do not have any common members. * {Prince Edward Island, British Columbia, Ontario, Alberta, Saskatchewan, Manitoba}

257-I Yes * Yes * Yes

257-J $S \cup T$

257-K $Y \cup Z$ * Yes

258-L No * Sets A and B are not disjoint sets. These two sets have two common members, 18 and 19.

258-A Yes

258-B (9,8)

258-C Yes * Yes * 9 + 10, or 19 * 10 + 9, or 19

258-D (4,3) (3,4) (6,1) (1,6)
↓ ↓ ↓ ↓
4 + 3, 3 + 4, 6 + 1, 1 + 6,
or 7 or 7 or 7 or 7

258-E Yes

258-F The sum of the components of (7,1), (1,7), and (6,2) is 8. * Examples are: (2,6), (3,5), and (4,4).

258-G The sum of the components of both (4,0) and (1,3) is 4. * Examples are: (0,4), (3,1), and (2,2).

258-H No

258-I An example of a true statement that you can obtain from $a + b = c$ by replacing each of the variables by a member of N is $5 + 9 = 14$. * Using

the same replacements, you obtain (5,9) from (a,b). * Yes

259-J The sum of the components of (5,1) is 6. The sum of the components of (1,5) is 6. The sum of the components of (9,3) is 12. The sum of the components of (3,9) is 12.

259-K Yes. Natural numbers are associated only with finite sets, and each finite set has a natural number associated with it. Therefore, the sum of two natural numbers is always associated with the union of two finite sets. Since the union of two finite sets is also a finite set, the sum of the components of each pair of natural numbers is a natural number.

259-A 5 * Yes

259-B 12 * 12

259-C Yes * 8 = 2 + 6 tells you that if you add 6 to 2, you obtain 8. From this, you also know that if you subtract 2 from 8, you obtain 6. Therefore, 6 is the difference of 8 and 2.

259-D An example of a true statement that you can obtain from $a - b = c$ by replacing each of the variables by a member of N is 13 - 9 = 4. * Using the same replacements, you obtain 13 = 9 + 4 from $a = b + c$. * Yes

259-E If a is the sum of b and c, then c is the difference of a and b. Therefore, if 9 is the sum of 2 and 7, then 7 is the difference of 9 and 2. a is replaced by 9, b by 2, and c by 7.

259-F Yes * 2

259-G If you subtract 1 from 5, you obtain 4. Therefore, 4 is the difference of 5 and 1. * 4

259-H 6 * 6

260-I No * There is no natural number that you can add to 5 to obtain a sum of 1.

260-J No * There is no natural number that you can add to 9 to obtain a sum of 3.

260-K Examples are: (10,4), (50,18), and (32,12). * 6 is the difference of (10,4). 32 is the difference of (50,18). 20 is the difference of (32,12).

260-L Examples are: (6,6), (93,93), and (14,14). * 0 is the difference of the components of each ordered pair named.

260-M Examples are: (7,19), (0,5), and (43,75). * No * For each of these ordered pairs, there is no natural number that you can add to the second component to obtain the first component.

260-N No * There is no natural number that is the difference of the components of an ordered pair whose first component is less than its second component.

260-O The replacement for b. Since you know that the difference of a and b is a natural number, you also know that the sum of b and the natural number is a. Therefore, a must be equal to or greater than b.

On your own

260-1 8 + 20, or 28

260-2 19 + 8, or 27

260-3 8 + 70, or 78

260-4 20 + 70, or 90

260-5 20 + 8, or 28

260-6 20 + 19, or 39

260-7 70 + 19, or 89

260-8 19 + 20, or 39

260-9 8 + 19, or 27

260-10 70 + 50, or 120

260-11 29 + 54, or 83

260-12 36 + 105, or 141

260-13 99 + 2, or 101

260-14 63 + 18, or 81

260-15 118 + 189, or 307

260-16 25

260-17 38

260-18 34

260-19 72

260-20 68

260-21 16

260-22 12, 15, 18, 21, and 24

260-23 Yes. The sum of any two members of M is also a member of M. * No. The difference of two members of M is not always a member of M. For example: (3,6) cannot be mapped onto a member of M by subtraction.

260-24 Closed under addition * Not closed under subtraction

260-25 Not closed under addition * Not closed under subtraction

260-26 Not closed under addition * Not closed under subtraction

260-27 Not closed under addition * Not closed under subtraction

260-28 Closed under addition * Not closed under subtraction

260-29 Closed under addition * Not closed under subtraction

260-30 Not closed under addition * Not closed under subtraction

260-31 Not closed under addition * Not closed under subtraction

Keeping Skilful

261-1 58,608

261-2 1056

261-3 13,498

261-4 17,550

261-5 51,184

261-6 42,714

261-7 57,963

261-8 513,240

261-9 214,428

261-10 428,945

261-11 96, remainder 30

261-12 86, remainder 0

261-13 368, remainder 0

261-14 611, remainder 2

261-15 902, remainder 0

261-16 113, remainder 100

261-17 92, remainder 110

261-18 34, remainder 111

261-19 201, remainder 0

261-20 746, remainder 0

261-21 {3}

261-22 {12}

261-23 {50}

261-24 {8}

261-25 {8}

261-26 {36}

261-27 {2}

261-28 {75}

261-29 {63}

261-30 {9}

261-31 {80}

261-32 {19}

261-33 {175}

261-34 {184}

261-35 {90}

261-36 {5}

261-37 {10}

261-38 {18}

261-39 {195}

261-40 {147}

261-41 G = {3/4, 6/8, 9/12}.
H = {9/12}.
G ∪ H = {3/4, 6/8, 9/12}.
G ∩ H = {9/12}.

261-42 G = {3/5, ...}.
H = {3/5, 6/10, 9/15, 12/20}.
G ∪ H = {3/5, ...}.
G ∩ H = {3/5, 6/10, 9/15, 12/20}.

261-43 G = {16/14}.
H = {7/8, 14/16}.
G ∪ H = {7/8, 14/16, 16/14}.
G ∩ H = { }.

261-44 G = {2/3, ...}.
H = {2/3, ...}.
G ∪ H = {2/3, ...}.
G ∩ H = {2/3, ...}.

261-45 G = {6/15}.
H = {6/15}.
G ∪ H = {6/15}.
G ∩ H = {6/15}.

261-46 G = {3/2}.
H = { }.
G ∪ H = {3/2}.
G ∩ H = { }.

261-47 Rays AF, AE (or AD), AB (or AC), and AJ

261-48 ∠ABE

261-49 The sides are rays DA and DC. The vertex is D.

261-50 The sides are rays ED and EB. The vertex is E.

261-51 ∠FAD and ∠DAC, ∠DAC and ∠CAJ, ∠CAJ and ∠JAF, ∠JAF and ∠FAD

261-52 ∠FAD, or ∠BAJ * ∠FAD, or ∠JAB

261-53 The empty set

261-54 ∠FAD, ∠DAC, ∠CAJ, ∠JAF

261-55 Segments AB, BE, and AE

261-56 The sides are segments AD, DC, and AC. The vertices are A, D, and C.

261-57 The sides are segments EB, BC, CD, and ED. The vertices are E, B, C, and D.

261-58 Segment EB

261-59 AE ∪ AB. The intersection is the set that contains all the points in segments AE and AB and no other points.

261-60 Points R and M

261-61 Points F, K, A, and J

261-62 Points K, E, R, and M

261-63 Points K, R, and M

Lesson 61

262-A No * Yes

262-B Yes * Yes

262-C Yes * Set F is the same set as D × E.

262-D Yes

262-E {(0,0), (0,1), (0,2), (0,3), (0,4), (0,5), (1,0), (1,1), (1,2), (1,3), (1,4), (1,5), (2,0), (2,1), (2,2), (2,3), (2,4), (2,5), (3,0), (3,1), (3,2), (3,3), (3,4), (3,5)}

262-F 4 * 6

262-G Yes * Since 4 is associated with set G and 6 is associated with set H, the product 4 × 6 is the number

associated with the Cartesian set G x H.

262-H The Cartesian set S x T

262-I An example is: A = {2, 4, 6, 8, 19}. * An example is: B = {3, 6}. * A x B = {(2,3), (2,6), (4,3), (4,6), (6,3), (6,6), (8,3), (8,6), (19,3), (19,6)}.

262-J A x B

263-K 0 * 4

263-L The empty set * There are no members in Q to match with the members in set R.

263-M Yes * The Cartesian set of { } and {2, 3, 4, 5} is the empty set. Therefore, the product of 0 and 4 is the number associated with the empty set, which is 0.

263-N No

263-O Yes * Yes * Yes

263-P 8 x 2, or 16 * 4 x 4, or 16 * 1 x 16, or 16 * (16,1) and (2,8)

263-Q *a* x *b*, or *c*

263-R Yes * Two numbers are used to obtain a product.

263-S Yes

263-T The product of the components of (12,2) is 12 x 2, or 24. The product of the components of (3,4) is 3 x 4, or 12. The product of the components of (11,4) is 11 x 4, or 44. * Yes

263-U Yes

263-V Yes

263-A 3 * Yes

263-B 3 * 12

264-C Since you know that 21 = 3 x 7, you also know that 3 = 21 ÷ 7. From 3 = 21 ÷ 7, you know that 3 is the quotient of 21 and 7.

264-D 5

264-E 4 * Yes

264-F 1 * 1

264-G There is no natural number by which you can multiply 4 to get a product of 3.

264-H No * No

264-I No

On your own

264-1 5 x 6, or 30

264-2 6 x 5, or 30

264-3 5 x 5, or 25

264-4 100 x 6, or 600

264-5 100 x 100, or 10,000

264-6 5 x 100, or 500

264-7 6 x 0, or 0

264-8 6 x 6, or 36

264-9 100 x 0, or 0

264-10 70 x 35, or 2450

264-11 50 x 3, or 150

264-12 20 x 3, or 60

264-13 3 x 3, or 9

264-14 1 x 4, or 4

264-15 10 x 8, or 80

264-16 39 x 5, or 195

264-17 12 x 0, or 0

264-18 15 x 6, or 90

264-19 9 x 1, or 9

264-20 13 x 2, or 26

264-21 14 x 7, or 98

264-22 2

264-23 5

264-24 6

264-25 10

264-26 1

264-27 5

264-28 3

264-29 2

264-30 1

264-31 4

264-32 7

264-33 9

264-34 A natural number

264-35 Not a natural number

264-36 Not a natural number

264-37 A natural number

264-38 A natural number

264-39 A natural number

264-40 Closed under multiplication

264-41 Closed under multiplication

264-42 Closed under multiplication

264-43 Not closed under multiplication * An example is 6 x 8, or 48. 48 is not a member of the set of even numbers less than 16.

264-44 Closed under multiplication

264-45 Not closed under multiplication * An example is 4 x 7, or 28. 28 is not a member of the set of natural numbers less than 20.

264-46 Not closed under multiplication * An example is 1052 x 1053, or 1,107,756. The product 1,107,756 is not a member of the set of natural numbers from 1051 through 1060.

Lesson 62

265-A Yes

265-B Yes

265-C Yes * Sets C and D have no common members.

265-D C ∪ D = {1, 3, 5, 7, 12, 14, 16}.

265-E D ∪ C = {12, 14, 16, 1, 3, 5, 7}.

265-F Yes

265-G Set C is equivalent to {0, 1, 2, 3}, which is the standard set of 4. * Set D is equivalent to

{0, 1, 2}, which is the standard set of 3.

265-H Yes * 4 + 3 is the number of members in C ∪ D. 3 + 4 is the number of members in D ∪ C.

265-I Yes

265-J Yes * Yes

265-K A ∪ B * B ∪ A * Yes * Yes

266-L For the statement expressed by sentence A, *a* was replaced by 6 and *b* by 9. For the statement expressed by sentence B, *a* was replaced by 52 and *b* by 30. For the statement expressed by sentence C, *a* was replaced by 89 and *b* by 102. * Yes

266-M Yes

266-N Yes

266-O {(13,0), (13,1), (13,2), (14,0), (14,1), (14,2)} * Set R * Set S

266-P {(0,13), (0,14), (1,13), (1,14), (2,13), (2,14)} * Set S * Set R

266-Q No * The sets do not have the same members. For example: (13,0), which is a member of R × S, is not the same as (0,13), which is a member of S × R.

266-R (13,0) is mapped onto (0,13); (13,1) onto (0,14); (13,2) onto (1,13); (14,0) onto (1,14); (14,1) onto (2,13), and (14,2) onto (2,14). * (0,13) is mapped onto (13,0); (0,14) onto (13,1); (1,13) onto (13,2); (1,14) onto (14,0); (2,13) onto (14,1), and (2,14) onto (14,2).

266-S Yes * Yes

266-T Yes. Two sets that are equivalent have the same number of members. * Yes. The same number is associated with equivalent sets.

266-U Yes * Yes * Yes

266-V A × B * B × A * Yes. A × B and B × A are equivalent sets. * Yes. The same number is associated with equivalent sets.

267-W Without a symbol for multiplication, you might mistake the product 2(4) for the number 24 (twenty-four).

267-X No * The product 7 · 8 is the same as the number 56, not 78.

267-Y For the statement expressed by sentence D, *a* was replaced by 7 and *b* by 8. For the statement expressed by sentence E, *a* was replaced by 29 and *b* by 16. For the statement expressed by sentence F, *a* was replaced by 115 and *b* by 21. * Yes * The commutative property of multiplication of natural numbers

267-A For the statement expressed by sentence G, *a* was replaced by 1 and *b* by 0. For the statement expressed by sentence H, *a* was replaced by 52 and *b* by 8. For the statement expressed by sentence I, *a* was replaced by 120 and *b* by 97.

267-B No * You know that $1 - 0 = 1$ because 1 is equal to the sum of 0 and 1. You also know that $0 - 1 = 1$ is a false statement because 0 is not equal to the sum of 1 and 1.

268-C Yes

268-D No

268-E One * If you obtain one false statement from $a - b = b - a$, then you know that, for each replacement of the variables in $a - b = b - a$, you do not always obtain a true statement. This means that $a - b = b - a$ is not a property.

268-F No * Yes

268-G No * You know that, for each replacement of the variables in $a \div b = b \div a$, you do not always obtain a true statement.

On your own

268-1 True statement

268-2 False statement

268-3 True statement

268-4 False statement

268-5 False statement

268-6 True statement

268-7 False statement

268-8 True statement

268-9 True statement

268-10 False statement

268-11 {15}

268-12 {0, 1, 2, ...}

268-13 {0}

268-14 {0, 1, 2, ..., 8}

268-15 {0, 1}

268-16 {0, 1, 2, ...}

268-17 {3, 4, 5, ...}

268-18 {14, 15, 16, ...}

Keeping Skilful

268-1 161

268-2 11

268-3 561

268-4 280

268-5 4123

268-6 59

268-7 116

268-8 2061

268-9 431

268-10 21121_{three}

268-11 1042_{five}

268-12 21313_{four}

268-13 30635_{seven}

268-14 1011111_{two}

268-15 13014_{nine}

268-16 22_{twelve}

268-17 $4 \pounds 0_{twelve}$

268-18 338_{twelve}

268-19 36_{twelve}

268-20 290_{twelve}

268-21 1482_{twelve}

Lesson 63

269-A 4 and 3 * 7

269-B By finding the sum of 4 and 3, which is 4 + 3, or 7 * 7 + 6, or 13

269-C Yes

270-D 3 and 6 * No

270-E 3 + 6, or 9

270-F By finding the sum of 3 and 6, which is 3 + 6, or 9 * 4 + 9, or 13

270-G Yes

270-H Yes

270-I For the statement expressed by sentence I, a was replaced by 9, b by 8, and c by 16. For the statement expressed by sentence J, a was replaced by 1, b by 45, and c by 24. For the statement expressed by sentence K, a was replaced by 13, b by 11, and c by 19. * Yes * The associative property of addition of natural numbers

270-J Because of the associative property of addition of natural numbers, you know that the way in which you group the numbers in 4 + 3 + 6 does not affect the sum. Therefore, you know that the symbol 4 + 3 + 6 names the number 13.

270-A 2 and 5 * 2 × 5, or 10

271-B (Column 1) 10 × 8, or 80

271-C (Column 1) 5 and 8

271-D 2 × 40, or 80

271-E Yes

271-F Yes

271-G Yes

271-H For the statement expressed by sentence G, a was replaced by 9, b by 12, and c by 17. For the statement expressed by sentence H, a was replaced by 82, b by 13, and c by 5. For the statement expressed by sentence I, a was replaced by 239, b by 4, and c by 11. * Yes * The associative property of multiplication of natural numbers

271-I Because of the associative property of multiplication of natural numbers, you know that the way in which you group the numbers in 2 · 5 · 8 does not affect the product. Therefore, you know that the symbol 2 · 5 · 8 names the number 80.

271-A a was replaced by 4, b by 3, and c by 1.

271-B (Column 2) No

271-C (Column 2) No * Since you obtained one false statement from $(a - b) - c = a - (b - c)$, you know that, for each replacement of the variables, it is not true that $(a - b) - c = a - (b - c)$. Therefore, you know that $(a - b) - c = a - (b - c)$ is not a property.

272-D (Column 1) No

272-E (Column 1) No * If division of natural numbers had the associative property, then you would obtain a true statement from $(a \div b) \div c = a \div (b \div c)$ for each replacement of a, b, and c. You know that for one set of replacements for a, b, and c, you do not obtain a true statement from $(a \div b) \div c = a \div (b \div c)$. Therefore, you know that division of natural numbers does not have the associative property.

272-A Yes * The associative property of addition of natural numbers

272-B Yes * The commutative property of addition of natural numbers. This property tells you that (36 + 83) is the same as (83 + 36).

272-C Yes. If a first number equals a second number and the second number equals a third number, then the first number equals the third number.

272-D (Column 2) Yes * The commutative property of multiplication of natural numbers. This property tells you that the product of 8 · 73 and 59 is equal to the product of 59 and 8 · 73.

272-E (Column 2) Yes * The associative property of multiplication of natural numbers

272-F Yes * Yes

272-G Yes * You know a first number, (8 · 73)59, is the same as a second number, 59(8 · 73). You also know that the second number, 59(8 · 73), is the same as a third number, (59 · 8)73. Therefore, you know that the first number is the same as the third number. (8 · 73)59 = (59 · 8)73.

On your own

272-1 True statement * The associative property of addition of natural numbers

272-2 False statement

272-3 False statement

272-4 True statement * The associative property of multiplication of natural numbers

272-5 False statement

272-6 False statement

272-7 True statement * The commutative property of addition of natural numbers

272-8 False statement

272-9 True statement * The associative property of addition of natural numbers

272-10 False statement

273-11 {10}

273-12 {12}

273-13 {19}

273-14 {0, 1, 2, ...}

273-15 {0}

273-16 {4, 5, 6, ...}

273-17 {11}

273-18 {0, 1, 2, ...}

273-19 {1, 2, 3, ...}

273-20 {1, 2, 3, ...}

273-21 {47}

273-22 {0, 1, 2, 3, 4}

273-23 {0, 1, 2, ..., 31}

273-24 {9}

273-25 37 + (76 + 92) = 37 + (92 + 76). The commutative property of addition.
37 + (92 + 76) = (37 + 92) + 76. The associative property of addition

273-26 14(7 · 9) = (7 · 9)14. The commutative property of multiplication

273-27 19(23 · 8) = 19(8 · 23). The commutative property of multiplication.
19(8 · 23) = (19 · 8)23. The associative property of multiplication

273-28 (21 · 19)6 = (19 · 21)6. The commutative property of multiplication.
(19 · 21)6 = 19(21 · 6). The associative property of multiplication

273-29 (37 · 28) + 5 = 5 + (37 · 28). The commutative property of addition

273-30 (24 + 18) + 12 = 12 + (24 + 18). The commutative property of addition.
12 + (24 + 18) = (12 + 24) + 18. The associative property of addition

273-31 3(7 + 41) = (7 + 41)3. The commutative property of multiplication.
(7 + 41)3 = (41 + 7)3. The commutative property of addition

273-32 (17 + 2)8 = 8(17 + 2). The commutative property of multiplication

273-33 (45 · 6)8 = 8(45 · 6). The commutative property of multiplication.
8(45 · 6) = (8 · 45)6. The associative property of multiplication

273-34 13 + (9 · 2) = (9 · 2) + 13. The commutative property of addition.
(9 · 2) + 13 = (2 · 9) + 13. The commutative property of multiplication

Keeping Skilful

273-1 {4, 5, 9, 13}

273-2 {13}

273-3 {4, 5, 9, 13, 23}

273-4 {23}

273-5 { }

273-6 {4, 5}

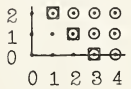
273-7 {13}

273-8 {4, 5, 9}

273-9 {23}

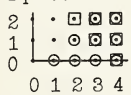
273-10 {(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)}

273-11 The graph of $\{(m,n) \mid m + n > 2\}$ is indicated by the encircled dots. The graph of $\{(m,n) \mid 3 - n = m\}$ is indicated by the dots enclosed by squares.



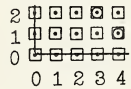
$\{(1,2), (2,1), (3,0)\}$

273-12 The graph of $\{(m,n) \mid m > n\}$ is indicated by the encircled dots. The graph of $\{(m,n) \mid m + n > 3\}$ is indicated by the dots enclosed by squares.



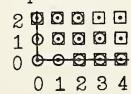
$\{(3,1), (3,2), (4,0), (4,1), (4,2)\}$

273-13 The graph of $\{(m,n) \mid 5 - m = n\}$ is indicated by the encircled dots. The graph of $\{(m,n) \mid m + 2 < 8\}$ is indicated by the dots enclosed by squares.



$\{(3,2), (4,1)\}$

273-14 The graph of $\{(m,n) \mid m + n < 5\}$ is indicated by the encircled dots. The graph of $\{(m,n) \mid m + n > 1\}$ is indicated by the dots enclosed by squares.



$\{(0,2), (1,1), (1,2), (2,0), (2,1), (2,2), (3,0), (3,1), (4,0)\}$

273-15 $\{5/7, 10/14, 15/21, 20/28\}$

273-16 $\{24/9\}$

273-17 $\{3/4, 6/8, 9/12, 12/16, 15/20\}$

273-18 $\{3/2, \ldots\}$

273-19 $\{(14,97)\}$

273-20 $\{(17,136)\}$

273-21 34 * 253

273-22 77 * 1372

273-23 191 * 8814

273-24 266 * 11,760

273-25 550 * 42,501

273-26 738 * 68,040

273-27 493 * 51,832

273-28 880 * 164,700

273-29 1109 * 187,408

Lesson 64

274-A Yes * 4 * Yes

274-B 8 * 12 * Yes

274-C 20

274-D Yes * Yes

274-E 5 * Yes

274-F 20

274-G Yes * Yes * Yes

274-H Yes

275-I 3 and 7 * 10

275-J 9 * Yes * $9(18 + 4) = 9(22)$. $9(22) = 198$. $(9 \cdot 18) + (9 \cdot 4) = 162 + 36$. $162 + 36 = 198$. Therefore, $9(18 + 4) = (9 \cdot 18) + (9 \cdot 4)$.

275-K Yes

275-L Yes

275-M One way is to first find the sum of b and c. Then multiply this sum by a. Another way is to first find the product of a and b and the product of a and c. Then find the sum of these products.

275-N For the statement expressed by sentence G, a was replaced by 3, b by 14, and c by 9. For the statement expressed by sentence H, a was replaced by 16, b by 12, and c by 8. For the statement expressed by sentence I, a was replaced by 41, b by 37, and c by 32. For the statement expressed by sentence J, a was replaced by 3, b by 3, and c by 3. * Yes * Because of the distributive property of multiplication over addition of natural numbers, you know that, for each replacement of a, b, and c, a true statement is obtained from $a(b + c) = (ab) + (bc)$.

275-O In $a(b + c) = (ab) + (ac)$, a is the multiplier of $(b + c)$, of b, and of c. In the condition expressed in D4, a has been multiplied by $(b + c)$, by b, and by c.

275-P N

275-Q Yes * Commutative property of multiplication

275-R Yes * Distributive property of multiplication over addition

275-S Yes * Commutative property of multiplication

275-T Yes

275-U Yes * You know that $a(b + c) = (ab) + (ac)$ is a property of natural numbers. Since you obtained $(b + c)a = (ba) + (ca)$ from $a(b + c) =$

$(ab) + (ac)$, you know that $(b + c)a = (ba) + (ca)$ is also a property of natural numbers. This means that for each replacement of a, b, and c, a true statement is obtained from $(b + c)a = (ba) + (ca)$.

276-A (Column 1, top) Yes. Since 53 is the same as 50 + 3, 6(53) is the same as 6(50 + 3).

276-B (Column 1) Yes * The distributive property of multiplication over addition

276-C (Column 1) Yes. $6 \cdot 50$ is the same as 300 and $6 \cdot 3$ is the same as 18.

276-D (Column 1) Yes

276-E (Column 1) Yes

276-F Yes * Distributive property of multiplication over addition

276-G Yes

276-H $8 \times 47 = 8(40 + 7)$.
$8(40 + 7) = (8 \cdot 40) + (8 \cdot 7)$.
$(8 \cdot 40) + (8 \cdot 7) = 320 + 56$.
$320 + 56 = 376$.

276-I $7 \cdot 92 = 7(90 + 2)$.
$7(90 + 2) = (7 \cdot 90) + (7 \cdot 2)$.
$(7 \cdot 90) + (7 \cdot 2) = 630 + 14$.
$630 + 14 = 644$.

276-J $3(79) = 3(70 + 9)$.
$3(70 + 9) = (3 \cdot 70) + (3 \cdot 9)$.
$(3 \cdot 70) + (3 \cdot 9) = 210 + 27$.
$210 + 27 = 237$.

276-A (Column 1, bottom) For any three natural numbers, the sum of the first number and the product of the second and third numbers is the same as the product of the sum of the first and second numbers and the sum of the first and third numbers.

276-B (Column 2) a was replaced by 3, b by 4, and c by 7.

276-C (Column 2) Yes

276-D (Column 2) No *
$3 + (4 \cdot 7) = 3 + 28$.
$3 + 28 = 31$.
$(3 + 4)(3 + 7) = 7(10)$.
$7(10) = 70$. Therefore, $3 + (4 \cdot 7)$ is not the same as $(3 + 4)(3 + 7)$.

276-E (Column 2) No. If addition did distribute over multiplication then, for each replacement of a, b, and c, you would obtain a true statement from $a + (bc) = (a + b)(a + c)$. Since you know of at least one replacement for which you do not obtain a true statement, you know that addition does not distribute over multiplication.

277-F a was replaced by 5, b by 6, and c by 2.

277-G Yes

277-H $4(3 - 7) = (4 \cdot 3) - (4 \cdot 7)$. * No * No

277-I Examples are:
$12(4 - 1) = (12 \cdot 4) - (12 \cdot 1)$, and
$8(10 - 9) = (8 \cdot 10) - (8 \cdot 9)$.

277-J Since you know that the difference of b and c is a natural number, you also know that b is the sum of c and the natural number. Therefore, b must be greater than or equal to c.

277-K Yes * Yes. When b is greater than or equal to c, you obtain a true statement from $a(b - c) = (ab) - (ac)$.

On your own

277-1 True statement * Distributive property

277-2 True statement * Commutative property of addition

277-3 True statement * Associative property of multiplication

277-4 False statement

277-5 True statement * Distributive property

277-6 False statement

277-7 True statement * Associative property of addition

277-8 True statement * Commutative property of multiplication

277-9 False statement

277-10 True statement * Commutative property of addition

277-11 False statement

277-12 False statement

277-13 True statement * Commutative property of multiplication

277-14 False statement

277-15 {0, 1, 2, ...}

277-16 {0, 1, 2, ...}

277-17 {0, 1, 2, ...}

277-18 {0, 1, 2, ...}

277-19 {2}

277-20 {8}

277-21 {2}

277-22 {4}

277-23 {0, 1, 2, 3, 4}

277-24 {0, 1, 2, ..., 8}

277-25 {7}

277-26 { }

277-27 {3}

277-28 $15 + 35 = (5 \cdot 3) + (5 \cdot 7)$. $(5 \cdot 3) + (5 \cdot 7) = 5(3 + 7)$. Therefore, $15 + 35 = 5(10)$.

277-29 $35 + 14 = (7 \cdot 5) + (7 \cdot 2)$. $(7 \cdot 5) + (7 \cdot 2) = 7(5 + 2)$. Therefore, $35 + 14 = 7(7)$.

277-30 $45 + 18 = (9 \cdot 5) + (9 \cdot 2)$. $(9 \cdot 5) + (9 \cdot 2) = 9(5 + 2)$. Therefore, $45 + 18 = 9(7)$. Or $45 + 18 = (3 \cdot 15) + (3 \cdot 6)$. $(3 \cdot 15) + (3 \cdot 6) = 3(15 + 6)$. Therefore, $45 + 18 = 3(21)$.

277-31 $35 + 42 = (7 \cdot 5) + (7 \cdot 6)$. $(7 \cdot 5) + (7 \cdot 6) = 7(5 + 6)$. Therefore, $35 + 42 = 7(11)$.

277-32 $46 + 32 = (2 \cdot 23) + (2 \cdot 16)$. $(2 \cdot 23) + (2 \cdot 16) = 2(23 + 16)$. Therefore, $46 + 32 = 2(39)$.

277-33 $154 + 24 = (2 \cdot 77) + (2 \cdot 12)$. $(2 \cdot 77) + (2 \cdot 12) = 2(77 + 12)$. Therefore, $154 + 24 = 2(89)$.

277-34 $22 + 121 = (11 \cdot 2) + (11 \cdot 11)$. $(11 \cdot 2) + (11 \cdot 11) = 11(2 + 11)$. Therefore, $22 + 121 = 11(13)$.

277-35 $75 + 110 = (5 \cdot 15) + (5 \cdot 22)$. $(5 \cdot 15) + (5 \cdot 22) = 5(15 + 22)$. Therefore, $75 + 110 = 5(37)$.

277-36 $51 + 34 = (17 \cdot 3) + (17 \cdot 2)$. $(17 \cdot 3) + (17 \cdot 2) = 17(3 + 2)$. Therefore, $51 + 34 = 17(5)$.

Applying mathematics

278-1 x, the number of dollars the discount was.
$937 - x = 750$, or $937 - 750 = x$.
{187}. $187

278-2 x, the number of incorrect answers.
$7/1 \sim 91/x$.
{13}. 13 incorrect answers

278-3 x, the number of magazines he gave away.
y, the number of magazines he kept.
$5 - x = y$.
{(1,4), (2,3), (3,2), (4,1)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (2,3) is Ralph gave 2 magazines away and kept 3 magazines.

278-4 x, the number of persons under 20 who entered this tournament.
$25/100 \sim x/84$.
{21}. 21 persons

278-5 x, the number of cents the 5 bars of soap will cost.
$3/51 \sim 5/x$.
{85}. 85¢

278-6 x, the number of miles in all he drove on 3 days.
$235 + 299 + 317 = x$.
{851}. 851 miles
THINK The order in which you group natural numbers does not affect the sum.

278-7 x, the number of dollars the regular price of the skirt was.
$20/100 \sim 4/x$.
{20}. $20

278-8 x, the number of pounds Bob weighed at the beginning of the season.
$x > 184 \wedge x - 14 < 176$.
{185, 186, 187, 188, 189}
From 185 through 189 lb.

278-9 x, the number of dollars her yearly take-home pay amounts to.
$82/100 \sim x/6500$.
{5330}. $5330

278-10 x, the number of blouses Ruth has.
y, the number of sweaters she has.
$x > y \wedge x + 3 = 8$.
{(5,0), (5,1), (5,2), (5,3), (5,4)}. Ruth has 5 blouses and from 0 through 4 sweaters.

278-11 x, the number of handbills Jack delivered.
y, the number of handbills Ross delivered.
$7/4 \sim x/y \wedge x - 75 = 86$.
{161/92}. Jack delivered 161 handbills, and Ross delivered 92 handbills.

278-12 x, the number of posters Ellen made.
y, the number of posters Marie made.
$x - y = 6 \wedge x + y = 20$.
{(13,7)}. Ellen made 13 posters and Marie made 7.

278-13 x, the number of women enrolled at the university. y, the number of men enrolled at the university. $x + y = 600 \wedge 47/100 \sim x/600$. {(282, 318)}. 318 men

278-14 x, the number of motor vehicles registered in Quebec and Prince Edward Island. y, the number of motor vehicles registered in provinces other than Quebec and Prince Edward Island. $x + y = 5{,}017{,}686 \wedge 1{,}040{,}366 + 27{,}502 = x$. {(1067868, 3949818)} 3,949,818 vehicles

278-15 $x > 15 \wedge 92 - x > 65$. {16, 17, 18, ..., 26}

278-16 $16/8 \sim x/y \wedge x < 11$. {2/1, 4/2, 6/3, 8/4, 10/5}

278-17 $x + y < 6 \wedge y > 2$. {(0,3), (0,4), (0,5), (1,3), (1,4), (2,3)}

Checking up

Test 67

279-1 True

279-2 True

279-3 False

279-4 False

279-5 True

Test 68

279-6 Distributive property of multiplication over addition

279-7 Commutative property of multiplication

279-8 Associative property of addition

279-9 Associative property of multiplication

279-10 True * $(3 \cdot 5) + (3 \cdot 8) = 3(5 + 8)$. Distributive property of multiplication over addition. $3(5 + 8) = (5 + 8)3$. Commutative property of multiplication

279-11 True * $(4 + 10) + 2 = 2 + (4 + 10)$. Commutative property of addition. $2 + (4 + 10) = 2 + (10 + 4)$. Commutative property of addition

279-12 False

279-13 True * $5(31 + 7) = 5(7 + 31)$. Commutative property of addition. $5(7 + 31) = (5 \cdot 7) + (5 \cdot 31)$. Distributive property of multiplication over addition

Lesson 65

279-A Numeral B tells you that the number $8 \cdot 2$ was obtained by multiplying 8 and 2. Numeral C tells you that the number $19 - 3$ was obtained by subtracting 3 from 19. Numeral D tells you that the number $32 \div 2$ was obtained by dividing 32 by 2.

279-B No

279-C 8 and 2 * A symbol for multiplication (·)

280-D 19 and 3 * The symbol for subtraction (-)

280-E The standard name of $9 + 7$ is 16. The standard name of $8 \cdot 2$ is 16. The standard name of $19 - 3$ is 16. The standard name of $32 \div 2$ is 16. * Yes

280-F $15 - 9 = 6$.

280-G $42 \div 7 = 6$.

280-H $23 + 8 = 31$.

280-I $6 \cdot 50 = 300$.

280-J Yes * The numeral $(4 \cdot 5) + (4 \cdot 3)$ contains both standard names and symbols for operations. * A symbol for multiplication (·) and the symbol for addition (+)

280-K The numeral 32

280-L Yes

280-M A symbol for multiplication (·) and the symbol for subtraction (-) * $(6 \cdot 7) - (6 \cdot 2) = 42 - 12$. $42 - 12 = 30$.

280-N The symbol for addition (+) and a symbol for multiplication (·)

280-O $(4 + 3)6 = (7)6$. $(7)6 = 42$.

280-P $4 + (3 \cdot 6) = 4 + 18$. $4 + 18 = 22$.

280-Q No

281-R 18

281-S 22

281-T Yes * To obtain the number expressed by $4 + 3 \cdot 6$, you perform the operation of multiplication before the operation of addition. Therefore, $4 + 3 \cdot 6$ is the same as $4 + 18$, or 22.

281-U To find the number expressed by either $9 + 24 \div 4$, or $9 + (24 \div 4)$, you perform the operation of division before the operation of addition. * $9 + 24 \div 4 = 9 + 6$. $9 + 6 = 15$.

281-V Expression I: $4 \cdot 2 + 7 = 8 + 7$. $8 + 7 = 15$.

Expression J:
25 − 5 · 2 = 25 − 10.
25 − 10 = 15.
Expression K:
3 · 2 + 3 · 3 = 6 + 9.
6 + 9 = 15.
Expression L:
42 ÷ 2 − 18 ÷ 3 = 21 − 6.
21 − 6 = 15.
Expression M:
3 · 4 + 12 ÷ 4 = 12 + 3.
12 + 3 = 15.

281-W. Yes

On your own

281-1 Examples are: 2 + 4, 12 · 1 ÷ 2, 11 − 5, 1 + 1 · 5, and 21 ÷ 3 − 1.

281-2 Examples are: 2 · 9, 3 · 3 · 2, 3(3 + 3), 6(5 − 2), and 5 × 5 − 56 ÷ 8.

281-3 Examples are: 100 ÷ 4, 10 + 3 · 5, 78 ÷ 2 − 14, 5 · 5, and 11 + 14.

281-4 Examples are: 54 ÷ 2, 3 · 3 · 3, 40 − (6 + 7), 19 + 2 · 4, and 5 × 6 − 3.

281-5 9 + 6 · 2 = 9 + 12.
9 + 12 = 21.

281-6 8 + 7 − 13 = 15 − 13.
15 − 13 = 2.

281-7 18 ÷ 6 + 7 = 3 + 7.
3 + 7 = 10.

281-8 16 − 5 · 3 = 16 − 15.
16 − 15 = 1.

281-9 13 − 24 ÷ 8 = 13 − 3.
13 − 3 = 10.

281-10 24 ÷ 2 · 3 = 12 · 3.
12 · 3 = 36.

281-11 14 − 3 + 6 = 11 + 6.
11 + 6 = 17.

281-12 8 · 9 + 8 · 3 = 72 + 24.
72 + 24 = 96.

281-13 (2 + 3)(2 + 7) = 5(9).
5(9) = 45.

281-14 9 · 7 ÷ 3 = 63 ÷ 3.
63 ÷ 3 = 21.

281-15 7 + 8 · 3 + 6 =
7 + 24 + 6.
7 + 24 + 6 = 31 + 6.
31 + 6 = 37.

281-16 16 + (6 + 2)5 = 16 + (8)5.
16 + (8)5 = 16 + 40.
16 + 40 = 56.

281-17 True statement

281-18 True statement

281-19 False statement

281-20 False statement

281-21 False statement

281-22 True statement

281-23 False statement

281-24 True statement

281-25 False statement

281-26 True statement

281-27 (5 + 4)·(3 + 1) = 36.

281-28 13 − (8 − 3) = 8.

281-29 60 = 12 · (7 − 2).

281-30 36 ÷ (3 · 3) + 1 = 5.

281-31 3 · (6 + 4) − 2 = 28.

281-32 13 = 14 ÷ (2 − 1) − 1.

281-33 (6 + 3)·(2 + 4) = 54.

281-34 (18 + 2) ÷ 5 + 4 = 8.

281-35 42 ÷ (6 + 1) = 6.

Keeping Skilful

281-1 3 = n{0, 1, 2}.

281-2 11 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

281-3 0 = n{ }.

281-4 7 = n{0, 1, 2, 3, 4, 5, 6}.

281-5 5 = n{0, 1, 2, 3, 4}.

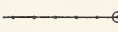
281-6 1 = n{0}.

281-7 8 = n{0, 1, 2, 3, 4, 5, 6, 7}.

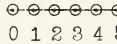
281-8 14 = n{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}.

281-9 {0, 1, 2, ..., 7}

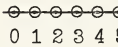
281-10 {5}



281-11 {0, 1, 2, ..., 8}

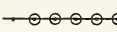


281-12 {0, 1, 2, 3, 4, 5}



281-13 {7}

281-14 {1, 2, 3, ...}



281-15 20

281-16 25

281-17 38

281-18 19

281-19 33

281-20 68

281-21 84

281-22 76

281-23 154

Lesson 66

282-A The set of natural numbers

282-B Addition and multiplication * Yes

282-C $ab = ba$.

282-D Addition * Multiplication

282-E Distributive property

282-F Yes * Yes * The set of even natural numbers

283-G Yes * 2 × 3 = 6. 2 × 7 = 14. 2 is a factor of each of the numbers 6 and 14.

283-H Yes. 6 + 14 = 20 and 2 × 10 = 20. * Yes. 6 × 14 = 84 and 2 × 42 = 84.

283-I For example: replace a by 24 and b by 16. Then $a + b$ becomes 24 + 16, or 40. * Yes. Since

$2 \times 20 = 40$, 40 is an even number.

283-J ab becomes $24 \cdot 16$, or 384. * Yes. Since $2 \times 192 = 384$, 384 is an even number.

283-K E is closed under addition and multiplication. * When you add two even numbers, the sum is always an even number. When you multiply two even numbers, the product is always an even number.

283-L $2m + 2n = 2(m + n)$. The distributive property. $2(m + n)$ is an even number because it has a factor of 2.

283-M $(2m)(2n) = 2(m)(2n)$. The associative property of multiplication. $2(m)(2n) = 2(m \cdot 2n)$. The associative property of multiplication. $2(m \cdot 2n)$ is an even number because it has a factor of 2.

283-N Yes

283-O Addition and multiplication are commutative in set E.

283-P Yes

283-Q Addition and multiplication are associative in set E.

283-R Yes

283-S Multiplication distributes over addition in the set of even numbers.

284-T Yes * Set E is a number system because it behaves like the natural-number system with respect to two operations and their properties.

On your own

284-1 True statement * Commutative property of addition

284-2 True statement * Associative property of multiplication

284-3 True statement * Distributive property

284-4 True statement * Closure property of addition

284-5 True statement * Commutative property of multiplication

284-6 True statement * Associative property of addition

284-7 False statement

284-8 True statement * Closure property of addition

284-9 False statement

284-10 True statement * Commutative property of addition

284-11 True statement * Associative property of addition

284-12 True statement * Closure property of multiplication

284-13 True statement * Commutative property of multiplication

284-14 False statement

284-15 True statement * Associative property of multiplication

284-16 {5}

284-17 {4}

284-18 { }

284-19 {0, 1, 2, ...}

284-20 {0, 1, 2, ...}

284-21 {3}

284-22 {0, 1, 2, 3}

284-23 {8}

284-24 {0}

284-25 Not a number system * The set is not closed under addition.

284-26 Is a number system

284-27 Not a number system * The set is not closed under addition.

Keeping Skilful

284-1 27,280

284-2 20,099

284-3 40,930

284-4 242,109

284-5 221,100

284-6 2,021,561

284-7 2809

284-8 78

284-9 520

284-10 22,119

284-11 14,326

284-12 54,587

284-13 68,445

284-14 281,725

284-15 1296

284-16 4239

284-17 47,704

284-18 478,743

284-19 241,280

284-20 533,280

284-21 151,434

284-22 364,224

284-23 56, remainder 0

284-24 77, remainder 25

284-25 284, remainder 2

284-26 306, remainder 0

284-27 20, remainder 237

284-28 79, remainder 0

284-29 362, remainder 7

284-30 318, remainder 0

Lesson 67

285-A (Column 1) Yes

285-B (Column 1) Yes

285-C (Column 1) Yes

285-D (Column 1) Examples are: $42 \div 5 = 8$, remainder 2. $196 \div 5 = 39$, remainder 1.

28 ÷ 5 = 5, remainder 3.

285-E Yes

285-F 0, 1, 2, 3, 4

285-G Yes * Yes. Set T has 5 members.

285-A (Column 2) Yes

285-B (Column 2) Yes * Yes * Yes

285-C (Column 2) Yes * 0 * Yes

285-D (Column 2) Sentence C expresses a true statement because you obtain a remainder of 1 when you divide the sum of 2 and 4 by 5. The remainder, 1, is the addiv of 2 and 4. Sentence D expresses a true statement because you obtain a remainder of 3 when you divide the sum of 4 and 4 by 5. The remainder, 3, is the addiv of 4 and 4.

286-E 0

286-F 0 and 0; 3 and 2; 2 and 3; 1 and 4

286-G Yes * Yes

286-H Yes

286-I Yes * Yes

286-J Yes * Yes

286-K You obtain a remainder of 2 when you divide the product of 3 and 4 by 5.

286-L 3

286-M

⊗	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

286-N Yes * Yes

286-A 0 * 0

287-B Yes * The addiv of 4 and 1 is the same as the addiv of 1 and 4.

287-C 2 * 2

287-D Yes * The muldiv of 3 and 4 is the same as the muldiv of 4 and 3.

287-E No * No

287-F Addivision is commutative in set T because you always obtain a true statement from $a \oplus b = b \oplus a$ when the universe for a and b is T. * Muldivision is commutative in set T because you always obtain a true statement from $a \otimes b = b \otimes a$ when the universe for a and b is T.

287-G Yes * The sum of 4 and 2 is 6. When 6 is divided by 5, there is a remainder of 1. * 4

287-H 0 * 4

287-I Yes

287-J Yes * The product of 2 and 4 is 8. When 8 is divided by 5, there is a remainder of 3. * 1 * $2 \otimes (4 \otimes 2) = 2 \otimes 3$. $2 \otimes 3 = 1$.

287-K Yes

287-L No * No

287-M Addivision is associative in set T. For each replacement from set T, a true statement is obtained from $(a \oplus b) \oplus c = a \oplus (b \oplus c)$. * Muldivision is associative in set T. For each replacement from set T, a true statement is obtained from $(a \otimes b) \otimes c = a \otimes (b \otimes c)$.

287-N $2 \otimes (4 \oplus 3) = 2 \otimes 2$. $2 \otimes 2 = 4$. * $(2 \otimes 4) \oplus (2 \otimes 3) = 3 \oplus 1$. $3 \oplus 1 = 4$. * Yes

287-O No

287-P Muldivision distributes over addivision in set T. * For each replacement of the variables from set T, a true statement is obtained from $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$.

287-Q Yes * Set T, together with the operations of addivision and muldivision, behaves like the natural-number system with respect to two operations and their properties.

On your own

288-1 0, 1, 2 * Yes

288-2 Yes * The addiv of any two members of F is also a member of F.

288-3 Yes * The muldiv of any two members of F is also a member of F.

288-4 Yes * Examples are:
$1 \oplus 2 = 0$. $2 \oplus 1 = 0$.
Therefore, $1 \oplus 2 = 2 \oplus 1$.
$2 \oplus 0 = 2$. $0 \oplus 2 = 2$.
Therefore, $2 \oplus 0 = 0 \oplus 2$.

288-5 Yes * Examples are:
$0 \otimes 1 = 0$. $1 \otimes 0 = 0$.
Therefore, $0 \otimes 1 = 1 \otimes 0$.
$2 \otimes 1 = 2$. $1 \otimes 2 = 2$.
Therefore, $2 \otimes 1 = 1 \otimes 2$.

288-6 Yes * Examples are:
$(1 \oplus 2) \oplus 2 = 2$.
$1 \oplus (2 \oplus 2) = 2$.
Therefore, $(1 \oplus 2) \oplus 2 = 1 \oplus (2 \oplus 2)$.
$(0 \oplus 2) \oplus 1 = 0$.
$0 \oplus (2 \oplus 1) = 0$.
Therefore, $(0 \oplus 2) \oplus 1 = 0 \oplus (2 \oplus 1)$.

288-7 Yes * Examples are:
$(2 \otimes 1) \otimes 2 = 1$.
$2 \otimes (1 \otimes 2) = 1$.
Therefore, $(2 \otimes 1) \otimes 2 = 2 \otimes (1 \otimes 2)$.
$(1 \otimes 0) \otimes 2 = 0$.
$1 \otimes (0 \otimes 2) = 0$.
Therefore, $(1 \otimes 0) \otimes 2 = 1 \otimes (0 \otimes 2)$.

288-8 Yes * Examples are:
$2 \otimes (1 \oplus 2) = 0$.
$(2 \otimes 1) \oplus (2 \otimes 2) = 0$.
Therefore, $2 \otimes (1 \oplus 2) = (2 \otimes 1) \oplus (2 \otimes 2)$.
$1 \otimes (2 \oplus 0) = 2$.
$(1 \otimes 2) \oplus (1 \otimes 0) = 2$.
Therefore, $1 \otimes (2 \oplus 0) = (1 \otimes 2) \oplus (1 \otimes 0)$.

288-9 Yes * Set F, together with the operations of addivision and muldivision, behaves like the natural-number system with respect to two operations and their properties.

288-10

$\oplus$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

288-11

$\otimes$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

288-12 Yes * The addiv of any two members of G is also a member of G.

288-13 Yes * The muldiv of any two members of G is also a member of G.

288-14 Yes * Examples are:
$3 \oplus 5 = 1$. $5 \oplus 3 = 1$.
Therefore, $3 \oplus 5 = 5 \oplus 3$.
$4 \oplus 1 = 5$. $1 \oplus 4 = 5$.
Therefore, $4 \oplus 1 = 1 \oplus 4$.

288-15 Yes * Examples are:
$2 \otimes 3 = 6$. $3 \otimes 2 = 6$.
Therefore, $2 \otimes 3 = 3 \otimes 2$.
$5 \otimes 4 = 6$. $4 \otimes 5 = 6$.
Therefore, $5 \otimes 4 = 4 \otimes 5$.

288-16 Yes * Examples are:
$2 \oplus (3 \oplus 4) = 2$.
$(2 \oplus 3) \oplus 4 = 2$.
Therefore, $2 \oplus (3 \oplus 4) = (2 \oplus 3) \oplus 4$.
$(1 \oplus 6) \oplus 3 = 3$.
$1 \oplus (6 \oplus 3) = 3$.
Therefore, $(1 \oplus 6) \oplus 3 = 1 \oplus (6 \oplus 3)$.

288-17 Yes * Examples are:
$(2 \otimes 5) \otimes 3 = 2$.
$2 \otimes (5 \otimes 3) = 2$.
Therefore, $(2 \otimes 5) \otimes 3 = 2 \otimes (5 \otimes 3)$.
$(4 \otimes 1) \otimes 6 = 3$.
$4 \otimes (1 \otimes 6) = 3$.
Therefore, $(4 \otimes 1) \otimes 6 = 4 \otimes (1 \otimes 6)$.

288-18 Yes * Examples are:
$3 \otimes (4 \oplus 2) = 4$.
$(3 \otimes 4) \oplus (3 \otimes 2) = 4$.
Therefore, $3 \otimes (4 \oplus 2) = (3 \otimes 4) \oplus (3 \otimes 2)$.
$6 \otimes (2 \oplus 1) = 4$.
$(6 \otimes 2) \oplus (6 \otimes 1) = 4$.
Therefore, $6 \otimes (2 \oplus 1) = (6 \otimes 2) \oplus (6 \otimes 1)$.

288-19 Yes * Set G, together with the operations of addivision and muldivision, behaves like the natural-number system with respect to two operations and their properties.

Lesson 68

289-A 0 * 4 * 5 * {5, 6, 7, 8}

289-B Yes

289-C Yes * Yes * Yes

289-D {30, 40, 50, 60, 70} * Yes

289-E Yes * Yes * Yes

289-F Yes

289-G Yes * Yes * Yes

289-H For the statement expressed by sentence A, a was replaced by 8. For the statement expressed by sentence B, a was replaced by 99. For the statement expressed by sentence C, a was replaced by 0. * Yes

289-I Yes * You know that $0 + a = a$ is a property. You also know that $0 + a = a + 0$ because of the commutative property of addition. Therefore, you can obtain $a + 0 = a$ from $0 + a = a$. Since $0 + a = a$ is a property, $a + 0 = a$ is also a property.

289-J {4} * {4}

290-K {0, 1, 2, ...}

290-L Yes * You know that $a + 0 = a$ is a property. $a + 0 = a$ is, of course, the same as $a = a + 0$. Since $a - a = 0$ also means $a = a + 0$, then $a - a = 0$ must be a property.

290-M Yes * You know that $a = 0 + a$ is a property. Since $a - 0 = a$ also

means $a = 0 + a$, then $a - 0 = a$ must also be a property.

290-N No * No

290-A 1 * 3 * 6

290-B {(6,7), (6,14), (6,21)} * Yes

290-C Yes * D5 expresses one way that M × K and K can be put in one-to-one correspondence. Two sets that can be put in one-to-one correspondence are equivalent sets.

290-D Yes

290-E {(6,1), (6,3), (6,5), (6,7), (6,9), (6,11)} * Yes

290-F Yes * Yes

290-G Yes

290-H B × A * Yes * Yes

291-I For the statement expressed by sentence G, a was replaced by 16. For the statement expressed by sentence H, a was replaced by 43. For the statement expressed by sentence I, a was replaced by 1. * Yes

291-J Yes * You know that $1 \cdot a = a$ is a property. You also know that $1 \cdot a = a \cdot 1$ because of the commutative property of multiplication. Therefore, since $1 \cdot a = a$ is a property, $a \cdot 1 = a$ is also a property.

291-K {3} * {3}

291-L Yes * You know that $a = 1 \cdot a$ is a property. Since $a/a = 1$ also means $a = 1 \cdot a$, you also know that $a/a = 1$ is a property.

291-M Since you know that $a = a \cdot 1$ is a property, you also know that $a/1 = a$ is a property.

291-N No * No

291-A 0 * 2

291-B Yes * Yes

291-C Yes * Yes

291-D Yes

291-E { } × A * Yes

292-F Yes * Yes * Yes

292-G You know that $0 \cdot a = 0$ is a property. You also know that $0 \cdot a = a \cdot 0$ because of the commutative property of multiplication. Since you can obtain $a \cdot 0 = 0$ from $0 \cdot a = 0$, you know that $a \cdot 0 = 0$ is also a property.

292-H Yes * You know that you obtain a true statement from $0 = 0 \cdot a$ for each replacement of a from {1, 2, 3, ...}. Since $0/a = 0$ also means $0 = 0 \cdot a$, you know that a true statement is obtained from $0/a = 0$ for each replacement of a from {1, 2, 3, ...}.

292-I No

292-J No

292-K Yes * Yes

292-L Yes * Yes. Since $0/0 = b$ also means $0 = b \times 0$, then the solution set of $0/0 = b$ is the same as the solution set of $0 = 0 \times b$. Since the solution set of $0 = b \times 0$ is {0, 1, 2, ...}, the solution set of $0/0 = b$ is also {0, 1, 2, ...}.

On your own

292-1 16

292-2 126

292-3 12

292-4 3

292-5 0

292-6 157

292-7 33

292-8 72

292-9 {1}

292-10 {5}

292-11 {1}

292-12 {0}

292-13 Yes * No

292-14 No * Yes

Keeping Skilful

292-1 3

292-2 20

292-3 4

292-4 15

292-5 11

292-6 3

292-7 36

292-8 51

292-9 28

292-10 {72}

292-11 {200}

292-12 {68}

292-13 {10}

292-14 {80}

292-15 {300}

292-16 {102}

292-17 {37}

292-18 {240}

292-19 {272}

Applying mathematics

293-1 x, the number of dollars deducted each month from Mrs Rowe's salary.
$482 - x = 395$.
{87}. $87

293-2 x, the numbers of acres Mr. Ferguson had in pasture last year.
$4/1 \sim 268/x$.
{67}. 67 acres

293-3 x, the number of dollars commission for the month.
$7/100 \sim x/4200$.
{294}. $294

293-4 x, the number of stamps the stamp collector had before he sold the 420 stamps.
$35/100 \sim 420/x$.
{1200}. 1200 stamps

293-5 x, the number of years old Mark is now.
$x - 7 < 18 \wedge x + 9 > 28$.
{20, 21, 22, 23, 24}. From 20 through 24 years old

293-6 x, the number of votes Bill received for each 100 votes cast.
$136/200 \sim x/100$.
{68}. 68%

293-7 x, the number of storm windows Mike painted on Friday.
y, the number of storm windows Mike painted on Saturday.
$x + y < 12 \wedge x = y$.
{(1,1), (2,2), (3,3), (4,4), (5,5)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (2,2) is Mike painted 2 storm windows on Friday and 2 storm windows on Saturday.

293-8 x, the number of envelopes the machine folds in 8 minutes.
$10,800/12 \sim x/8$.
{7200}. 7200 envelopes

294-9 x, the number of points Chuck scored.
y, the number of points Steve scored.
$x + y = 19 \wedge y - 7 = x$.
{(6,13)}. Chuck scored 6 points and Steve scored 13 points.

294-10 x, the number of colleges in Alberta.
y, the number of colleges in Newfoundland.
$x + y = 12 \wedge 5/1 \sim x/y$.
{(10,2)}. There are 10 colleges in Alberta and 2 colleges in Newfoundland.

294-11 x, number of dollars interest.
y, number of dollars Mr. Larson paid at the end of the year.
$6/100 \sim x/1650 \wedge 1650 + x = y$.
{(99,1749)}. $1749

294-12 x, the number of dollars Lois spent for the skirt, sweater, and blouse.
y, the number of dollars she had left.
$41 - x = y \wedge x = 13 + 9 + 4$.
{(26,15)}. $15

294-13 $6/100 \sim 51/x$.
{850}

294-14 $x > 63(19)$.
{1198, 1199, 1200, ...}

294-15 $y - x = 917 \wedge 796 + x = 2243$.
{(1447,2364)}

294-16 $147/100 \sim x/4000$.
{5880}

294-17 $x + y = 8158 \wedge x = 214(38)$.
{(8132,26)}

294-18 $x + y < 12 \wedge x - y = 3$.
{(3,0), (4,1), (5,2), (6,3), (7,4)}

294-19 $x/100 \sim 235/188$.
{125}

294-20 $28/32 \sim x/y \wedge x + y < 15$.
{ }

294-21 $x + y = 349 \wedge 7y = 791$.
{(236,113)}

Checking up

Test 69

294-1 $(15 \div 3) \times 4 = 5 \times 4$.
$5 \times 4 = 20$.

294-2 $6(7 + 2) + 6(7 - 2) = 6(9) + 6(5)$.
$6(9) + 6(5) = 54 + 30$.
$54 + 30 = 84$.

294-3 $9 \div 9 + 10 \div 10 = 1 + 1$.
$1 + 1 = 2$.

294-4 $7 \times 3 + 3 \times 2 = 21 + 6$.
$21 + 6 = 27$.

294-5 $13 + 7 \times 2 = 13 + 14$.
$13 + 14 = 27$.

294-6 $48 \div 8 + 3(5 + 2) = 6 + 3(7)$.
$6 + 3(7) = 6 + 21$.
$6 + 21 = 27$.

294-7 Commutative property of addition

294-8 Identity element property of multiplication

294-9 Associative property of multiplication

294-10 Closure property of addition

294-11 Identity element property of addition

294-12 Commutative property of multiplication

294-13 {27}

294-14 {2}

294-15 {0}

294-16 {0, 1, 2, ...}

Test 70

294-17 First find the product of the two members of Q in the usual way. Then divide the product by 4. The remainder is the muldiv of the two members.

294-18

⊕	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

294-19

⊗	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

294-20 Yes * Set Q is closed under both addivision and muldivision. Both addivision and muldivision are commutative and associative, and muldivision distributes over addivision. In other words, set Q, together with the operations of addivision and muldivision, behaves like the natural-number system with respect to two operations and their properties.

Lesson 69

295-A For the statement expressed by sentence A, a was replaced by 3, b by 2, and c by 6. For the statement expressed by sentence B, a was replaced by 6, b by 1, and c by 6.

295-B Since you know that $2 \times 3 = 6$ is a true statement, you also know that 2 is a factor of 6. * Since you know that $6 \times 1 = 6$ is a true statement, you also know that 6 is a factor of 6.

295-C Yes * The only true statements that can be obtained from $a \times b = c$, when the replacement for c is 6 are $1 \times 6 = 6$, $2 \times 3 = 6$, $6 \times 1 = 6$, and $3 \times 2 = 6$.

295-D {1, 2, 4, 8} * {1, 2, 3, 4, 6, 12} * {1, 13}

295-E Yes * Yes

295-F Yes * Every counting number is a factor of itself.

295-G Yes. Since you know that $2 \times 2 = 4$ is a true statement, you also know that 2 is a factor of 2×2, or 4. * Yes. Since you know that $2 \times 3 = 6$ is a true statement, you also know that 2 is a factor of 2×3, or 6.

295-H Yes * Yes * Yes

295-I Yes

295-J No

295-K Yes * No

295-L Yes * Yes * Yes * Yes

295-M Yes * No

296-N Both 2 and 3 are factors of 6 * 12, 18, 24, 30

296-O Yes * {1, 2, 4}

296-P The set of factors of 6 is {1, 2, 3, 6}. The set of factors of 8 is {1, 2, 4, 8}. The set of factors of 9 is {1, 3, 9}. The set of factors of 10 is {1, 2, 5, 10}. The set of factors of 12 is {1, 2, 3, 4, 6, 12}.

296-Q You know that 1 is a factor of every member of C and that every member of C is a factor of itself.

296-A Your chart should be similar to the one shown at the top of the next column.

1	$\not{2}\not{6}_{2,13}$
2	$\not{2}\not{7}_{3,9}$
3	$\not{2}\not{8}_{2,4,7,14}$
$\not{4}_2$	29
5	$\not{3}\not{0}_{2,3,5,6,10,15}$
$\not{8}_{2,3}$	31
7	$\not{3}\not{2}_{2,4,8,16}$
$\not{8}_{2,4}$	$\not{3}\not{3}_{3,11}$
$\not{9}_3$	$\not{3}\not{4}_{2,17}$
$\not{1}\not{0}_{2,5}$	$\not{3}\not{5}_{5,7}$
11	$\not{3}\not{6}_{2,3,4,6,9,12,18}$
$\not{1}\not{2}_{2,3,4,6}$	37
13	$\not{3}\not{8}_{2,19}$
$\not{1}\not{4}_{2,7}$	$\not{3}\not{9}_{3,13}$
$\not{1}\not{5}_{3,5}$	$\not{4}\not{0}_{2,4,5,8,10,20}$
$\not{1}\not{6}_{2,4,8}$	41
17	$\not{4}\not{2}_{2,3,6,7,14,21}$
$\not{1}\not{8}_{2,3,6,9}$	43
19	$\not{4}\not{4}_{2,4,11,22}$
$\not{2}\not{0}_{2,4,5,10}$	$\not{4}\not{5}_{3,5,9,15}$
$\not{2}\not{1}_{3,7}$	$\not{4}\not{6}_{2,23}$
$\not{2}\not{2}_{2,11}$	47
23	$\not{4}\not{8}_{2,3,4,6,8,12,16,24}$
$\not{2}\not{4}_{2,3,4,6,8,12}$	$\not{4}\not{9}_{7}$
$\not{2}\not{5}_{5}$	$\not{5}\not{0}_{2,5,10,25}$

296-B See the chart given for 296-A

296-C See the chart given for 296-A.

296-D Yes * 2 is a factor of 4. * Yes * Each number that has 4 as a factor also has 2 as a factor. * See the chart given for 296-A.

296-E To check your work, see the chart given for 296-A.

297-F (Column 1) You know that each numeral crossed out names a number that has

a factor other than itself and 1. * 25

297-G {2, 3, 5, 7} * No * The number 1 has only one factor. The only factor of 1 is 1.

297-H You know that a prime number has exactly two factors and that 1 has only one factor. * Yes * Yes

297-I No * No * No * Every even number greater than 2 has at least three factors, itself, 1, and 2.

297-J Yes * 1

297-K Yes * {1, 5, 25}

297-L 49 * {1, 7, 49} * Three

297-A 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

297-B No * For example: There are 4 primes between 1 and 10, 4 primes between 10 and 20, 2 primes between 20 and 30, 2 primes between 30 and 40, 3 primes between 40 and 50, and 2 primes between 50 and 60.

297-C 3 and 5

297-D Yes. They are separated by only one other number, the number 30. * No. They are separated by 38, 39, and 40. * Besides 3 and 5, there are 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, and 71 and 73.

297-E Yes * 3 and 5 * Yes * Yes. 4 is the sum of 2 and 2. * Yes. 10 is the sum of 5 and 5, or the sum of 3 and 7. * Yes. 12 is the sum of 5 and 7. * Yes. 14 is the sum of 3 and 11, or the sum of 7 and 7. * Yes

297-F (Column 2) 16 = 3 + 13, or 16 = 5 + 11.
18 = 5 + 13, or
18 = 7 + 11.
20 = 3 + 17, or
20 = 7 + 13.
22 = 3 + 19, or
22 = 5 + 17, or
22 = 11 + 11.
24 = 5 + 19, or
24 = 7 + 17, or
24 = 11 + 13.
26 = 3 + 23, or
26 = 7 + 19, or
26 = 13 + 13.
28 = 5 + 23, or
28 = 11 + 17.
30 = 7 + 23, or
30 = 11 + 19, or
30 = 13 + 17.
32 = 3 + 29, or
32 = 13 + 19.
34 = 3 + 31, or
34 = 5 + 29, or
34 = 11 + 23, or
34 = 17 + 17.
36 = 5 + 31, or
36 = 7 + 29, or
36 = 13 + 23, or
36 = 17 + 19.

298-G 5 * 5 and 7 * 7

298-H 17, 19, 23 * 17, 19, 23, 29 * 17, 19, 23, 29, 31

298-I See the chart below.

x	$2x$	Prime numbers between x and $2x$
6	12	7, 11
7	14	11, 13
8	16	11, 13
9	18	11, 13, 17
10	20	11, 13, 17, 19
11	22	13, 17, 19
12	24	13, 17, 19, 23
13	26	17, 19, 23

On your own

298-1 {1, 3, 9, 27}

298-2 {1, 3}

298-3 {1, 19}

298-4 {1, 2, 4, 8, 16, 32}

298-5 {1, 3, 19, 57}

298-6 {1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120}

298-7 {1, 167}

298-8 {1, 227}

298-9 3, 19, 167, and 227 are primes. * 27, 32, 57, and 120 are composite numbers.

298-10 Two

298-11 Yes * Each factor of 4 is also a factor of 16.

298-12 Even number

298-13 Even number

298-14 Even number

298-15 Even number

298-16 Odd number

298-17 Odd number

298-18 Odd number

298-19 Even number

298-20 Even number

298-21 1, 7

298-22 1, 2

298-23 1, 2, 4, 5, 10, 20, 25, 50, 100

298-24 1, 2

298-25 1, 3

298-26 1, 2, 13, 26

298-27 1, 2, 7, 14

298-28 1, 2, 3, 6, 9, 18, 27, 54

298-29 1, 3, 13, 39

298-30 3, 17

298-31 2, 3, 7

298-32 89

298-33 2, 11, 13

298-34 2, 3, 5, 11

298-35 7, 11, 13

298-36 2, 89

298-37 2, 3, 11

298-38 3, 5, 13

298-39 30
298-40 1
298-41 10
298-42 1
298-43 5
298-44 16
298-45 18
298-46 25
298-47 3
298-48 7
298-49 120
298-50 8

Lesson 70

299-A No * 2 and 2 * 3 and 2
299-B Yes * Yes
299-C Yes. 2^3 is the same as $2 \times 2 \times 2$. 3^1 is the same as 3. Therefore, $2^3 \times 3^1$ means the same as $2 \times 2 \times 2 \times 3$. * Yes. Sentence E expresses 24 as the product of primes.
299-D 20 * Think of 20 as 2×10.
299-E 10 * Think of 10 as 2×5.
299-F Yes * Yes
299-G Yes * 2^2 is the same as 2×2. 5^2 is the same as 5×5. Therefore, $2^2 \times 5^2$ means the same as $2 \times 2 \times 5 \times 5$. * Yes
299-H No
299-I Think of 10 as 2×5. * 35 * 5 and 7
299-J Yes * 350 = $2^1 \times 5^2 \times 7^1$.
300-K Sentence M and Q * No
300-L Yes
300-M Yes * Yes * Yes * No
300-N Three times
300-O There is just one factor of 3 in the product 21. * Yes
300-P Yes * $2^3 \times 3^1 \times 7^1$
300-Q $16 = 2^4$. * $144 = 2^4 \times 3^2$.
300-R Yes * Yes * No
301-S No * If 1 were considered a prime, every counting number greater than 1 could be factored into primes in infinitely many ways. For example: The number 6 could be expressed as 2×3, $1 \times 2 \times 3$, $1 \times 1 \times 2 \times 3$, $1 \times 1 \times 1 \times 2 \times 3$, and so on.
301-A Set M * Set N
301-B Yes * Yes * Yes
301-C Each of the numbers, 1, 2, and 4, is a factor of 8 and of 20.
301-D {1, 2, 4} * Yes * Yes
301-E {1, 2, 3, 4, 6, 12} * {1, 2, 3, 5, 6, 10, 15, 30} * {1, 2, 3, 6}
301-F {1} * {1} * {1}
301-G Every prime has just two factors, itself and 1. Therefore, you know that 1 is the only common factor of any pair of prime numbers.
301-H Yes * 4
301-I Yes * 14 * A number greater than 14 cannot be a factor of 14.
301-J Yes * Yes * Yes
301-K 2 and 5 * 10 * Yes
301-L $300 = 2^2 \times 3^1 \times 5^2$. * $550 = 2^1 \times 5^2 \times 11^1$.
301-M 2 and 5
302-N 50 * Yes
302-O Yes
302-P Yes
302-Q Yes
302-R 12 * 12
302-S $14 = 2^1 \times 7^1$. $21 = 3^1 \times 7^1$. $28 = 2^2 \times 7^1$.
302-T 7
302-U $100 = 2^2 \times 5^2$. $125 = 5^3$. $75 = 3^1 \times 5^2$. The greatest common factor of 100, 125, and 75 is 5^2, or 25.

On your own

302-1 For 6: {1, 2, 3, 6}. For 8: {1, 2, 4, 8} * {1, 2}
302-2 For 15: {1, 3, 5, 15}. For 30: {1, 2, 3, 5, 6 10, 15, 30} * {1, 3, 5, 15}
302-3 For 27: {1, 3, 9, 27}. For 15: {1, 3, 5, 15} * {1, 3}
302-4 For 36: {1, 2, 3, 4, 6, 9, 12, 18, 36}. For 27: {1, 3, 9, 27} * {1, 3, 9}
302-5 For 30: {1, 2, 3, 5, 6, 10, 15, 30}. For 72: {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72} * {1, 2, 3, 6}
302-6 For 16: {1, 2, 4, 8, 16}. For 12: {1, 2, 3, 4, 6, 12} * {1, 2, 4}
302-7 2 is the greatest common factor of 6 and 8.
15 is the greatest common factor of 15 and 30.
3 is the greatest common factor of 27 and 15.
9 is the greatest common factor of 36 and 27.
6 is the greatest common factor of 30 and 72.
4 is the greatest common factor of 16 and 12.
302-8 1 * 1 * 1 * 1
302-9 Yes * 1
302-10 An example of a set of three composite numbers whose only common factor is 1 is {6, 35, 143}. Another example is {4, 6, 9}.

302-11 $24 = 2^3 \times 3^1$.
$60 = 2^2 \times 3^1 \times 5^1$. *
$2^2 \times 3^1$, or 12

302-12 $36 = 2^2 \times 3^2$.
$90 = 2^1 \times 3^2 \times 5^1$.
$2^1 \times 3^2$, or 18

302-13 $72 = 2^3 \times 3^2$.
$108 = 2^2 \times 3^3$. *
$2^2 \times 3^2$, or 36

302-14 $25 = 5^2$.
$125 = 5^3$. *
5^2, or 25

302-15 $60 = 2^2 \times 3^1 \times 5^1$.
$84 = 2^2 \times 3^1 \times 7^1$. *
$2^2 \times 3^1$, or 12

302-16 $165 = 3^1 \times 5^1 \times 11^1$.
$234 = 2^1 \times 3^2 \times 13^1$. *
3^1, or 3

302-17 $252 = 2^2 \times 3^2 \times 7^1$.
$672 = 2^5 \times 3^1 \times 7^1$. *
$2^2 \times 3^1 \times 7^1$, or 84

302-18 $306 = 2^1 \times 3^2 \times 17^1$.
$1173 = 3^1 \times 17^1 \times 23^1$. *
$3^1 \times 17^1$, or 51

302-19 $2040 = 2^3 \times 3^1 \times 5^1 \times 17^1$.
$2184 = 2^3 \times 3^1 \times 7^1 \times 13^1$.
* $2^3 \times 3^1$, or 24

302-20 5

302-21 3

302-22 3

302-23 2

302-24 7

302-25 11

302-26 2

302-27 2

302-28 14

302-29 6

302-30 44

302-31 12

302-32 9

Applying mathematics

302-1 x, the number of tons of copper produced in Ontario in the year 1960.
$x - 75{,}313 = 128{,}808$
{204121}
204,121 tons

302-2 x, the number of miles Mr. Peters can drive in 3 hours.
$104/2 \sim x/3$.
{156}
156 miles

302-3 x, the number of dimes Helen has.
y, the number of quarters Helen has.
$x + y < 5$.
{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)}
Each member of the solution set may be used to obtain an answer.
The answer obtained from (1,3) is 1 dime and 3 quarters.

302-4 x, the number of ounces 6 cans of orange juice contain.
$1/46 \sim 6/x$.
{276}. 276 ounces

303-5 x, the number of boys who are members of the Spanish Club.
$x + 9 < 15$.
{1, 2, 3, 4, 5}
From 1 through 5 boys

303-6 x, the number of stamps Mr. Curtis had before he sold 39 of them.
$x < 150 \wedge x - 39 > 62$.
{102, 103, 104, ..., 149}
From 102 through 149 stamps

303-7 x, the number of cents he spent for the candy.
y, the number of cents he had left.
$82 - x = y \wedge 2/3 \sim 12/x$.
{(18,64)}. 64 cents

303-8 x, the number of pieces of antique glassware Mrs. Roberts has in her collection.
$x + 18 > 100 \wedge$
$x - 27 < 66$.
{83, 84, 85, ..., 92}
From 83 through 92 pieces of antique glassware

303-9 x, the number of dollars interest the bank charges per $100.
$135/2250 \sim x/100$.
{6}. 6%

303-10 x, the number of yards of material Sue bought.
y, the number of yards of material Carol bought.
$x + y < 12 \wedge x = 3 + 4$.
{(7,1), (7,2), (7,3), (7,4)}. From 1 through 4 yards

303-11 x, the number of dollars deducted each month for insurance and taxes.
y, the number of dollars his monthly take-home pay is.
$550 - x = y \wedge$
$18/100 \sim x/550$.
{(99,451)}. $451

303-12 x, the number of persons who lived in New York, Illinois, and California.
y, the number of persons in the United States who lived outside these three states.
$179323000 - x = y \wedge$
$x = 16782000 + 10081000 + 15717000$.
{(42580000,136743000)}
136,743,000 persons

303-13 x, the number of questions Ellen answered on the test.
y, the number of incorrect

answers she had on the test.
$x - 72 = y \wedge 75/100 \sim 72/x$.
{(96,24)}. 24 incorrect answers

303-14 x, the number of trees Mr. Cass sold.
y, the number of trees he had left.
$10 - x = y \wedge y < x$.
{(6,4), (7,3), (8,2), (9,1)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (7,3) is Mr. Cass sold 7 trees and had 3 trees left.

303-15 x, the number of games the Danville hockey team won last year.
y, the number of games they lost last year.
$x + y = 35 \wedge 3/2 \sim x/y$.
{21/14}. Last year the Danville hockey team won 21 games and lost 14.

303-16 x, the number of sweaters Mrs. Nichols knitted last year.
y, the number of pairs of socks she knitted last year.
$x - y = 4 \wedge x + y < 16$.
{(5,1), (6,2), (7,3), (8,4), (9,5)}. Each member of the solution set may be used to obtain an answer. The answer obtained from (7,3) is Mrs. Nichols knitted 7 sweaters and 3 pairs of socks last year.

303-17 $x = 2467 + 38961 + 15890$.
{57318}

303-18 $17/425 \sim x/100$.
{4}

303-19 $x + y < 24 \wedge 2y = 32$.
{(0,16), (1,16), (2,16), (3,16), (4,16), (5,16), (6,16), (7,16)}

303-20 $x < 32(17) \wedge x > 912 - 862$.
{ }

303-21 $25/20 \sim x/y \wedge x + y < 75$.
{5/4, 10/8, 15/12, 20/16, 25/20, 30/24, 35/28, 40/32}

303-22 $x + y = 21 \wedge x = 5y$.
{ }

303-23 $x < 528 \div 48$.
{0, 1, 2, ..., 10}

303-24 $x - y = 27 \wedge x + 37 = 112$.
{(75,48)}

303-25 $x > 1001(10)$.
{10011, 10012, 10013, ...}

Checking up

Test 71

304-1 $x + y = y + x$.

304-2 $rs = sr$.

304-3 $(a + b) + c = a + (b + c)$.

304-4 $(pq)r = p(qr)$.

304-5 $m(n + p) = mn + mp$.

Test 72

304-6 24

304-7 factors

304-8 zero, or 0

304-9 successor

304-10 sum

304-11 quotient

304-12 many-to-one

304-13 product

304-14 binary

304-15 operation

304-16 factorization

Test 73

304-17 F

304-18 T

304-19 F

304-20 F

304-21 T

304-22 T

304-23 F

304-24 T

304-25 F

Test 74

304-26 (d) 25

304-27 (c) 2^3

305-28 (c) 10

305-29 (b) multiplication

305-30 (a) {7, 9, 11, ...}

305-31 (d) composite

305-32 (b) 8

305-33 (b) $x + y$

305-34 (c) 210

305-35 (d) $x/y \sim 2/3 \wedge x + y < 30$.

Test 75

305-36 { }, {red}, {black}

305-37 $5 < 32$.

305-38 19

305-39 700

305-40 28

305-41 Only one, the number 2

305-42 17. A prime number is a complete factorization of itself.

305-43 {1, 2, 3, 4, 6, 8, 12, 24}

Test 76

305-1 T

305-2 F

305-3 F

305-4 T

305-5 T

305-6 F

305-7 T

305-8 T

305-9 T

305-10 T

305-11 F

Test 77

306-12 base

306-13 numbers

306-14 trillions

306-15 power
306-16 expanded form
306-17 1, 2, 4, 8, 16, 32
306-18 2^6
306-19 $5(10^4) + 6(10^3) + 0(10^2) + 7(10^1) + 3(1)$
306-20 exponent
306-21 3×10^4
306-22 10
306-23 base
306-24 factors
306-25 126,000,000,058,369
306-26 216
306-27 44,000,000,000,000,000
306-28 2

Test 78

306-29 Y
306-30 X
306-31 Z
306-32 X
306-33 Z
306-34 Y
306-35 X
306-36 Y
306-37 Y
306-38 X

Test 79

307-39 x, the number of boys in Bell School who do not take part in after-school sports.
$230 - 161 = x$.
{69}. 69 boys

307-40 x, the number that was Frank's bowling score.
$136 - x < 12 \wedge x < 128$.
{125, 126, 127}. His score could have been 125, 126, or 127.

307-41 x, the number of dollars Miss Lee saved last year.
y, the number of dollars she spent.
$x + y = 4500 \wedge$
$3/100 \sim x/4500$.
{(135,4365)}. $4365

307-42 x, the number of dollars Lucy spent for clothes.
$125/100 \sim x/24$.
{30}. $30

307-43 x, the number of dollars discount Mr. Jackson received.
y, the number of dollars he paid for the drums.
$60 - x = y \wedge$
$15/100 \sim x/60$.
{(9,51)}. $51

307-44 $x + y = 12 \wedge x = 3y$.
{(9,3)}

307-45 $352/550 \sim x/100$.
{64}

307-46 $16/100 \sim 80/x$.
{500}

307-47 $24/100 \sim x/950$.
{228}

307-48 $1/100 \sim 10/x$.
{1000}

307-49 $5/10 \sim x/y \wedge xy < 35$.
{1/2, 2/4, 3/6, 4/8}

307-50 $3/1 \sim x/y \wedge x + y < 20$.
{3/1, 6/2, 9/3, 12/4}

Lesson 71

308-A Two

308-B 6 * 5 * The number of parts into which the whole piece is marked off * The number of these parts that you are to think about

309-C (1,3)

309-D 5 * The piece is marked off into 5 parts and you are to think about all 5 of these parts.

309-E 5 * 5 * 8

309-F The number of these parts that you are to think about

309-G 1. The whole piece of paper is the 1 part. * Yes

309-H 3 * The second component, 1, refers to the number of parts in a whole piece of paper. The first component, 3, refers to the number of parts that you are to think about.

309-I No. The part is not gray. * Yes * The second component, 1, refers to the number of parts in the whole piece of paper. The first component, 0, refers to the number of parts that you are to think about.

309-J The piece of paper is marked off into 8 parts, and you are to think about 0 of these parts.

309-K The numerator of (2,4) is 2, of (5,6) is 5, of (1,3) is 1, of (5,5) is 5, of (8,5) is 8, of (1,1) is 1, of (3,1) is 3, of (0,1) is 0, and of (0,8) is 0. * The denominator of (2,4) is 4, of (5,6) is 6, of (1,3) is 3, of (5,5) is 5, of (8,5) is 5, of (1,1) is 1, of (3,1) is 1, of (0,1) is 1, and of (0,8) is 8.

310-L $\frac{2}{4}$, $\frac{5}{6}$, $\frac{1}{3}$, $\frac{5}{5}$, $\frac{8}{5}$, $\frac{1}{1}$, $\frac{3}{1}$, $\frac{0}{1}$, $\frac{0}{8}$

310-M One half

310-N Two thirds

310-O Seven fifths

310-P Nine fifteenths

310-Q Three fourths

310-R Eight sixths

310-A 12 * 12

310-B Yes * Yes

310-C 24 * 24 * Yes * Yes

310-D Yes

311-E ---

311-F $\frac{1}{3}$ is equivalent to $\frac{2}{6}$ because $1 \times 6 = 2 \times 3$.

311-G $\frac{1}{3}$ is equivalent to $\frac{4}{12}$ because $1 \times 12 = 4 \times 3$.

311-H $\frac{1}{3}$ is equivalent to $\frac{10}{30}$ because $1 \times 30 = 10 \times 3$.

311-I $\frac{1}{3}$ is equivalent to $\frac{15}{45}$ because $1 \times 45 = 15 \times 3$.

311-J $\frac{1}{3}$ is equivalent to $\frac{30}{90}$ because $1 \times 90 = 30 \times 3$.

311-K $\frac{1}{3}$ is equivalent to $\frac{5}{15}$ because $1 \times 15 = 5 \times 3$.

311-L ---

311-M True statement

311-N True statement

311-O False statement

311-P False statement

311-Q True statement

311-R False statement

311-S False statement

311-T False statement

311-U True statement

311-V Yes * For each replacement of b and d, $0 \times d = 0 \times b$. Since $0 \times d = 0$ and $0 \times b = 0$, each product is zero no matter what replacements are made for d and b.

On your own

312-1 Five eighths

312-2 Zero halves

312-3 Three thirds

312-4 Seventeen elevenths

312-5 Four fourths

312-6 Fifty-one thirtieths

312-7 Fourteen twenty-eighths

312-8 Zero one-hundredths

312-9 $\frac{2}{8}$ is equivalent to $\frac{1}{4}$.

312-10 $\frac{5}{1}$ is not equivalent to $\frac{1}{4}$ because 5×4 is not equal to 1×1.

312-11 $\frac{10}{40}$ is equivalent to $\frac{1}{4}$.

312-12 $\frac{4}{16}$ is equivalent to $\frac{1}{4}$.

312-13 $\frac{25}{100}$ is equivalent to $\frac{1}{4}$.

312-14 $\frac{20}{100}$ is not equivalent to $\frac{1}{4}$ because 20×4 is not equal to 1×100.

312-15 False statement

312-16 True statement

312-17 True statement

312-18 False statement

312-19 False statement

312-20 True statement

312-21 False statement

312-22 True statement

312-23 True statement

312-24 True statement

312-25 False statement

312-26 True statement

Keeping Skilful

312-1 {(14,0), (13,1), (12,2), (11,3), (10,4), (9,5), (8,6)}

312-2 {(4006,3297)}

312-3 {(0,8), (1,9), (2,10), (3,11), (4,12)}

312-4 {(15,0), (14,1), (13,2), (12,3)}

312-5 {(27,0), (27,1), (27,2), (27,3), (27,4)}

312-6 {(18,0), (19,1), (20,2), (21,3), (22,4), (23,5)}

312-7 {(3,9)}

312-8 { }

312-9 135

312-10 481

312-11 503

312-12 391

312-13 45

312-14 59

312-15 390

312-16 153

312-17 2, 3, 7

312-18 5, 13

312-19 37

312-20 2, 3, 5, 7

312-21 2, 3, 113

312-22 2, 3, 19

312-23 2, 3, 89

312-24 3, 37

Lesson 72

312-A 3 * 3 * Multiply both the numerator and denominator of $\frac{2}{3}$ by 3.

312-B Multiply both the numerator and denominator of $\frac{2}{3}$ by 5.

312-C Yes. By using the definition of equivalent fractions, you obtain $6 \times 3 = 2 \times 9$ from $\frac{6}{9} \sim \frac{2}{3}$. * Yes. By using the definition of equivalent fractions, you obtain $10 \times 3 = 2 \times 15$ from $\frac{10}{15} \sim \frac{2}{3}$.

312-D $\frac{12}{18}$ * $\frac{14}{21}$ * $\frac{20}{30}$ * $\frac{200}{300}$ * Yes * By using the definition of equivalent fractions, you can decide that each fraction you obtain is equivalent to $\frac{2}{3}$. For example, $\frac{12}{18}$ is equivalent to $\frac{2}{3}$ because $12 \times 3 = 2 \times 18$.

312-E If you multiply both the numerator and denominator of $\frac{2}{3}$ by the same member of C, you will obtain a fraction that is equivalent to $\frac{2}{3}$.

313-F 6 * 6 * Divide both the numerator and denominator of $\frac{6}{12}$ by 6.

313-G Yes * Yes * Divide both the numerator and denominator of $\frac{6}{12}$ by 2.

313-H Yes. By using the definition of equivalent fractions, you obtain $1 \times 12 = 6 \times 2$ from $\frac{1}{2} \sim \frac{6}{12}$. Since you know that $1 \times 12 = 6 \times 2$ is a true statement, you also know that $\frac{1}{2} \sim \frac{6}{12}$ is a true statement. *

Yes. By using the definition of equivalent fractions, you obtain the true statement $3 \times 12 = 6 \times 6$ from $\frac{3}{6} \sim \frac{6}{12}$.

313-I Yes

313-J Yes

313-A 4 * Yes. By the associative property of multiplication of natural numbers, you know that $5(4 \cdot 7) = (5 \cdot 4)7$. By the commutative property of multiplication of natural numbers, you know that $(5 \cdot 4)7 = (4 \cdot 5)7$. * Yes

313-B kb * $\frac{ka}{kb}$

313-C By using the definition of equivalent fractions * For each replacement of the variables, $a(kb) = (ka)b$.

313-D Associative property of multiplication of natural numbers

314-E Commutative property of multiplication of natural numbers

314-F After replacement of the variables, sentence A tells you that a first number, $a(kb)$, is equal to a second number, $(ak)b$. Sentence B tells you that the second number is equal to a third number, $(ka)b$. Therefore, the first number must be equal to the third number. This is the condition expressed by sentence C. * Yes

314-G Yes * Yes

314-H Yes. 8 is the same as $2 \cdot 4$, and 6 is the same as $2 \cdot 3$. * Yes

314-I $\frac{4}{3}$ * Yes * Yes

314-J Yes * Yes

314-K $\frac{3}{5}$ * Yes * Yes

314-L k * Yes

314-M $\frac{a}{b}$ * Yes. Any fraction whose numerator and denominator have a common factor can be expressed as $\frac{ka}{kb}$ where k is the common factor. You know that if you divide both the numerator and denominator of $\frac{ka}{kb}$ by this common factor you will obtain $\frac{a}{b}$. You also know that $\frac{a}{b}$ is equivalent to $\frac{ka}{kb}$.

314-N True statement

314-O True statement

314-P False statement

314-Q True statement

314-R False statement

314-S False statement

314-T True statement

314-U True statement

314-V False statement

314-W True statement

314-X False statement

314-Y True statement

315-A Examples are $\frac{2}{10}$, $\frac{3}{15}$, and $\frac{10}{50}$. * No * Yes

315-B No * Yes

315-C Yes * Yes

315-D For set D, examples are $\frac{32}{36}$ and $\frac{40}{45}$. For set E, examples are $\frac{30}{25}$ and $\frac{42}{35}$. For set F, examples are $\frac{16}{24}$ and $\frac{20}{30}$. For set G, examples are $\frac{33}{44}$ and $\frac{300}{400}$.

315-E 1, 2, 4, 8 * 1, 3, 9 * 1 is the only common factor of 8 and 9.

315-F 1, 2, 3, 6 * 1, 5 * 1

315-G 1, 2 * 1, 3 * 1

315-H 1

315-I No * No

315-J 5 * Yes * No

315-K No

315-L 1, 2, 4 * 1, 3, 5, 15 * The numerator and denominator of $\frac{4}{15}$ are relatively prime.

315-M Examples are $\frac{8}{30}$, $\frac{12}{45}$, and $\frac{40}{150}$.

315-N 1, 2, 3, 4, 6, 12 * 1, 3 * The numerator and denominator of $\frac{12}{3}$ are not relatively prime. Both 1 and 3 are common factors of 12 and 3.

315-O $\frac{4}{1}$. Since 3 is the only common factor of 12 and 3 that is greater than 1, you can obtain the basic fraction that is equivalent to $\frac{12}{3}$ by dividing both the numerator and denominator of $\frac{12}{3}$ by 3.

316-A For picture A, $\frac{1}{4}$. For picture B, $\frac{2}{8}$. For picture C, $\frac{3}{12}$. For picture D, $\frac{4}{16}$ * Yes

316-B Yes * Yes * Yes

316-C Yes * Yes * Examples are 28 and 40.

316-D Yes * 24

316-E Yes * Yes

316-F 6 * 8 * Yes

317-G The set is a rational number because it is a set of equivalent fractions that contains the basic fraction $\frac{3}{5}$ and all fractions equivalent to $\frac{3}{5}$.

317-H The set is not a rational number because it is not

a set of equivalent fractions. $\frac{2}{7}$ is not equivalent to either $\frac{6}{14}$ or $\frac{9}{21}$.

317-I The set is not a rational number because it is not a set of equivalent fractions. No two of the fractions $\frac{1}{2}$, $\frac{2}{2}$, or $\frac{3}{2}$ are equivalent.

317-J The set is a rational number because it is a set of equivalent fractions that contains the basic fraction $\frac{2}{3}$ and all fractions equivalent to $\frac{2}{3}$.

317-K No * Two different basic fractions cannot be equivalent to each other.

317-L Yes. By replacing n with each member of C in turn, you can obtain all the fractions equivalent to the basic fraction $\frac{2}{3}$ from the fraction $\frac{2n}{3n}$. * Yes. From the reduction property of fractions, you know that for each replacement of n, the fraction $\frac{2n}{3n}$ is equivalent to the fraction $\frac{2}{3}$. Therefore, the fraction $\frac{2n}{3n}$ indicates the same rational number as the fraction $\frac{2}{3}$, and the rational number $\frac{2n}{3n}$ is equal to the rational number $\frac{2}{3}$.

317-M True statement

317-N False statement

317-O True statement

317-P True statement

317-Q False statement

317-R False statement

On your own

317-1 Divide both the numerator and denominator of $\frac{30}{50}$ by 2.

317-2 Divide both the numerator and denominator of $\frac{30}{50}$ by 5.

317-3 Multiply both the numerator and denominator of $\frac{30}{50}$ by 2.

317-4 Divide both the numerator and denominator of $\frac{30}{50}$ by 10.

317-5 Multiply both the numerator and denominator of $\frac{30}{50}$ by 3.

317-6 Divide both the numerator and denominator of $\frac{70}{35}$ by 35.

317-7 Multiply both the numerator and denominator of $\frac{70}{35}$ by 2.

317-8 Divide both the numerator and denominator of $\frac{70}{35}$ by 7.

317-9 Divide both the numerator and denominator of $\frac{70}{35}$ by 5.

317-10 Multiply both the numerator and denominator of $\frac{70}{35}$ by 3.

317-11 $\frac{1}{6}$

317-12 $\frac{3}{4}$

317-13 $\frac{9}{2}$

317-14 $\frac{7}{8}$

317-15 $\frac{12}{5}$

317-16 $\frac{10}{1}$

317-17 $\frac{17}{10}$

317-18 $\frac{4}{1}$

317-19 True * $\frac{5}{15} = \frac{5 \cdot 1}{5 \cdot 3}$. From the reduction property, you know that $\frac{1}{3} = \frac{5 \cdot 1}{5 \cdot 3}$.

317-20 True * $\frac{10}{4} = \frac{2 \cdot 5}{2 \cdot 2}$. From the reduction property, you know that $\frac{5}{2} = \frac{2 \cdot 5}{2 \cdot 2}$.

317-21 False * You cannot replace k, a, and b so that you will obtain $\frac{3}{4} = \frac{4}{3}$ from $\frac{a}{b} = \frac{ka}{kb}$.

317-22 True * $\frac{70}{80} = \frac{10 \cdot 7}{10 \cdot 8}$. From the reduction property, you know that $\frac{70}{80} = \frac{10 \cdot 7}{10 \cdot 8}$.

317-23 False * You cannot replace k, a, and b so that you will obtain $\frac{5}{3} = \frac{3}{5}$ from $\frac{a}{b} = \frac{ka}{kb}$.

317-24 False * You cannot replace k, a, and b so that you will obtain $\frac{6}{5} = \frac{36}{25}$ from $\frac{a}{b} = \frac{ka}{kb}$.

Keeping Skilful

317-1 1805

317-2 900

317-3 2124

317-4 7246

317-5 7025

317-6 91082

317-7 34, remainder 0

317-8 70, remainder 36

317-9 207, remainder 0

317-10 119, remainder 6

317-11 92, remainder 8

317-12 362, remainder 0

317-13 128, remainder 22

317-14 140, remainder 0

317-15 60 * 611

317-16 59 * 780

317-17 138 * 1512

317-18 380 * 23,556

317-19 471 * 8588

317-20 155 * 5194

317-21 329 * 24,408

317-22 798 * 150,920

317-23 482 * 55,056

Lesson 73

318-A Yes. Because of the reduction property of rational numbers, you know that the rational number $\frac{2}{2}$ is equal to the rational number $\frac{1}{1}$. * Yes. The fraction numeral $\frac{1}{1}$ names the rational number $\frac{2}{2}$.

318-B Yes. The fraction numeral $\frac{1}{2}$ names the rational number $\frac{2}{4}$. * Yes. The fraction numeral $\frac{3}{1}$ names the rational number $\frac{9}{3}$.

318-C No * No

318-D Examples are $\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{4}$, $\frac{5}{6}$, $\frac{7}{1}$, $\frac{20}{21}$.

318-E Yes. You know that there are infinitely many prime numbers. Therefore, there are infinitely many pairs of these primes that are relatively prime and are basic fractions. There are also the infinitely many basic fractions, each of whose components is not prime. * Yes. Since a rational number contains one, and only one, basic fraction, each basic fraction indicates a different rational number.

318-F Yes * Since each basic fraction indicates a different rational number and there are infinitely many basic fractions, the set of all possible rational numbers of arithmetic is an infinite set.

318-G $\frac{0}{1}$ * $\frac{1}{1}$

318-H $\frac{2}{1}$

318-I $\frac{3}{1}$

319-J Locate point $\frac{4}{1}$ in the line and to the right of point $\frac{3}{1}$ so that the segment determined by points $\frac{3}{1}$ and $\frac{4}{1}$ is congruent to the segment determined by points $\frac{0}{1}$ and $\frac{1}{1}$. Locate point $\frac{5}{1}$ in the line and to the right of point $\frac{4}{1}$ so that the segment determined by points $\frac{4}{1}$ and $\frac{5}{1}$ is congruent to the segment determined by points $\frac{0}{1}$ and $\frac{1}{1}$.

319-K $\frac{2}{1}$

319-L $\frac{1}{4}$ * $\frac{2}{4}$

319-M Since point S is the right endpoint of the third of the four segments, you should assign $\frac{3}{4}$ to point S.

319-N Because of the reduction property of rational numbers, you know that $\frac{4}{4}$ is the same rational number as $\frac{1}{1}$.

319-O To the right endpoint of the second of the three segments

319-P Segment PT was separated into three congruent segments. Then point U was located in the line and to the right of point T so that the segment determined by points T and U was congruent to each of these three segments.

319-Q Segment PT was separated into two congruent segments, segments PR and RT. Then point $\frac{3}{2}$ was located in the line and to the right of point $\frac{1}{1}$ so that the segment determined by points $\frac{1}{1}$ and $\frac{3}{2}$ was congruent to each of segments PR and RT. * Segment PT was separated into 5 congruent segments. Then more points were located in the line and to the right of point $\frac{1}{1}$ so that the segments determined were congruent to each of these five segments. The right endpoint of the thirteenth segment from point $\frac{0}{1}$ is point $\frac{13}{5}$.

319-R First separate the segment determined by points $\frac{0}{1}$ and $\frac{1}{1}$ into b congruent segments. Then locate points in the line and to the right of point $\frac{1}{1}$ so that they determine segments congruent to each of the b segments. You assign the rational number $\frac{a}{b}$ to the right endpoint of the ath of b segments.

319-S $\{\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\}$ * No * There are infinitely many rational numbers associated with points between point $\frac{0}{1}$ and point $\frac{1}{1}$.

320-T Yes * Yes

320-U Examples are the fraction numerals $\frac{6}{9}$, $\frac{8}{12}$, and $\frac{10}{15}$. * Examples are the fraction numerals $\frac{9}{15}$, $\frac{12}{20}$, and $\frac{15}{25}$.

320-V First separate the segment determined by points $\frac{0}{1}$ and $\frac{1}{1}$ into ten congruent segments. Then locate point $\frac{11}{10}$ in the line and to the right of point $\frac{1}{1}$ so that the segment determined by points $\frac{1}{1}$ and $\frac{11}{10}$ is congruent to each of the ten segments. * Examples are the fraction numerals $\frac{22}{20}$, $\frac{33}{30}$, and $\frac{44}{40}$.

320-W Yes. There are infinitely many rational numbers. Each of these

rational numbers is associated with a point in the rational-number line. Therefore, there are infinitely many points in the rational-number line. * Yes. A rational number is an infinite set of equivalent fractions. The name of each fraction in a rational number may be used to name the point associated with that rational number.

320-A Yes

320-B Yes * Yes. Since point $\frac{1}{2}$ is to the left of point $\frac{4}{3}$, the rational number $\frac{1}{2}$ is less than the rational number $\frac{4}{3}$.

320-C Yes * Yes * No * No

320-D No * No

320-E $\frac{3}{4}$

320-F 20

320-G The numerator of $\frac{3}{4}$ and the denominator of $\frac{5}{8}$ are to be multiplied. * 24

320-H Yes

321-I $\frac{7}{6}$ and $\frac{5}{5}$ * 7 and 5. 5 and 6 * No

321-J *a*, *b*, *c*, and *d* * Yes. 0 is a member of N which is the universe for *a* and *c*. * No. 0 is not a member of C which is the universe for *b* and *d*.

321-K *a* was replaced by 2, *b* by 3, *c* by 4, and *d* by 5. * Yes

321-L Yes

321-M False statement * $4 \times 4 < 5 \times 3$ is a false statement.

321-N True statement * $5 \times 8 < 14 \times 3$ is a true statement.

321-O False statement * $12 \times 1 < 6 \times 2$ is a false statement.

321-P False statement * $116 \times 40 < 77 \times 59$ is a false statement.

321-Q True statement * $3 \times 5 < 7 \times 8$ is a true statement.

321-R True statement * $3 \times 4 < 3 \times 5$ is a true statement.

321-S True statement * $7 \times 20 < 9 \times 26$ is a true statement.

321-T True statement * $15 \times 41 < 19 \times 60$ is a true statement.

321-U The replacement for *a* must be less than the replacement for *c*.

321-A Yes

322-B $\frac{9}{2} > \frac{4}{3}$. * $\frac{3}{6} > \frac{2}{7}$. * $\frac{40}{3} > \frac{17}{5}$.

322-C Point $\frac{a}{b}$ is to the right of point $\frac{c}{d}$.

322-D The number obtained from *ad* is greater than the number obtained from *cb*.

On your own

322-1 Examples are $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{12}{24}$.

322-2 Examples are $\frac{1400}{2}$, $\frac{2100}{3}$, and $\frac{7000}{10}$.

322-3 Examples are $\frac{6}{14}$, $\frac{12}{28}$, and $\frac{30}{70}$.

322-4 Examples are $\frac{2}{1}$, $\frac{4}{2}$, and $\frac{40}{20}$.

322-5 Examples are $\frac{0}{1}$, $\frac{0}{5}$, and $\frac{0}{12000}$.

322-6 Examples are $\frac{1}{1}$, $\frac{5}{5}$, and $\frac{100}{100}$.

322-7 Examples are $\frac{1}{3}$, $\frac{3}{9}$, and $\frac{32}{96}$.

322-8 Examples are $\frac{2}{10}$, $\frac{10}{50}$, and $\frac{15}{75}$.

322-9 $\frac{2}{1}$

322-10 $\frac{3}{4}$

322-11 $\frac{3}{2}$

322-12 $\frac{5}{3}$

322-13 Examples are $\frac{8}{6}$, $\frac{12}{9}$, and $\frac{16}{12}$.

322-14 Examples are $\frac{2}{8}$, $\frac{3}{12}$, and $\frac{10}{40}$.

322-15 $\frac{0}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{6}{3}$, $\frac{8}{3}$

322-16 $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$, $\frac{12}{12}$

322-17 $\frac{0}{10}$, $\frac{4}{10}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, $\frac{12}{10}$

322-18 $\frac{74}{100}$, $\frac{82}{100}$, $\frac{99}{100}$, $\frac{106}{100}$, $\frac{110}{100}$

322-19 True statement

322-20 False statement

322-21 True statement

322-22 True statement

322-23 False statement

322-24 True statement

322-25 True statement

322-26 False statement

322-27 True statement

Keeping Skilful

322-1 $12 = 2^2 \times 3^1$.
$40 = 2^3 \times 5^1$. * 2^2, or 4

322-2 $24 = 2^3 \times 3^1$.
$84 = 2^2 \times 3^1 \times 7^1$. * $2^2 \times 3^1$, or 12

322-3 $45 = 3^2 \times 5^1$.
$72 = 2^3 \times 3^2$. * 3^2, or 9

322-4 $85 = 5^1 \times 7^1$.
$145 = 5^1 \times 29^1$. * 5^1, or 5

322-5 $70 = 2^1 \times 5^1 \times 7^1$.
$102 = 2^1 \times 3^1 \times 17^1$. * 2^1, or 2

322-6 $132 = 2^2 \times 3^1 \times 11^1$.
$150 = 2^1 \times 3^1 \times 5^2$. * $2^1 \times 3^1$, or 6

322-7 {(140,7)}

322-8 {7/9, 14/18, 21/27}

322-9 {(90,14)}

322-10 {9/15}

322-11 {(56,100)}

Checking up

Test 80

323-1 Picture A

323-2 Picture D

323-3 Picture C

323-4 Picture B

323-5 Picture C

323-6 Picture D

323-7 Picture A

323-8 Picture B

323-9 Picture B

323-10 Picture D

323-11 3

323-12 356

323-13 False statement

323-14 True statement

323-15 True statement

323-16 False statement

323-17 True statement

323-18 False statement

323-19 True statement

323-20 False statement

323-21 False statement

323-22 Divide both the numerator and denominator of $\frac{36}{72}$ by 12.

323-23 Multiply both the numerator and denominator of $\frac{36}{72}$ by 2.

323-24 Divide both the numerator and denominator of $\frac{36}{72}$ by 9.

323-25 Multiply both the numerator and denominator of $\frac{36}{72}$ by 3.

323-26 Divide both the numerator and denominator of $\frac{36}{72}$ by 3.

323-27 Divide both the numerator and denominator of $\frac{36}{72}$ by 36.

323-28 Multiply both the numerator and denominator of $\frac{36}{72}$ by 4.

323-29 Divide both the numerator and denominator of $\frac{36}{72}$ by 18.

323-30 Divide both the numerator and denominator of $\frac{36}{72}$ by 4.

323-31 Multiply both the numerator and denominator of $\frac{36}{72}$ by 5.

Test 81

323-32 Yes * The numerator and denominator of $\frac{14}{5}$ are relatively prime.

323-33 $\frac{5}{6}$

323-34 $\frac{4}{7}$

323-35 $\frac{11}{20}$

323-36 $\frac{3}{2}$

323-37 $\frac{3}{4}$

323-38 True statement

323-39 False statement

323-40 False statement

323-41 False statement

Lesson 74

324-A The first component of $(\frac{2}{3}, \frac{3}{4})$ is $\frac{2}{3}$ and the second component is $\frac{3}{4}$. The first component of $(\frac{8}{10}, \frac{2}{1})$ is $\frac{8}{10}$ and the second component is $\frac{2}{1}$. The first component of $(\frac{4}{5}, \frac{5}{4})$ is $\frac{4}{5}$ and the second component is $\frac{5}{4}$. The first component of $(\frac{3\frac{2}{3}}{}, \frac{0}{8})$ is $3\frac{2}{3}$ and the second component is $\frac{0}{8}$.

324-B $(\frac{8}{10}, \frac{2}{1})$ is mapped onto the product $\frac{8}{10} \times \frac{2}{1}$. $(\frac{4}{5}, \frac{5}{4})$ is mapped onto the product $\frac{4}{5} \times \frac{5}{4}$. $(3\frac{2}{3}, \frac{0}{8})$ is mapped onto the product $3\frac{2}{3} \times \frac{0}{8}$.

324-C Yes * Yes

324-D The rational numbers $\frac{8}{10}$ and $\frac{2}{1}$ * Yes

325-E Yes * Yes * The product of two rational numbers is a rational number indicated by a fraction. The numerator of this fraction is the product of the two numerators, and the denominator is the product of the two denominators.

325-F Yes * Yes

325-G *a* was replaced by 5, *b* by 8, *c* by 1, and *d* by 5. * Yes * You know from the definition of the product of two rational numbers that, for each replacement of *a*, *b*, *c*, and *d*, a true statement is obtained from $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

325-H For the statement expressed by sentence G, *a* was replaced by 7, *b* by 6, *c* by 2, and *d* by 3. * For the statement expressed by sentence H, *a* was replaced by 75, *b* by 1, *c* by 1, and *d* by 4. * For the statement expressed by sentence I, *a* was replaced by 3, *b* by 4, *c* by 4, and *d* by 3. * Yes

325-I True statement

325-J False statement

325-K True statement

325-L False statement

325-M False statement

325-N True statement

325-A Yes

325-B Yes. You know that the product of two natural numbers is always a natural number. * Yes. Since neither *b* nor *d* is replaced by 0, you know that the product of *b* and *d* is not 0. You also know that the product of *b* and *d* is a natural number because counting numbers are natural numbers, and the product of two natural numbers is always a natural number. * Yes.

Since ac is a natural number and bd is a natural number, $\frac{ac}{bd}$ is a rational number.

325-C Yes. Every rational number can be indicated by a fraction. Therefore, you can think of $\frac{a}{b}$ as indicating x and $\frac{c}{d}$ as indicating y. You already know that the product of $\frac{a}{b}$ and $\frac{c}{d}$ is a rational number.

326-D Multiplication of rational numbers is a binary operation because it involves two rational numbers. That is, multiplication of rational numbers is a mapping of ordered pairs of rational numbers onto rational numbers.

326-E Yes

326-F $\frac{3}{6}$ and $\frac{2}{3}$

326-G Yes * Yes

326-H $\frac{6}{9}$ * Yes

326-I $\frac{6}{16}$

326-J Yes

326-K An example is $\frac{8}{10} \times \frac{6}{27} = \frac{48}{270}$. $\frac{48}{270}$ is the same number as $\frac{8}{45}$.

326-L An example is $\frac{1}{4} \times \frac{10}{4} = \frac{10}{16}$. $\frac{10}{16}$ is the same number as $\frac{15}{24}$.

326-M An example is $\frac{10}{20} \times \frac{24}{42} = \frac{240}{840}$. $\frac{240}{840}$ is the same number as $\frac{4}{14}$.

326-N An example is $\frac{2}{6} \times \frac{14}{12} = \frac{28}{72}$. $\frac{28}{72}$ is the same number as $\frac{7}{18}$.

326-O An example is $\frac{20}{22} \times \frac{20}{50} = \frac{400}{1100}$. $\frac{400}{1100}$ is the same number as $\frac{20}{55}$.

326-P An example is $\frac{15}{25} \times \frac{10}{40} = \frac{150}{1000}$. $\frac{150}{1000}$ is the same number as $\frac{3}{20}$.

326-Q An example is $\frac{4}{20} \times \frac{30}{70} = \frac{120}{1400}$. $\frac{120}{1400}$ is the same number as $\frac{3}{35}$.

326-R An example is $\frac{18}{16} \times \frac{22}{6} = \frac{396}{96}$. $\frac{396}{96}$ is the same number as $\frac{99}{24}$.

326-S An example is $\frac{5}{2} \times \frac{3}{3} = \frac{15}{6}$. $\frac{15}{6}$ is the same number as $\frac{60}{24}$.

326-T Yes. Think of $\frac{a}{b}$ and $\frac{c}{d}$ as any two rational numbers. If the universe for k and m is C, then $\frac{ka}{kb}$ represents each rational number that is equal to $\frac{a}{b}$, and $\frac{mc}{md}$ represents each rational number that is equal to $\frac{c}{d}$. You know that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ by the definition of the product of two rational numbers. By the same property, you know that $\frac{ka}{kb} \times \frac{mc}{md} = \frac{(ka)(mc)}{(kb)(md)}$. Using the associative, and commutative properties of natural numbers along with the reduction property of rational numbers, you can show that $\frac{(ka)(mc)}{(kb)(md)} = \frac{ac}{bd}$.

326-U Yes * Yes * More than one ordered pair of rational numbers can be mapped onto the same rational number by multiplication.

On your own

326-1 True statement

326-2 False statement

326-3 False statement

326-4 True statement

326-5 True statement

326-6 False statement

326-7 True statement

326-8 True statement

327-9 $\frac{7}{16}$

327-10 $\frac{8}{15}$

327-11 $\frac{0}{60}$

327-12 $\frac{36}{14}$

327-13 $\frac{300}{2}$

327-14 $\frac{180}{128}$

327-15 $\frac{27}{27}$

327-16 $\frac{24}{24}$

327-17 $\frac{37}{24}$

327-18 $\frac{25}{4}$

327-19 $\frac{0}{9}$

327-20 $\frac{21}{4}$

327-21 $\frac{200}{90}$

327-22 $\frac{27}{27}$

327-23 $\frac{450}{20}$

327-24 Examples other than those given in the text are:
$\frac{30}{40} \times \frac{10}{80} = \frac{300}{3200}$.
$\frac{9}{12} \times \frac{3}{24} = \frac{27}{288}$.
$\frac{15}{20} \times \frac{5}{40} = \frac{75}{800}$.

327-25 Examples are:
$\frac{0}{1} \cdot \frac{2}{4} = \frac{0}{4}$.
$\frac{0}{9} \cdot \frac{1}{2} = \frac{0}{18}$.
$\frac{0}{3} \cdot \frac{3}{6} = \frac{0}{18}$

327-26 Examples are:
$\frac{1}{2}(\frac{9}{12}) = \frac{9}{24}$.
$\frac{3}{6}(\frac{3}{4}) = \frac{9}{24}$.
$\frac{2}{4}(\frac{6}{8}) = \frac{12}{32}$.

327-27 Examples are:
$\frac{2}{4}(\frac{2}{1}) = \frac{4}{4}$.
$\frac{1}{2}(\frac{6}{3}) = \frac{6}{6}$.
$\frac{4}{6}(\frac{10}{5}) = \frac{40}{40}$.

327-28 Examples are:
$\frac{1}{1} \cdot \frac{1}{5} = \frac{1}{5}$.
$\frac{4}{4} \cdot \frac{2}{10} = \frac{8}{40}$.
$\frac{3}{3} \cdot \frac{4}{20} = \frac{12}{60}$.

327-29 Examples are:
$\frac{0}{2} \times \frac{6}{1} = \frac{0}{2}$.
$\frac{0}{3} \times \frac{12}{2} = \frac{0}{6}$.
$\frac{0}{5} \times \frac{18}{3} = \frac{0}{15}$.

Keeping Skilful

327-1 164

327-2 595

327-3 2193

327-4 164

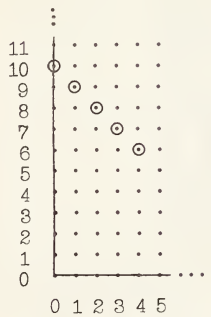
327-5 146

327-6 4515

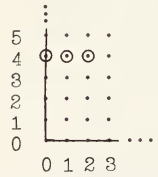
327-7 289

327-8 52

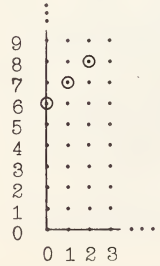
327-9 {(0,10), (1,9), (2,8), (3,7), (4,6)}



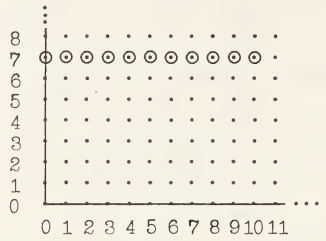
327-10 {(0,4), (1,4), (2,4)}



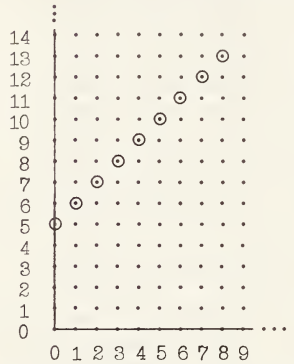
327-11 {(0,6), (1,7), (2,8)}



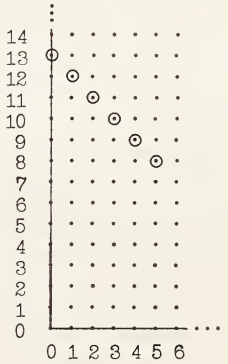
327-12 {(0,7), (1,7), (2,7), (3,7), (4,7), (5,7), (6,7), (7,7), (8,7), (9,7), (10,7)}



327-13 {(0,5), (1,6), (2,7), (3,8), (4,9), (5,10), (6,11), (7,12), (8,13)}



327-14 {(0,13), (1,12), (2,11), (3,10), (4,9), (5,8)}



Lesson 75

327-A The first component of $(\frac{1}{5},\frac{2}{5})$ is $\frac{1}{5}$ and the second component is $\frac{2}{5}$. The first component of $(\frac{2}{9},\frac{5}{9})$ is $\frac{2}{9}$ and the second component is $\frac{5}{9}$. The first component of $(\frac{5}{6},\frac{1}{6})$ is $\frac{5}{6}$ and the second component is $\frac{1}{6}$. The first component of $(\frac{2}{7},\frac{6}{7})$ is $\frac{2}{7}$ and the second component is $\frac{6}{7}$.

327-B $(\frac{2}{9},\frac{5}{9})$ is mapped onto the sum $\frac{2}{9} + \frac{5}{9}$. $(\frac{5}{6},\frac{1}{6})$ is mapped onto the sum $\frac{5}{6} + \frac{1}{6}$. $(\frac{2}{7},\frac{6}{7})$ is mapped onto the sum $\frac{2}{7} + \frac{6}{7}$.

327-C Yes * 3

327-D $\frac{2}{9}$ and $\frac{5}{9}$ * 9 * 9 * $\frac{7}{9}$

328-E Yes * 1 + 2 is the same as 3. * Yes * 2 + 5 is the same as 7.

328-F Yes

328-G Yes * Yes

328-H Yes * From the definition of the sum of two rational numbers, you know that for each replacement of a, b, and c, $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$.

328-I For the statement expressed by sentence G, a was replaced by 7, b by 8, and c by 1. * For the statement expressed by sentence H, a was replaced by 5, b by 9, and c by 2. * Yes

328-J True statement

328-K False statement

328-L True statement

328-M False statement

328-N False statement

328-O True statement

328-P False statement

328-Q True statement

328-R False statement

328-S True statement.

328-T True statement

328-U False statement

329-A (Column 1) Yes

329-B (Column 1) Since the universe for a and c is N, a and c must be replaced by natural numbers. From the closure property of addition of natural numbers, you know that the sum of two natural numbers is always a natural number. * The universe for b is the set of counting numbers.

329-C Yes

329-D Yes. If $\frac{a}{b}$ and $\frac{c}{b}$ are any two rational numbers, then their sum is $\frac{a+c}{b}$. You already know that for each replacement of a, b, and c, $\frac{a+c}{b}$ is a rational number.

329-E Since an ordered pair of rational numbers is used to obtain a sum, addition of rational numbers is a binary operation.

329-F Yes * $\frac{7}{14}$ and $\frac{21}{14}$

329-G Yes * $\frac{7}{14}$ is the same rational number as $\frac{7}{7} \times \frac{1}{2}$. From the reduction property of rational numbers, you know that $\frac{7}{7} \times \frac{1}{2}$ is the same rational number as $\frac{1}{2}$. * $\frac{21}{14}$ is the same rational number as $\frac{7}{7} \times \frac{3}{2}$. From the reduction property of rational numbers, you know that $\frac{7}{7} \times \frac{3}{2}$ is the same rational number as $\frac{3}{2}$. * Yes. $\frac{28}{14}$ is the same rational number as $\frac{7}{7} \times \frac{4}{2}$. From the reduction property of rational numbers, you know that $\frac{7}{7} \times \frac{4}{2}$ is the same rational number as $\frac{4}{2}$.

329-H $\frac{2}{3}$ and $\frac{1}{3}$ * Yes * $\frac{2}{3}$ is the same rational number as $\frac{12}{18}$. $\frac{1}{3}$ is the same rational number as $\frac{6}{18}$.

329-I Yes

329-J Yes

329-A (Column 2) No * 2 * 3

329-B (Column 2) Yes * Yes * From the reduction property of rational numbers, you know that $\frac{1}{2}$ is the same rational number as $\frac{1}{2} \times \frac{3}{3}$.

330-C Yes * Yes * From the reduction property of rational numbers, you know that $\frac{2}{3} = \frac{2}{2} \times \frac{2}{3}$.

330-D Yes * Yes * If the numbers used in forming the sum of two rational numbers are replaced by equal rational numbers, the sum is still the same rational number. This is the well-defined property of addition of rational numbers.

330-E Yes * Yes * Yes * 2 x 3, or 6

330-F The numerator of $\frac{(1 \times 3) + (2 \times 2)}{2 \times 3}$ was obtained by finding the sum of the numerators of $\frac{1}{2} \times \frac{3}{3}$ and $\frac{2}{2} \times \frac{2}{3}$. The denominator of $\frac{(1 \times 3) + (2 \times 2)}{2 \times 3}$ is the same as the denominators of $\frac{1}{2} \times \frac{3}{3}$ and $\frac{2}{2} \times \frac{2}{3}$.

330-G Because of the well-defined property of addition of rational numbers * Yes

330-H When you simplify the numerator and denominator of $\frac{(1 \times 3) + (2 \times 2)}{2 \times 3}$, you obtain $\frac{7}{6}$. * Yes

330-I Yes * Yes * Because of the reduction property of rational numbers

330-J 7 * Because of the reduction property of rational numbers

330-K Because of the well-defined property of addition of rational numbers * Because of the well-defined property of addition of rational numbers

330-L 7 x 5, or 35 * Yes

330-M The numerator of $\frac{(2 \times 5) + (7 \times 3)}{7 \times 5}$ was obtained by finding the sum of the numerators of $\frac{2}{7} \times \frac{5}{5}$ and $\frac{7}{7} \times \frac{3}{5}$. The denominator of $\frac{(2 \times 5) + (7 \times 3)}{7 \times 5}$ is the same as the denominators of $\frac{2}{7} \times \frac{5}{5}$ and $\frac{7}{7} \times \frac{3}{5}$.

330-N Yes

330-O Yes

330-P $\frac{3}{2} + \frac{4}{5}$

$\frac{3}{2} \times \frac{5}{5} + \frac{2}{2} \times \frac{4}{5} =$

$\frac{(3 \times 5) + (2 \times 4)}{2 \times 5}$.

$\frac{3}{2} + \frac{4}{5} = \frac{23}{10}$.

330-Q $\frac{7}{11} + \frac{2}{3}$

$\frac{7}{11} \times \frac{3}{3} + \frac{11}{11} \times \frac{2}{3} =$

$\frac{(7 \times 3) + (11 \times 2)}{11 \times 3}$.

$\frac{7}{11} + \frac{2}{3} = \frac{43}{33}$.

330-R $\frac{9}{5} + \frac{5}{13}$

$\frac{9 \times 13}{5 \times 13} + \frac{5 \times 5}{5 \times 13} =$

$\frac{(9 \times 13) + (5 \times 5)}{5 \times 13}.$

$\frac{9}{5} + \frac{5}{13} = \frac{142}{65}.$

330-S $\frac{4}{3} + \frac{8}{7}$

$\frac{4 \times 7}{3 \times 7} + \frac{3 \times 8}{3 \times 7} =$

$\frac{(4 \times 7) + (3 \times 8)}{3 \times 7}.$

$\frac{4}{3} + \frac{8}{7} = \frac{52}{21}.$

331-A Yes. The reduction property of rational numbers tells you that $\frac{a}{b}$ is the same as $\frac{ad}{bd}$. * Yes. The reduction property of rational numbers tells you that $\frac{c}{d}$ is the same as $\frac{bc}{bd}$.

331-B The well-defined property of addition of rational numbers tells you that if the numbers used in forming the sum of two rational numbers are replaced by equal rational numbers, the sum is still the same rational number. * Because of the well-defined property of addition of rational numbers

331-C bd

331-D The numerator of $\frac{ad + bc}{bd}$ was obtained by finding the sum of the numerators of $\frac{ad}{bd}$ and $\frac{bc}{bd}$. The denominator of $\frac{ad + bc}{bd}$ is the same as the denominators of $\frac{ad}{bd}$ and $\frac{bc}{bd}$.

331-E Yes

331-F Yes

331-G For the statement expressed by sentence U, a was replaced by 2, b by 3, c by 5, and d by 7. For the statement expressed by sentence V, a was replaced by 1, b by 8, c by 2, and d by 9. For the statement expressed by sentence W, a was replaced by 3, b by 5, c by 2, and d by 11.

331-H Yes * From the sum property of rational numbers, you know that for each replacement of a, b, c, and d, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.

331-I False statement

331-J False statement

331-K True statement

331-L True statement

331-M True statement

331-N False statement

331-O True statement

331-P False statement

331-Q True statement

331-R More than one ordered pair of rational numbers can be mapped by addition onto the same rational number.

On your own

331-1 Not equal to

331-2 Equal to

331-3 Equal to

331-4 Not equal to

331-5 Equal to

331-6 Equal to

331-7 Equal to

331-8 Not equal to

331-9 True statement

331-10 False statement

331-11 False statement

331-12 True statement

331-13 False statement

331-14 False statement

331-15 False statement

331-16 True statement

331-17 True statement

331-18 True statement

332-19 $\frac{6}{7}$

332-20 $\frac{59}{42}$

332-21 $\frac{22}{15}$

332-22 $\frac{12}{12}$

332-23 $\frac{67}{90}$

332-24 $\frac{43}{12}$

332-25 $\frac{132}{96}$

332-26 $\frac{177}{132}$

332-27 $\frac{17}{15}$

332-28 $\frac{27}{12}$

332-29 $\frac{182}{147}$

332-30 $\frac{113}{110}$

332-31 $\frac{670}{4500}$

332-32 $\frac{25}{1}$

332-33 $\frac{886}{768}$

332-34 $\frac{498}{360}$

332-35 Yes * The sum of any two rational numbers is a rational number.

332-36 Examples other than those given in the text are:

$\frac{6}{9} + \frac{1}{8} = \frac{57}{72}.$

$\frac{2}{3} + \frac{2}{16} = \frac{38}{48}.$

$\frac{6}{9} + \frac{4}{32} = \frac{228}{288}.$

332-37 Examples are:

$\frac{9}{15} + \frac{2}{7} = \frac{93}{105}.$

$\frac{6}{10} + \frac{8}{28} = \frac{248}{280}.$

$\frac{3}{5} + \frac{4}{14} = \frac{62}{70}.$

332-38 Examples are:

$\frac{15}{18} + \frac{3}{21} = \frac{369}{378}.$

$\frac{5}{6} + \frac{2}{14} = \frac{82}{84}.$

$\frac{10}{12} + \frac{1}{7} = \frac{82}{84}.$

332-39 Examples are:

$\frac{6}{9} + \frac{3}{2} = \frac{39}{18}.$

$\frac{2}{3} + \frac{12}{8} = \frac{52}{24}.$

$\frac{4}{6} + \frac{9}{6} = \frac{13}{6}.$

332-40 Examples are:

$\frac{14}{24} + \frac{2}{5} = \frac{118}{120}.$

$\frac{7}{12} + \frac{4}{10} = \frac{118}{120}.$

$\frac{35}{60} + \frac{24}{60} = \frac{59}{60}.$

332-41 Examples are:
$\frac{8}{9} + \frac{3}{6} = \frac{75}{54}$.
$\frac{16}{18} + \frac{1}{2} = \frac{50}{36}$.
$\frac{8}{9} + \frac{1}{2} = \frac{25}{18}$.

Keeping Skilful

332-1 {(247, 45)}
332-2 {3/4, 6/8, 9/12, 12/16, 15/20}
332-3 {33/36}
332-4 {(25, 26)}
332-5 {7/8, 14/16, 21/24, 28/32, 35/40}
332-6 {(9, 12)}
332-7 {3/11, 6/22}
332-8 {(26, 30)}
332-9 $\frac{21}{8}$
332-10 $\frac{0}{16}$
332-11 $\frac{7}{27}$
332-12 $\frac{51}{51}$
332-13 $\frac{36}{11}$
332-14 $\frac{12}{104}$
332-15 $\frac{0}{5}$
332-16 $\frac{35}{115}$
332-17 $\frac{816}{13}$

Lesson 76

332-A Yes * Yes * Yes
332-B $\frac{0}{7}$
332-C Yes
332-D N
332-E Yes * If the difference of a first and second number is the same as a third number, then the first number is the same as the sum of the second and third numbers.
332-F From the definition of the sum of two rational numbers, you know that for each replacement of a, b, and c, $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$. * No * No
333-G No * There is no rational number of arithmetic that can be added to $\frac{1}{9}$ to get a sum of $\frac{5}{9}$.
333-H No * No * If R_a were closed under subtraction, the difference of any two members of R_a would be a member of R_a. This is not true since you know that the difference of $\frac{5}{9}$ and $\frac{1}{9}$ is not a member of R_a.
333-I Yes * Yes * No * Yes
333-J Yes. You know that $a - c$ is a natural number if a is greater than c. Therefore, $\frac{a - c}{b}$ is a member of R_a.
333-K Yes. If a is equal to c, then $a - c$ is equal to the natural number 0. Therefore, $\frac{a - c}{b}$ is the same as $\frac{0}{b}$, which is a member of R_a.
333-L No. If a is less than c, then $a - c$ is not a natural number, and $\frac{a - c}{b}$ is not a member of R_a.
333-M The universe for a and c is N. The universe for b is C.
333-N For the statement expressed by sentence F, a was replaced by 4, b by 5, and c by 3. * For the statement expressed by sentence G, a was replaced by 6, b by 7, and c by 4. * For the statement expressed by sentence H, a was replaced by 25, b by 21, and c by 17.
333-O Yes * Because of the definition of the difference of two rational numbers. For each replacement of a, b, and c, if a is greater than or equal to c, then $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$.
333-P False statement
333-Q True statement
333-R True statement
333-S False statement
333-T False statement
333-U True statement
333-V False statement
333-W True statement
333-X False statement
334-A Yes. Because of the reduction property of rational numbers * Yes. Because of the reduction property of rational numbers * By using the definition of the difference of two rational numbers * Yes * For each replacement of a, b, and c, if a is greater than or equal to c, than $\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$.
334-B Yes. If the rational numbers that form a difference are replaced by equal rational numbers, then the difference is still the same rational number. Since $\frac{2 \times 4}{3 \times 4} = \frac{2}{3}$ and $\frac{3 \times 1}{3 \times 4} = \frac{1}{4}$, you know that the difference of $\frac{2 \times 4}{3 \times 4}$ and $\frac{3 \times 1}{3 \times 4}$ is the same as the difference of $\frac{2}{3}$ and $\frac{1}{4}$. * Yes. $(2 \times 4) - (3 \times 1)$ is the same as $8 - 3$, or 5. 3×4 is the same as 12

Therefore, $\frac{(2 \times 4) - (3 \times 1)}{(3 \times 4)}$ is the same as $\frac{5}{12}$.

334-C Yes. Because of the reduction property of rational numbers * Yes. Because of the reduction property of rational numbers

334-D Yes. $\frac{ad}{bd} - \frac{bc}{bd}$ is the same as $\frac{a}{b} - \frac{c}{d}$ because, if the rational numbers that form a difference are replaced by equal rational numbers, then the difference is still the same rational number.

334-E bd * bd * Yes * Yes

334-F The numerator of $\frac{ad - bc}{bd}$ is the difference of the numerators of $\frac{ad}{bd}$ and $\frac{bc}{bd}$. The denominator of $\frac{ad - bc}{bd}$ is the common denominator of $\frac{ad}{bd}$ and $\frac{bc}{bd}$.

334-G Yes * Yes

334-H For the statement expressed by sentence O, a was replaced by 7, b by 9, c by 2, and d by 5. * For the statement expressed by sentence P, a was replaced by 5, b by 11, c by 3, and d by 8.

334-I Yes

334-J False statement

334-K False statement

334-L True statement

334-M False statement

334-N False statement

334-O True statement

On your own

335-1 True statement

335-2 False statement

335-3 False statement

335-4 True statement

335-5 True statement

335-6 False statement

335-7 False statement

335-8 False statement

335-9 True statement

335-10 False statement

335-11 True statement

335-12 False statement

335-13 $\frac{2}{8}$

335-14 $\frac{5}{12}$

335-15 $\frac{0}{6}$

335-16 $\frac{5}{6}$

335-17 $\frac{27}{40}$

335-18 $\frac{2}{30}$

335-19 $\frac{3}{42}$

335-20 $\frac{3}{27}$

335-21 $\frac{11}{60}$

335-22 $\frac{4}{21}$

335-23 $\frac{23}{104}$

335-24 $\frac{3}{110}$

335-25 $\frac{3}{30}$

335-26 $\frac{45}{23}$

335-27 $\frac{14}{24}$

335-28 $\frac{35}{42}$

335-29 $\frac{0}{16}$

335-30 $\frac{39}{88}$

335-31 Not a member

335-32 Not a member

335-33 Is a member * $\frac{1}{6}$

335-34 Is a member * $\frac{1}{42}$

335-35 Not a member

335-36 Is a member * $\frac{0}{9}$

335-37 Not a member

335-38 Is a member * $\frac{40}{21}$

335-39 Is a member * $\frac{2}{15}$

335-40 Not a member

335-41 Is a member * $\frac{11}{45}$

335-42 Is a member * $\frac{27}{5}$

335-43 Is a member * $\frac{15}{18}$

335-44 Is a member * $\frac{25}{66}$

335-45 Not a member

335-46 Is a member * $\frac{25}{150}$

335-47 Is a member * $\frac{78}{18}$

335-48 Is a member * $\frac{4}{32}$

Applying mathematics

335-1 x, the number of dollars deducted from Mr. Howe's monthly salary for taxes and insurance.
$750 - x = 607$.
{143}. $143

335-2 x, the number of questions Jerry answered correctly.
$x < 50 \wedge x > 37$.
{38, 39, 40, ..., 49}.
From 38 through 49 questions

335-3 x, the number of minutes it took the girls to ride the 7 miles.
$2/24 \sim 7/x$.
{84}. 84 minutes

335-4 x, the number of tankers Peter saw.
y, the number of freighters he saw.
$x + y = 6$.
{(1,5), (2,4), (3,3), (4,2), (5,1)}.
Each member of the solution set may be used to obtain an answer. The answer obtained from (4,2) is 4 tankers and 2 freighters.

335-5 x, the number of persons who went to the bazaar on Friday.
$3/1 \sim x/177$.
{531}. 531 persons

335-6 x, the number of windows Bob washed.
y, the number of windows Terry washed.
$x + y < 27 \wedge 5/3 \sim x/y$.
{5/3, 10/6, 15/9}
Each member of the solu-

tion set may be used to obtain an answer. The answer obtained from 10/6 is Bob washed 10 windows and Terry washed 6.

335-7 x, the number of plain doughnuts Mrs. Gordon bought.
y, the number of chocolate doughnuts she bought.
$x - y = 8 \wedge x + y = 24$.
{(16,8)}. 16 plain doughnuts and 8 chocolate doughnuts

335-8 x, the number of dollars the bank charges for each \$100 it lends.
y, the number of dollars interest the manufacturer paid at the end of one year.
$x/100 \sim y/7200 \wedge$
$7560 - 7200 = x$.
{(5,360)}. 5%

336-9 x, the number of West High graduates who will enroll in university.
$25/100 \sim x/384$.
{96}. 96 graduates

336-10 x, the number of students who will graduate from university for each 100 West students who enrolled.
$72/96 \sim x/100$.
{75}. 75%

336-11 x, the number of kinds of animals that are either reptiles or amphibians.
$x = 4000 + 2000$.
{6000}. 6000 animals

336-12 x, the number of kinds of vertebrates other than reptiles.
$40000 - 4000 = x$.
{36000}. 36,000 kinds of vertebrates other than reptiles

336-13 x, the number of neckties sold during the three weeks.
y, the number of neckties left at the end of the third week.
$250 - x = y \wedge$
$x = 104 + 97 + 49$.
{(250,0)}. 0 neckties

336-14 x, the number of cents the buttons cost.
y, the number of cents in all that the buttons and thread cost.
z, the number of cents Marilyn should have received in change.
$15/1 \sim x/4 \wedge x + 7 = y \wedge$
$75 - y = z$.
Marilyn should have received 8¢ in change.

Checking up

Test 82

336-1 T
336-2 F
336-3 T
336-4 F
336-5 F
336-6 F
336-7 T

Test 83

336-8 T
336-9 F
336-10 T
336-11 F
336-12 T
336-13 F
336-14 F
336-15 T
336-16 $\frac{13}{9}$
336-17 $\frac{40}{48}$
336-18 $\frac{15}{55}$
336-19 $\frac{8}{63}$
336-20 $\frac{-2}{15}$
336-21 $\frac{27}{40}$
336-22 $\frac{0}{2}$
336-23 $\frac{1}{42}$
336-24 $\frac{58}{48}$

Lesson 77

337-A Yes * No * No * No * No
337-B Yes * No * No * Yes * Yes * Yes
337-C Yes * $2 \cdot 2 \cdot 3 \cdot 5$
337-D 2 and 3
337-E 3^1
337-F Yes
337-G Yes
337-H 14 * Yes. $\frac{5}{4}$ is equivalent to $\frac{70}{56}$.
338-I (Column 1) Yes. When you divide both the numerator and denominator of a fraction by their greatest common factor, you obtain a fraction whose numerator and denominator are relatively prime.
338-J 18 * $\frac{1}{3}$
338-K 9 * $\frac{3}{4}$
338-L 7 * $\frac{5}{7}$
338-M 15 * $\frac{2}{5}$
338-N 22 * $\frac{3}{4}$
338-O 24 * $\frac{3}{1}$
338-P 1. The numerator and denominator of a basic fraction are relatively prime. Two numbers that are relatively prime have no common factors other than 1.
338-Q 10 * 8
338-A Yes
338-B $\frac{7}{6}$
338-C Each of the other sentences in D8 expresses a true statement. * $\frac{11}{20}$ is equivalent to $\frac{22}{40}$. $\frac{4}{3}$ is equivalent to $\frac{56}{42}$. $\frac{3}{2}$ is equivalent to $\frac{81}{54}$. $\frac{5}{6}$ is equivalent to $\frac{75}{90}$. $\frac{7}{4}$ is

equivalent to $\frac{42}{24}$. $\frac{47}{20}$ is equivalent to $\frac{188}{80}$. $\frac{3}{2}$ is equivalent to $\frac{231}{154}$.

338-D Yes * You know that for each replacement of a, b, c, and d, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. For the statement expressed by sentence I, a was replaced by 10, b by 3, c by 9, and d by 14.

338-E Yes * Each of the numbers 10, 9, 3, and 14 is expressed as a product of primes.

338-F In the symbol for the numerator, a numeral 2 and a numeral 3 have been crossed out and the numeral 1 written above them. In the symbol for the denominator, a numeral 2 and a numeral 3 have been crossed out and the numeral 1 written below them.

338-G Yes. Since $2 \cdot 3 = 6$, dividing a number by 2 and then by 3 is the same as dividing the number by 6. * Yes. The only common prime factors of the numerator and denominator are 2 and 3. The greatest power of 2 that is a factor of both the numerator and denominator is 2^1. The greatest power of 3 that is a factor of both the numerator and denominator is 3^1. Therefore, $2^1 \times 3^1$, or 6, is the greatest common factor.

338-H There are no numerals for the common factors that have not been crossed out in both the symbol for the numerator and for the denominator.

338-I (Column 2) Yes * Yes * Yes

339-J Yes

339-K 2 * 3 * Yes

339-L Sentence M expresses a true statement because $\frac{5 \cdot 1}{1} \times \frac{3 \cdot 1}{7 \cdot 1} = \frac{5 \cdot 1 \cdot 3 \cdot 1}{1 \cdot 7 \cdot 1 \cdot 1}$, or $\frac{15}{7}$. Using the statements expressed by sentence K in D4 and sentence M in D5, you can obtain the statement expressed by sentence N in D5.

339-M Using one method, you can first find the fraction that indicates the product and then divide both the numerator and denominator by their greatest common factor. In the other method, you first divide both a numerator and a denominator by common prime factors of either numerator and either denominator. Then you find the product.

339-N $\frac{4}{9}$

339-O $\frac{1}{6}$

339-P $\frac{1}{6}$

339-Q $\frac{3}{14}$

339-R $\frac{3}{2}$

339-S $\frac{4}{3}$

339-T $\frac{1}{2}$

339-U $\frac{5}{2}$

339-V $\frac{176}{35}$

339-A Yes * Yes

339-B Yes * Yes * Yes * Yes * Yes

339-C Examples are 72, 84, 96, 108, and 120. * Yes

339-D Yes. 15 is a factor of 15. * Yes. 15 is a factor of 30. * Yes. 15 is a factor of each of the numbers 45, 60, and 75.

339-E 36, 18, 54

339-F Yes * Yes

340-G 2 and 7 * 2

340-H Yes

340-I No * Yes

340-J Yes

340-K Yes. Both 14 and 4 are factors of 28. * Yes. 28 is the least number that is a multiple of both 14 and 4.

340-L 24

340-M 12

340-N 14

340-O 18

340-P 100

340-Q 8

340-R Think of numbers 6 and 15.
$6 = 2^1 \times 3^1$.
$15 = 3^1 \times 5^1$.
You know that the product of 6 and 15 is a multiple of both 6 and 15. The product of 6 and 15 is $2^1 \times 3^2 \times 5^1$, or 90. You also know how to find the least common multiple of 6 and 15. $2^1 \times 3^1 \times 5^1$, or 30, is the least common multiple of 6 and 15. Notice that the least common multiple of 6 and 15 has 3^1 as a factor while the product of 6 and 15 has 3^2 as a factor. 3 is the greatest common factor of 6 and 15. If you divide $2^1 \times 3^2 \times 5^1$ (the product of 6 and 15)

by 3, you obtain $2^1 \times 3^1 \times 5^1$, which is the least common multiple.

340-S Yes * Yes

340-T Yes. Think of a and b as any two counting numbers. ab is a common multiple of a and b because both a and b are factors of ab. * No. Think of the numbers 4 and 8. The product $4 \cdot 8$ is a common multiple of both 4 and 8, but it is not the least common multiple. The least common multiple of 4 and 8 is 2^3, or 8.

340-U Yes * 4 is the least number that is a multiple of both 2 and 4.

341-V Yes * $\frac{5}{4}$

341-W $\frac{1}{5} + \frac{8}{9} = \frac{19}{18}$.
$\frac{7}{8} - \frac{11}{20} = \frac{13}{40}$.
$\frac{13}{30} + \frac{4}{25} = \frac{89}{150}$.
$\frac{15}{16} - \frac{1}{12} = \frac{41}{48}$.
$\frac{3}{14} - \frac{5}{28} = \frac{1}{28}$.

341-X 3 * $\frac{1}{3}$ * Yes * If the rational numbers that form a difference are replaced by equal rational numbers, then the difference is still the same rational number.

341-Y Yes * Yes * 10 * $\frac{7}{10}$ * Yes

On your own

341-1 2^3, or 8 * $\frac{5}{6}$

341-2 2^3, or 8 * $\frac{1}{1}$

341-3 $2 \cdot 3$, or 6 * $\frac{13}{6}$

341-4 $2 \cdot 7$, or 14 * $\frac{3}{4}$

341-5 $2^2 \cdot 3$, or 12 * $\frac{7}{5}$

341-6 12 * $\frac{7}{4}$

341-7 8 * $\frac{5}{9}$

341-8 32 * $\frac{7}{4}$

341-9 12 * $\frac{1}{10}$

341-10 28 * $\frac{7}{3}$

341-11 $\frac{9}{10}$

341-12 $\frac{4}{3}$

341-13 $\frac{2}{5}$

341-14 $\frac{7}{12}$

341-15 $\frac{11}{6}$

341-16 $\frac{3}{2}$

341-17 $\frac{4}{5}$

341-18 $\frac{2}{3}$

341-19 $\frac{10}{3}$

341-20 18 * $\frac{19}{18}$

341-21 30 * $\frac{23}{30}$

341-22 4 * $\frac{5}{4}$

341-23 33 * $\frac{17}{33}$

341-24 24 * $\frac{1}{24}$

341-25 28 * $\frac{43}{28}$

341-26 12 * $\frac{11}{4}$

341-27 84 * $\frac{17}{84}$

341-28 60 * $\frac{11}{30}$

341-29 $\frac{23}{15}$ * $\frac{2}{5}$

341-30 $\frac{7}{12}$ * $\frac{1}{12}$

341-31 $\frac{9}{10}$ * $\frac{1}{5}$

341-32 $\frac{11}{4}$ * $\frac{3}{2}$

341-33 $\frac{13}{18}$ * $\frac{1}{9}$

341-34 $\frac{5}{4}$ * $\frac{3}{8}$

341-35 $\frac{14}{9}$ * $\frac{16}{27}$

341-36 $\frac{104}{33}$ * $\frac{21}{11}$

341-37 $\frac{39}{35}$ * $\frac{2}{7}$

341-38 $\frac{56}{45}$ * $\frac{16}{45}$

341-39 $\frac{149}{70}$ * $\frac{1}{1}$

341-40 $\frac{61}{42}$ * $\frac{3}{14}$

Keeping Skilful

You know that a rational number is a set of equivalent fractions. You also know that any member of the set can be used to indicate the rational number. In the response to an exercise that involves rational numbers, we will often name more than one of the fractions that indicates the rational number. If you use a fraction whose name is not given, be sure that it is equivalent to the ones given in this response book. Be sure to follow any specific instructions given by your teacher. For example, he may or may not require you to give a response as a basic fraction.

341-1 $\frac{2}{12}$, or $\frac{1}{6}$

341-2 $\frac{72}{35}$

341-3 $\frac{48}{4}$, or $\frac{12}{1}$

341-4 $\frac{49}{9}$

341-5 $\frac{14}{40}$, or $\frac{7}{20}$

341-6 $\frac{0}{77}$, or $\frac{0}{1}$

341-7 $\frac{54}{6}$, or $\frac{9}{1}$

341-8 $\frac{75}{49}$

341-9 $\frac{0}{20}$, or $\frac{0}{1}$

341-10 $\frac{10}{16}$, or $\frac{5}{8}$

341-11 $\frac{17}{20}$

341-12 $\frac{45}{36}$, or $\frac{15}{12}$, or $\frac{5}{4}$

341-13 $\frac{31}{6}$

341-14 $\frac{23}{1}$

341-15 $\frac{115}{72}$

341-16 $\frac{56}{88}$, or $\frac{7}{11}$

341-17 $\frac{67}{84}$

341-18 $\frac{8035}{945}$

Lesson 78

342-A Yes * No

342-B Yes * No

342-C 0 * Yes

342-D Yes. Each member of set S is a rational number. * Yes. The members of set S go on and on. There is no last member.

342-E Yes * Yes

342-F Yes

342-G 4 * 105 * 17 * 1056

342-H Yes. $\frac{7}{1}$ and $\frac{14}{2}$ are the same rational number. * Yes. $\frac{7}{1}$ and $\frac{35}{5}$ are the same rational number. * Yes. For each replacement of n, $\frac{7n}{n}$ and $\frac{7}{1}$ are the same rational number.

343-I $\frac{20}{1}$ * $\frac{45}{1}$

343-J $\frac{a}{1}$

343-K Yes * Yes

343-L Yes

343-M Yes * Yes * Yes

343-N Yes. You know this because of the definition of the sum of two rational numbers. * Yes * Yes. Since $\frac{a}{1} + \frac{b}{1}$ is the same rational number as $\frac{a+b}{1}$ and $\frac{a+b}{1}$ is the mate of $a + b$, you know that $\frac{a}{1} + \frac{b}{1}$ and $a + b$ are mates.

343-O Yes * Yes

343-P Yes

343-Q Yes * Yes. The product of $\frac{2}{1}$ and $\frac{9}{1}$ is $\frac{18}{1}$. The product of 2 and 9 is 18. 18 is the mate of $\frac{18}{1}$.

343-R Yes. You know this because of the definition of the product of two rational numbers. * Yes * Yes. Since $\frac{a}{1} \cdot \frac{b}{1}$ is the same rational number as $\frac{a \cdot b}{1}$, and $\frac{a \cdot b}{1}$ is the mate of $a \cdot b$, you know that $\frac{a}{1} \cdot \frac{b}{1}$ and $a \cdot b$ are mates.

On your own

343-1 9

343-2 36

343-3 11

343-4 50

343-5 100

343-6 175

343-7 $\frac{15}{1}$

343-8 $\frac{6}{1}$

343-9 $\frac{38}{1}$

343-10 $\frac{59}{1}$

343-11 $\frac{505}{1}$

343-12 $\frac{0}{1}$

343-13 4

343-14 1

343-15 0

343-16 2

343-17 3

343-18 5

343-19 $\frac{12}{1} \cdot \frac{0}{1} = \frac{0}{1}$.

343-20 $\frac{0}{1} + \frac{1}{1} = \frac{1}{1}$.

343-21 $\frac{15}{1} \cdot \frac{20}{1} = \frac{300}{1}$.

343-22 $\frac{5}{1} + \frac{505}{1} = \frac{510}{1}$.

343-23 $\frac{3}{1} \cdot \frac{1}{1} = \frac{3}{1}$.

343-24 $\frac{10000}{1} + \frac{1}{1} = \frac{10001}{1}$.

344-25 62 + 70 = 132.

344-26 14 · 0 = 0.

344-27 25 · 1 = 25.

344-28 101 + 1 = 102.

344-29 0 + 15 = 15.

344-30 222 · 3 = 666.

Keeping Skilful

344-1 {140}

344-2 {1, 2, 3, 4, 5, 6}

344-3 {148}

344-4 {600}

344-5 {1, 2, 3, ..., 17}

344-6 {1690}

344-7 {1, 2, 3, ..., 104}

344-8 {20}

344-9 {1}

344-10 {25}

344-11 {44}

344-12 {993}

344-13 {75}

344-14 {292}

344-15 {76}

344-16 { }

344-17 {400}

344-18 {271, 272, 273, ...}

344-19 {1, 2, 3, ..., 97}

344-20 {21}

344-21 108 * 2340

344-22 187 * 4140

344-23 290 * 13,104

344-24 475 * 8226

344-25 192 * 9047

344-26 525 * 66,254

344-27 874 * 175,840

344-28 506 * 61,093

344-29 1075 * 284,350

344-30 $\frac{6}{7}$

344-31 $\frac{1}{6}$

344-32 $\frac{16}{48}$, or $\frac{4}{12}$, or $\frac{1}{3}$

344-33 $\frac{20}{63}$

344-34 $\frac{13}{30}$

344-35 $\frac{0}{7}$, or $\frac{0}{1}$

344-36 $\frac{7}{98}$, or $\frac{1}{14}$

344-37 $\frac{33}{54}$, or $\frac{11}{18}$

344-38 $\frac{18}{8}$, or $\frac{9}{4}$

344-39 $\frac{15}{42}$, or $\frac{5}{14}$

344-40 $\frac{35}{36}$

344-41 $\frac{18}{2}$, or $\frac{9}{1}$

Lesson 79

344-A Yes

344-B Yes

344-C Yes * Yes

345-D Yes * $8\frac{1}{3}$ means $8 + \frac{1}{3}$. $8 + \frac{1}{3}$ means the rational number 8, or $\frac{8}{1}$, plus the rational number $\frac{1}{3}$. Therefore, $8\frac{1}{3} = \frac{8}{1} + \frac{1}{3}$.

345-E The universe for a and c is N and the universe for b and d is C. For each replacement of a, b, c, and d, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$. For the statement expressed by sentence F, a was replaced by 8, b by 1, c by 1, and d by 3.

345-F Yes * Yes

345-G 24 * Yes * Yes

345-H Yes * Yes

345-I Yes * Yes

345-J 4 * Add the numerator of $\frac{3}{4}$ to this product. $5\frac{3}{4} = \frac{23}{4}$.

345-K $\frac{17}{8}$

345-L $\frac{18}{5}$

345-M $\frac{17}{2}$

345-N $\frac{10}{9}$

345-O $\frac{38}{7}$

345-A 5 * Yes * Yes * Yes

345-B Yes. You know this because of the reduction property of rational numbers. * Yes. You know that the rational number $\frac{3}{1}$ is the mate of

the natural number 3. $\frac{3}{1}$ and $1\frac{5}{5}$ are the same rational number. * The numerals 3 and $1\frac{5}{5}$ name the same rational number. Because of the well-defined property of addition, you know that $1\frac{5}{5} + \frac{1}{5}$ is the same as $\frac{3}{1} + \frac{1}{5}$.

345-C Yes * Yes

346-D (Top of Column 1) Yes

346-E (Top of Column 1) 12

346-F (Column 1) 24

346-G (Column 1) 27

346-H (Column 1) $1\frac{5}{7}$ * $6\frac{1}{2}$ * $3\frac{1}{8}$ * $3\frac{8}{9}$

346-I (Column 1) $2\frac{4}{5}$

346-J (Column 1) $1\frac{4}{13}$

346-K (Column 1) $3\frac{1}{3}$

346-L (Column 1) $3\frac{2}{9}$

346-M (Column 1) $6\frac{1}{6}$

346-N (Column 1) $7\frac{1}{7}$

346-O (Column 1) $3\frac{5}{12}$

346-P (Column 1) $3\frac{3}{20}$

346-Q (Column 1) $7\frac{4}{5}$

346-R (Column 1) $5\frac{1}{3}$

346-S $1\frac{10}{11}$

346-T $16\frac{4}{5}$

346-U $14\frac{3}{7}$

346-V $9\frac{1}{10}$

346-W $26\frac{2}{3}$

346-A Yes * Since the numerals 4 and $\frac{4}{1}$ name the same rational number, $4 + \frac{3}{2}$ and $\frac{4}{1} + \frac{3}{2}$ name the same sum.

346-B $\frac{11}{2}$ * $5\frac{1}{2}$

346-C Yes * Since the numerals 6 and $\frac{6}{1}$ name the same rational number, $6 - \frac{4}{3}$ and $\frac{6}{1} - \frac{4}{3}$ name the same difference.

346-D (Bottom of Column 1) $1\frac{4}{3}$ * $4\frac{2}{3}$

346-E (Bottom of Column 1) Yes

346-F (Column 2) Yes * From the definition of the product of two rational numbers, you know that the product of 5, or $\frac{5}{1}$, and $\frac{3}{4}$ is $\frac{5(3)}{1(4)}$. You also know that the product of 1 and any natural number is the natural number. Therefore, to find the product of 5 and $\frac{3}{4}$, you can multiply 5 and 3 to obtain the numerator and use 4 as the denominator for a fraction that indicates the product.

346-G (Column 2) $\frac{42}{8}$ * $5\frac{1}{4}$

346-H (Column 2) $\frac{9}{4}$ * $2\frac{1}{4}$

346-I (Column 2) $\frac{72}{5}$ * $14\frac{2}{5}$

346-J (Column 2) $\frac{20}{3}$ * $6\frac{2}{3}$

346-K (Column 2) $\frac{32}{9}$ * $3\frac{5}{9}$

346-L (Column 2) $\frac{39}{10}$ * $3\frac{9}{10}$

346-M (Column 2) $\frac{110}{13}$ * $8\frac{6}{13}$

346-N (Column 2) $\frac{42}{9}$ * $4\frac{2}{3}$

346-O (Column 2) $\frac{240}{7}$ * $34\frac{2}{7}$

346-P (Column 2) By replacing the rational number $2\frac{5}{8}$ by the equal rational number $\frac{21}{8}$ and replacing the rational number $3\frac{1}{9}$ by the equal rational number $\frac{28}{9}$ * $8\frac{1}{6}$

346-Q (Column 2) Yes * You know that $12 \times 1\frac{1}{7}$ is the same as $\frac{12}{1} \times \frac{8}{7}$. Using the definition of the product of two rational numbers, you know that $\frac{12}{1} \times \frac{8}{7} = \frac{96}{7}$. You also know that $\frac{96}{7}$ is the same rational number as $13\frac{5}{7}$.

346-R (Column 2) Yes * $6 \times \frac{2}{3} = \frac{6}{1} \times \frac{2}{3}$. $\frac{6}{1} \times \frac{2}{3} = \frac{12}{3}$. $\frac{12}{3} = 4$.

On your own

347-1 $5\frac{7}{8}$

347-2 $3\frac{4}{5}$

347-3 $6\frac{2}{3}$

347-4 $11\frac{7}{15}$

347-5 $10\frac{3}{4}$

347-6 $6\frac{3}{4}$

347-7 $6\frac{2}{7}$

347-8 $4\frac{3}{4}$

347-9 $\frac{29}{3}$

347-10 $\frac{65}{16}$

347-11 $\frac{113}{9}$

347-12 $\frac{31}{5}$

347-13 $\frac{63}{8}$

347-14 $\frac{23}{2}$

347-15 $\frac{169}{10}$

347-16 $\frac{77}{5}$

347-17 $\frac{83}{6}$

347-18 $\frac{131}{12}$

347-19 $2\frac{3}{4}$

347-20 $2\frac{2}{5}$

347-21 $1\frac{5}{8}$

347-22 $2\frac{2}{7}$

347-23 $2\frac{3}{11}$

347-24 $2\frac{8}{9}$

347-25 $8\frac{3}{4}$

347-26 $12\frac{2}{3}$

347-27 $\frac{29}{3}$ * $9\frac{2}{3}$

347-28 $\frac{56}{5}$ * $11\frac{1}{5}$

347-29 $\frac{55}{6}$ * $9\frac{1}{6}$

347-30 $\frac{35}{6}$ * $5\frac{5}{6}$

347-31 $\frac{25}{4}$ * $6\frac{1}{4}$

347-32 $\frac{91}{8}$ * $11\frac{3}{8}$

347-33 $\frac{26}{7}$ * $3\frac{5}{7}$

347-34 $\frac{185}{12}$ * $15\frac{5}{12}$

347-35 $\frac{98}{9}$ * $10\frac{8}{9}$

347-36 $\frac{243}{40}$ * $6\frac{3}{40}$

347-37 $\frac{693}{50}$ * $13\frac{43}{50}$

347-38 $\frac{35}{12}$ * $2\frac{11}{12}$

Keeping Skilful

347-1 $12 * \frac{11}{12}$

347-2 $9 * \frac{5}{9}$

347-3 $16 * \frac{5}{16}$

347-4 $12 * \frac{11}{12}$

347-5 $21 * \frac{4}{21}$

347-6 $42 * \frac{11}{14}$

347-7 40 * $\frac{33}{40}$

347-8 70 * $\frac{24}{35}$

347-9 36 * $\frac{11}{12}$

347-10 36 * $\frac{17}{3}$

347-11 102 * $\frac{5}{102}$

347-12 132 * $\frac{53}{66}$

347-13 78

347-14 275

347-15 242

347-16 0

347-17 196

347-18 1491

Checking up

Test 84

347-1 (c) $2 \cdot 3^2$

347-2 (d) $2^2 \cdot 3^4 \cdot 5$

347-3 (c) $\frac{5}{0}$

347-4 T

347-5 T

347-6 F

347-7 T

347-8 F

347-9 F

347-10 $4\frac{3}{7}$

347-11 $5\frac{7}{8}$

347-12 $7\frac{1}{4}$

347-13 $4\frac{1}{3}$

347-14 $\frac{17}{10}$

347-15 $\frac{41}{12}$

347-16 $\frac{18}{7}$

347-17 $\frac{63}{5}$

347-18 $\frac{39}{8}$

347-19 $\frac{13}{12}$

347-20 $\frac{42}{5}$

347-21 $\frac{223}{12}$

Test 85

347-22 $\frac{49}{30}$, or $1\frac{19}{30}$

347-23 $\frac{6}{5}$, or $1\frac{1}{5}$

347-24 $\frac{10}{1}$, or 10

347-25 $\frac{17}{16}$, or $1\frac{1}{16}$

347-26 $\frac{38}{9}$, or $4\frac{2}{9}$

347-27 $\frac{315}{105}$

347-28 $\frac{11}{3}$, or $3\frac{2}{3}$

347-29 $\frac{38}{9}$, or $4\frac{2}{9}$

347-30 $\frac{6}{5}$, or $1\frac{1}{5}$

347-31 $\frac{1}{1}$, or 1

347-32 $\frac{5}{48}$

347-33 $\frac{15}{1}$, or 15

347-34 $\frac{7}{5}$, or $1\frac{2}{5}$

347-35 $\frac{49}{4}$, or $12\frac{1}{4}$

347-36 $\frac{2145}{24}$, or $10\frac{5}{24}$

Lesson 80

348-A (Column 1) Yes

348-B Yes

348-C You replace y by $\frac{6}{4}$ and z by $\frac{3}{4} + \frac{3}{4}$. * You replace x by $\frac{1}{4} + \frac{5}{4}$ and z by $\frac{3}{4} + \frac{3}{4}$. These are the same replacements that you used for x and z to obtain the statements expressed by sentences A and B.

348-D R_a * Yes

348-E Yes

348-F Yes * The transitive property of equality

348-A (Column 2) Yes

349-B The transitive property of equality is: For each replacement of the variables, if $x = y$ and $y = z$, then $x = z$. However, $y = z$ also means $z = y$. Therefore, the transitive property can be stated as follows: For each replacement of the variables, if $x = y$ and $z = y$, then $x = z$. To show that $\frac{1}{5} \times \frac{2}{1} = \frac{2}{1} \times \frac{1}{5}$, use the transitive property and make replacements as follows:

$x = y$.

$\frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$.

$z = y$.

$\frac{2}{1} \times \frac{1}{5} = \frac{2}{5}$.

$x = z$.

$\frac{1}{5} \times \frac{2}{1} = \frac{2}{1} \times \frac{1}{5}$.

349-C No

349-D Yes * To show that $\frac{3}{8} \times \frac{1}{6} = \frac{1}{6} \times \frac{3}{8}$, use the transitive property and make replacements as follows:

$x = y$.

$\frac{3}{8} \times \frac{1}{6} = \frac{3}{48}$.

$z = y$.

$\frac{1}{6} \times \frac{3}{8} = \frac{3}{48}$.

$x = z$.

$\frac{3}{8} \times \frac{1}{6} = \frac{1}{6} \times \frac{3}{8}$.

349-E No

349-F Yes * From the transitive property of equality

349-G No

349-H The operation of multiplication of rational numbers has the commutative property.

349-I The universe for a and c is N. The universe for b and d is C.

349-J Yes * The definition of the product of two rational numbers

349-K Yes * The definition of the product of two rational numbers

349-L Yes * The commutative property of multiplication of natural numbers

349-M Yes * The transitive property of equality

349-N Yes * The transitive property of equality

350-O (Column 1) True statement

350-P (Column 1) False statement

350-Q (Column 1) True statement

350-R True statement

350-S False statement

350-T False statement

350-A Yes

350-B $x = y$.

$\frac{3}{4} + \frac{3}{8} = \frac{9}{8}$.

$y = z$.
$\frac{9}{8} = \frac{3}{8} + \frac{3}{4}$.

$x = z$.
$\frac{3}{4} + \frac{3}{8} = \frac{3}{8} + \frac{3}{4}$.

350-C No

350-D Yes * The transitive property

350-E No

350-F Yes * By using the transitive property

350-G No

350-H The operation of addition of rational numbers has the commutative property.

350-I True statement

350-J False statement

350-K True statement

350-L False statement

350-M The condition means that, upon replacement of the variables, $x - y$ is the same number as $y - x$. * R_a

350-N For the statement expressed by sentence X, x was replaced by $\frac{7}{10}$ and y by $\frac{3}{10}$. For the statement expressed by sentence Y, x was replaced by $\frac{3}{4}$ and y by $\frac{1}{2}$. For the statement expressed by sentence Z, x was replaced by $\frac{11}{12}$ and y by $\frac{0}{3}$.

350-O (Column 2) You know that $\frac{3}{10} - \frac{7}{10}$ is not a rational number of arithmetic because 3 is less than 7.

350-P (Column 2) No

350-Q (Column 2) No * Since you know of at least one set of replacements for which you do not obtain a true statement from $x - y = y - x$, you know that subtraction of rational numbers does not have the commutative property.

On your own

351-1 $\frac{1}{6} + \frac{2}{3} = \frac{1}{3} \times \frac{5}{2}$.

351-2 $\frac{3}{4} + \frac{1}{4} = \frac{2}{4} + \frac{2}{4}$.

351-3 $\frac{5}{8} + \frac{1}{2} = \frac{10}{4} - \frac{11}{8}$.

351-4 $\frac{11}{20} - \frac{1}{4} = \frac{3}{5} - \frac{6}{20}$.

351-5 $\frac{7}{10} - \frac{7}{10} = \frac{2}{5} \times \frac{0}{2}$.

351-6 $\frac{2}{9} + \frac{1}{4} = \frac{17}{36} - \frac{0}{12}$.

351-7 True statement

351-8 False statement

351-9 False statement

351-10 True statement

351-11 True statement

351-12 True statement

351-13 False statement

351-14 False statement

351-15 False statement

351-16 True statement

351-17 $\frac{1}{3} \times \frac{7}{8} = \frac{7}{24}$.
$\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$.

351-18 $\frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$. (or $\frac{4}{18}$)
$\frac{1}{2} \times \frac{4}{9} = \frac{2}{9}$. (or $\frac{4}{18}$)

351-19 $\frac{3}{3} \times \frac{7}{6} = \frac{7}{6}$. (or $\frac{21}{18}$, or $1\frac{1}{6}$)
$\frac{7}{6} \times \frac{3}{3} = \frac{7}{6}$. (or $\frac{21}{18}$, or $1\frac{1}{6}$)

351-20 $\frac{8}{10} \times \frac{9}{9} = \frac{4}{5}$. (or $\frac{72}{90}$, or $\frac{8}{10}$)
$\frac{9}{9} \times \frac{8}{10} = \frac{4}{5}$. (or $\frac{72}{90}$, or $\frac{8}{10}$)

351-21 $\frac{5}{2} \times \frac{0}{15} = 0$. (or $\frac{0}{30}$, or $\frac{0}{6}$)
$\frac{0}{15} \times \frac{5}{2} = 0$. (or $\frac{0}{30}$, or $\frac{0}{6}$)

351-22 $\frac{11}{12} \times \frac{1}{11} = \frac{1}{12}$. (or $\frac{11}{132}$)
$\frac{1}{11} \times \frac{11}{12} = \frac{1}{12}$. (or $\frac{11}{132}$)

351-23 $\frac{4}{5} \times \frac{5}{4} = 1$. (or $\frac{20}{20}$, or $\frac{1}{1}$)
$\frac{5}{4} \times \frac{4}{5} = 1$. (or $\frac{20}{20}$, or $\frac{1}{1}$)

351-24 $\frac{0}{8} \times \frac{8}{8} = 0$. (or $\frac{0}{64}$, or $\frac{0}{8}$)
$\frac{8}{8} \times \frac{0}{8} = 0$. (or $\frac{0}{64}$, or $\frac{0}{8}$)

351-25 $\frac{4}{9} \times \frac{2}{5} = \frac{8}{45}$.
$\frac{2}{5} \times \frac{4}{9} = \frac{8}{45}$.

351-26 $\frac{7}{2} \times \frac{3}{4} = \frac{21}{8}$. (or $2\frac{5}{8}$)
$\frac{3}{4} \times \frac{7}{2} = \frac{21}{8}$. (or $2\frac{5}{8}$)

351-27 $\frac{2}{9} \times \frac{0}{11} = 0$. (or $\frac{0}{99}$)
$\frac{0}{11} \times \frac{2}{9} = 0$. (or $\frac{0}{99}$)

351-28 $\frac{15}{8} \times \frac{7}{7} = \frac{15}{8}$.
(or $\frac{105}{56}$, or $1\frac{7}{8}$)
$\frac{7}{7} \times \frac{15}{8} = \frac{15}{8}$.
(or $\frac{105}{56}$, or $1\frac{7}{8}$)

351-29 $\frac{1}{4} + \frac{1}{12} = \frac{1}{3}$. (or $\frac{16}{48}$, or $\frac{4}{12}$)
$\frac{1}{12} + \frac{1}{4} = \frac{1}{3}$. (or $\frac{16}{48}$, or $\frac{4}{12}$)

351-30 $\frac{7}{5} + \frac{1}{8} = \frac{61}{40}$. (or $1\frac{21}{40}$)
$\frac{1}{8} + \frac{7}{5} = \frac{61}{40}$. (or $1\frac{21}{40}$)

351-31 $\frac{6}{3} + \frac{0}{5} = 2$.
(or $\frac{30}{15}$, or $\frac{6}{3}$, or $\frac{2}{1}$)
$\frac{0}{5} + \frac{6}{3} = 2$.
(or $\frac{30}{15}$, or $\frac{6}{3}$, or $\frac{2}{1}$)

351-32 $\frac{1}{10} + \frac{2}{3} = \frac{23}{30}$.
$\frac{2}{3} + \frac{1}{10} = \frac{23}{30}$.

351-33 $\frac{7}{7} + \frac{6}{6} = 2$. (or $\frac{84}{42}$, or $\frac{2}{1}$)
$\frac{6}{6} + \frac{7}{7} = 2$. (or $\frac{84}{42}$, or $\frac{2}{1}$)

351-34 $\frac{5}{4} + \frac{7}{12} = \frac{11}{6}$.
(or $\frac{88}{48}$, or $\frac{22}{12}$, or $1\frac{5}{6}$)
$\frac{7}{12} + \frac{5}{4} = \frac{11}{6}$.
(or $\frac{88}{48}$, or $\frac{22}{12}$, or $1\frac{5}{6}$)

351-35 $\frac{10}{2} + \frac{2}{10} = \frac{26}{5}$.
(or $\frac{104}{20}$, or $\frac{52}{10}$, or $5\frac{1}{5}$)
$\frac{2}{10} + \frac{10}{2} = \frac{26}{5}$.
(or $\frac{104}{20}$, or $\frac{52}{10}$, or $5\frac{1}{5}$)

351-36 $\frac{0}{8} + \frac{1}{6} = \frac{1}{6}$. (or $\frac{8}{48}$, or $\frac{4}{24}$)
$\frac{1}{6} + \frac{0}{8} = \frac{1}{6}$. (or $\frac{8}{48}$, or $\frac{4}{24}$)

351-37 $\frac{4}{5} + \frac{1}{2} = \frac{13}{10}$. (or $1\frac{3}{10}$)
$\frac{1}{2} + \frac{4}{5} = \frac{13}{10}$. (or $1\frac{3}{10}$)

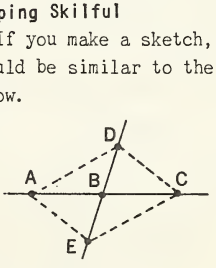
351-38 $\frac{12}{4} + \frac{1}{3} = \frac{10}{3}$.
(or $\frac{40}{12}$, or $3\frac{1}{3}$)
$\frac{1}{3} + \frac{12}{4} = \frac{10}{3}$.
(or $\frac{40}{12}$, or $3\frac{1}{3}$)

351-39 $\frac{8}{9} + \frac{5}{18} = \frac{7}{6}$.
(or $\frac{189}{162}$, or $\frac{21}{18}$, or $1\frac{1}{6}$)
$\frac{5}{18} + \frac{8}{9} = \frac{7}{6}$.
(or $\frac{189}{162}$, or $\frac{21}{18}$, or $1\frac{1}{6}$)

351-40 $\frac{2}{7} + \frac{11}{10} = \frac{97}{70}$. (or $1\frac{27}{70}$)
$\frac{11}{10} + \frac{2}{7} = \frac{97}{70}$. (or $1\frac{27}{70}$)

Keeping Skilful

If you make a sketch, it should be similar to the one below.



351-1 Infinitely many. Points A and C are in the same line. A given line is included in infinitely

many planes. * Infinitely many. Points B and E are in the same line. * One. Points B, C, and D are noncollinear points. There is exactly one plane that contains three noncollinear points.

351-2 Six. The segments that are determined by points A, B, C, D, and E and that contain point B are $\overline{AB}$, $\overline{BD}$, $\overline{BC}$, $\overline{BE}$, $\overline{AC}$, and $\overline{DE}$.

351-3 $\overline{AR} < \overline{AC}$.

351-4 $\overline{AR} > \overline{AC}$.

351-5 $\overline{AR} \cong \overline{AC}$.

351-6 ΔACD

351-7 $\angle AEC$

351-8 The set that contains only point B

351-9 $\angle ABD$ and $\angle DBC$, $\angle DBC$ and $\angle CBE$, $\angle CBE$ and $\angle EBA$, $\angle EBA$ and $\angle ABD$

351-10 The sides are $\overline{AD}$, $\overline{DC}$, $\overline{CE}$, and $\overline{EA}$. The angles are $\angle ADC$, $\angle DCE$, $\angle CEA$, and $\angle EAD$.

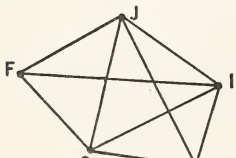
351-11 ΔECD

351-12 ΔEBC

351-13 Segments BC, CD, and DB

351-14 Eight * ΔABD, ΔABE, ΔACD, ΔACE, ΔBCD, ΔBCE, ΔADE, and ΔCDE

351-15 Ten. See the sketch below.



Points F, G, H, I, and J are any five points, no three of which are collinear. The triangles determined by these five points are ΔFGH, ΔFGI, ΔFGJ, ΔFHI, ΔFHJ, ΔFIJ, ΔGHI, ΔGHJ, ΔGIJ, and ΔHIJ.

Lesson 81

352-A $\frac{1}{2}$ and $\frac{2}{3}$ * Yes

352-B $\frac{6}{24}$ * Yes

352-C $\frac{2}{3}$ and $\frac{3}{4}$ * $\frac{6}{12}$

352-D $\frac{6}{24}$ * Yes

352-E Yes * No

352-F $\frac{1}{10}$ * $\frac{2}{50}$

352-G $\frac{2}{10}$ * $\frac{2}{50}$

352-H Yes * No

352-I $(\frac{3}{4} \cdot \frac{5}{8})\frac{4}{3} = \frac{90}{16}$.
$\frac{3}{4}(\frac{5}{8} \cdot \frac{4}{3}) = \frac{90}{16}$.
$(\frac{3}{4} \cdot \frac{5}{8})\frac{4}{3} = \frac{3}{4}(\frac{5}{8} \cdot \frac{4}{3})$. * No

352-J The operation of multiplication of rational numbers has the associative property.

352-K True statement

352-L False statement

352-M True statement

352-N True statement

353-A $\frac{3}{8}$ and $\frac{1}{5}$ * Yes

353-B $\frac{195}{200}$ * Yes * Yes

353-C $\frac{1}{5}$ and $\frac{2}{5}$ * $\frac{3}{5}$

353-D $\frac{39}{40}$ * Yes

353-E Yes * No

353-F $\frac{11}{12}$ * $\frac{80}{48}$, or $\frac{5}{3}$, or $1\frac{2}{3}$

353-G $\frac{4}{4}$, or 1 * $\frac{5}{3}$, or $1\frac{2}{3}$

353-H Yes * No

353-I $(\frac{2}{3} + \frac{1}{4}) + \frac{3}{4} = 1\frac{2}{3}$.
$\frac{2}{3} + (\frac{1}{4} + \frac{3}{4}) = 1\frac{2}{3}$.
$(\frac{2}{3} + \frac{1}{4}) + \frac{3}{4} = \frac{2}{3} + (\frac{1}{4} + \frac{3}{4})$.
* No

353-J True statement

353-K False statement

353-L True statement

353-M The condition requires that, upon replacement of the variables, the difference of $(x - y)$ and z is the same number as the difference of x and $(y - z)$. * R_a

353-N x was replaced by $2\frac{5}{2}$, y by $\frac{3}{4}$, and z by $\frac{3}{4}$. * No

353-O If a condition is a property, then you must obtain a true statement for each replacement of the variables. You know of one set of replacements for which you do not obtain a true statement. Therefore, you know that subtraction of rational numbers does not have the associative property.

354-A Yes * $3\frac{3}{4}$ means $3 + \frac{3}{4}$ and $2\frac{1}{8}$ means $2 + \frac{1}{8}$.

354-B Yes * Associative property of addition

354-C Yes * Commutative property of addition

354-D Yes * Associative property of addition

354-E Yes. You know this because of the transitive property of equality and the true statements expressed in D6. * Yes. 3 + 2 is the same as 5, and $\frac{3}{4} + \frac{1}{8}$ is the same as $\frac{7}{8}$. $5 + \frac{7}{8}$ is the same as $5\frac{7}{8}$. * Yes. You know this because of the transitive property and the true statements $3\frac{3}{4} + 2\frac{1}{8} = (3 + 2) + (\frac{3}{4} + \frac{1}{8})$ and $(3 + 2) + (\frac{3}{4} + \frac{1}{8}) = 5\frac{7}{8}$.

354-F $\frac{3}{8}$ * 1 * Yes

354-G Yes * Yes * Yes

354-H Yes * Yes

354-I $\frac{8}{12} - \frac{9}{12}$ is not a rational number of arithmetic but $\frac{20}{12} - \frac{9}{12}$ is. * Yes

354-J $\frac{11}{12}$ * 2 * Yes

On your own

355-1 True statement * Commutative property of addition

355-2 True statement * Associative property of multiplication

355-3 False statement

355-4 True statement * Commutative property of multiplication

355-5 The second and third sentences express false statements.

355-6 True statement * Transitive property of equality

355-7 False statement

355-8 True statement * Associative property of multiplication

355-9 False statement

355-10 True statement * Commutative property of multiplication

355-11 True statement * Associative property of addition

355-12 False statement

355-13 False statement

355-14 True statement * Commutative property of addition

For the responses to most of the remaining exercises that involve rational numbers, we will give only the numeral for a basic fraction or a mixed numeral. Again remember that the fraction you name must be equivalent to the basic fraction named in this response book. Be sure to follow any specific instructions given by your teacher.

355-15 $(\frac{3}{4} \times \frac{3}{8}) \times \frac{1}{5}$ and $\frac{3}{4} \times (\frac{3}{8} \times \frac{1}{5})$ * $\frac{9}{160}$

355-16 $(\frac{9}{10} \times \frac{10}{9}) \times \frac{1}{3}$ and $\frac{9}{10} \times (\frac{10}{9} \times \frac{1}{3})$ * $\frac{1}{3}$

355-17 $(\frac{6}{5} \times \frac{1}{2}) \times \frac{1}{4}$ and $\frac{6}{5} \times (\frac{1}{2} \times \frac{1}{4})$ * $\frac{3}{20}$

355-18 $(\frac{0}{6} \times \frac{3}{7}) \times \frac{5}{8}$ and $\frac{0}{6} \times (\frac{3}{7} \times \frac{5}{8})$ * 0

355-19 $(\frac{1}{25} \times \frac{25}{2}) \times \frac{1}{16}$ and $\frac{1}{25} \times (\frac{25}{2} \times \frac{1}{16})$ * $\frac{1}{32}$

355-20 $(\frac{4}{4} \times \frac{2}{3}) \times \frac{1}{2}$ and $\frac{4}{4} \times (\frac{2}{3} \times \frac{1}{2})$ * $\frac{1}{3}$

355-21 $(\frac{4}{11} \times \frac{1}{7}) \times \frac{0}{5}$ and $\frac{4}{11} \times (\frac{1}{7} \times \frac{0}{5})$ * 0

355-22 $(\frac{3}{3} \times \frac{4}{5}) \times \frac{2}{3}$ and $\frac{3}{3} \times (\frac{4}{5} \times \frac{2}{3})$ * $\frac{8}{15}$

355-23 $(\frac{1}{6} + \frac{2}{3}) + \frac{5}{2}$ and $\frac{1}{6} + (\frac{2}{3} + \frac{5}{2})$ * $3\frac{1}{3}$

355 24 $(\frac{5}{8} + \frac{3}{4}) + \frac{1}{5}$ and $\frac{5}{8} + (\frac{3}{4} + \frac{1}{5})$ * $1\frac{23}{40}$

355-25 $(\frac{3}{7} + \frac{2}{2}) + \frac{6}{7}$ and $\frac{3}{7} + (\frac{2}{2} + \frac{6}{7})$ * $2\frac{2}{7}$

355-26 $(\frac{3}{6} + \frac{13}{13}) + \frac{7}{6}$ and $\frac{3}{6} + (\frac{13}{13} + \frac{7}{6})$ * $2\frac{2}{3}$

355-27 $(\frac{2}{3} + \frac{1}{4}) + \frac{7}{12}$ and $\frac{2}{3} + (\frac{1}{4} + \frac{7}{12})$ * $1\frac{1}{2}$

355-28 $(\frac{6}{20} + \frac{3}{10}) + \frac{4}{5}$ and $\frac{6}{20} + (\frac{3}{10} + \frac{4}{5})$ * $1\frac{2}{5}$

355-29 18 * $2\frac{3}{4}$

355-30 $33\frac{5}{24}$ * $6\frac{11}{24}$

355-31 $23\frac{1}{2}$ * $16\frac{1}{2}$

355-32 $28\frac{27}{40}$ * $\frac{37}{40}$

355-33 $54\frac{3}{4}$ * $18\frac{3}{4}$

355-34 $6\frac{1}{15}$ * $3\frac{4}{15}$

355-35 $14\frac{5}{8}$ * $1\frac{7}{8}$

355-36 $112\frac{1}{2}$ * $89\frac{1}{2}$

355-37 $32\frac{3}{5}$ * $2\frac{4}{5}$

Lesson 82

355-A The product of $\frac{1}{2}$ and $(\frac{2}{3} + \frac{1}{4})$ * The sum of $\frac{1}{2}(\frac{2}{3})$ and $\frac{1}{2}(\frac{1}{4})$

355-B Yes * Yes

355-C $\frac{11}{24}$ * Yes

355-D Yes * Transitive property of equality

356-E Yes

356-F Yes * Yes * Yes

356-G Yes * Yes

356-H From the true statements expressed by sentences E and F and the transitive property of equality

356-1 Yes * Yes

356-J For sentence H:
$\frac{2}{3}(\frac{1}{5} + \frac{2}{5}) = \frac{2}{3}(\frac{3}{5})$.
$\frac{2}{3}(\frac{3}{5}) = \frac{2}{5}$. *
$\frac{2}{3}(\frac{1}{5}) + \frac{2}{3}(\frac{2}{5}) = \frac{2}{15} + \frac{4}{15}$.
$\frac{2}{15} + \frac{4}{15} = \frac{2}{5}$. *
True statement.
For sentence I:
$\frac{5}{6}(\frac{2}{3} + \frac{1}{9}) = \frac{5}{6}(\frac{7}{9})$.
$\frac{5}{6}(\frac{7}{9}) = \frac{35}{54}$. *
$\frac{5}{6}(\frac{2}{3}) + \frac{5}{6}(\frac{1}{9}) = \frac{5}{9} + \frac{5}{54}$.
$\frac{5}{9} + \frac{5}{54} = \frac{35}{54}$. *
True statement.
For sentence J:
$\frac{1}{4}(\frac{3}{4} + \frac{1}{2}) = \frac{1}{4}(\frac{5}{4})$.
$\frac{1}{4}(\frac{5}{4}) = \frac{5}{16}$. *
$\frac{1}{4}(\frac{3}{4}) + \frac{1}{4}(\frac{1}{2}) = \frac{3}{16} + \frac{1}{8}$.
$\frac{3}{16} + \frac{1}{8} = \frac{5}{16}$. *
True statement.
For sentence K:
$\frac{3}{4}(\frac{1}{6} + \frac{1}{3}) = \frac{3}{4}(\frac{1}{2})$.
$\frac{3}{4}(\frac{1}{2}) = \frac{3}{8}$. *
$\frac{3}{4}(\frac{1}{6}) + \frac{3}{4}(\frac{1}{3}) = \frac{1}{8} + \frac{1}{4}$.
$\frac{1}{8} + \frac{1}{4} = \frac{3}{8}$. *
True statement.

356-K For the statement expressed by sentence L, x was replaced by $\frac{3}{4}$, y by $\frac{1}{3}$, and z by $\frac{2}{7}$. For the statement expressed by sentence M, x was replaced by $\frac{1}{8}$, y by $\frac{9}{10}$, and z by $\frac{2}{5}$. For the statement expressed by sentence N, x was replaced by $\frac{4}{3}$, y by $\frac{1}{8}$, and z by $\frac{3}{7}$. * Because of the distributive property of multiplication over addition

of rational numbers

356-L $(y + z)x = yx + zx$. * R_a

356-M For the statement expressed by sentence O, x was replaced by $\frac{2}{3}$, y by $\frac{4}{6}$, and z by $\frac{1}{5}$. For the statement expressed by sentence P, x was replaced by $\frac{5}{8}$, y by $\frac{10}{3}$, and z by $\frac{1}{4}$. For the statement expressed by sentence Q, x was replaced by $\frac{2}{5}$, y by $\frac{8}{7}$, and z by $\frac{1}{9}$.

356-N Yes

356-O For each replacement of the variables:
$(y + z)x = x(y + z)$.
Commutative property of multiplication.
$x(y + z) = xy + xz$.
Distributive property of multiplication over addition.
$(y + z)x = xy + xz$.
Transitive property of equality.
$xy + xz = yx + zx$.
Commutative property of multiplication.
$(y + z)x = yx + zx$.
Transitive property of equality

357-P 2600 * Yes * Yes

357-Q Yes * $104\frac{1}{3}$ means $104 + \frac{1}{3}$.

357-R Yes * The distributive property

357-S Yes

357-T $4(2\frac{1}{3}) = 4(2 + \frac{1}{3})$.
$4(2 + \frac{1}{3}) = 4(2) + 4(\frac{1}{3})$.
$4(2) + 4(\frac{1}{3}) = 8 + 1\frac{1}{3}$.
$8 + 1\frac{1}{3} = 9\frac{1}{3}$.

357-U $(80\frac{1}{2})6 = (80 + \frac{1}{2})6$.
$(80 + \frac{1}{2})6 = (80)6 + (\frac{1}{2})6$.
$(80)6 + (\frac{1}{2})6 = 480 + 3$.
$480 + 3 = 483$.

357-V $(14\frac{2}{7})35 = (14 + \frac{2}{7})35$.
$(14 + \frac{2}{7})35 =$
$(14)35 + (\frac{2}{7})35$.
$(14)35 + (\frac{2}{7})35 = 490 + 10$.
$490 + 10 = 500$.

357-W $47(11\frac{1}{9}) = 47(11 + \frac{1}{9})$.
$47(11 + \frac{1}{9}) =$
$47(11) + 47(\frac{1}{9})$.
$47(11) + 47(\frac{1}{9}) = 517 + 5\frac{2}{9}$.
$517 + 5\frac{2}{9} = 522\frac{2}{9}$.

357-A $x + (yz) = (x + y)(x + z)$.

357-B x was replaced by $\frac{1}{2}$, y by $\frac{2}{3}$, and z by $\frac{3}{4}$.

357-C No * $\frac{1}{2} + (\frac{2}{3} \cdot \frac{3}{4})$ is the same as 1. $(\frac{1}{2} + \frac{2}{3})(\frac{1}{2} + \frac{3}{4})$ is the same as $1\frac{11}{24}$.

357-D No. A condition is a property only when a true statement is obtained from the condition for each replacement of the variables. * No

358-E (Column 1) $x(y - z) = xy - xz$. * R_a

358-F (Column 1) x was replaced by $\frac{4}{5}$, y by $\frac{2}{3}$, and z by $\frac{1}{8}$. * x was replaced by $\frac{4}{5}$, y by $\frac{1}{4}$, and z by $\frac{5}{2}$.

358-G Yes * No. $\frac{1}{4} - \frac{5}{2}$ is not a rational number of arithmetic.

358-H If the replacement for y is greater than or equal to the replacement for z, then $x(y - z) = xy - xz$ and multiplication distributes over subtraction in the set of rational numbers.

358-A R_a

358-B Addition and multiplication * Yes

358-C Addition * Multiplication

358-D Addition * Multiplication

358-E (Column 2) Distributive property, or $x(y + z) = xy + xz$.

358-F (Column 2) Yes. The set of rational numbers forms a number system because it behaves like the natural-number system with respect to two operations (addition and multiplication) and their properties.

359-G The set of rational numbers that are indicated by basic fractions having a numerator of 1.

359-H Yes * Two examples are: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. $\frac{1}{6}$ is a member of set M. $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$. $\frac{1}{35}$ is a member of set M.

359-I No * Two examples are: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{5} + \frac{1}{7} = \frac{12}{35}$. $\frac{5}{6}$ and $\frac{12}{35}$ cannot be indicated by basic fractions having numerators of 1.

359-J No. From your answer to exercise I, you know that set M is not closed under addition. Therefore, set M does not form a number system because it does not behave like the natural-number system with respect to two operations (addition and multiplication) and their properties.

On your own

359-1 True statement * Commutative property of multiplication

359-2 False statement

359-3 You cannot decide if the sentence expresses a true statement because $\frac{1}{3} - \frac{2}{3}$ is not a rational number of arithmetic.

359-4 True statement * Commutative property of addition

359-5 True statement * Associative property of addition

359-6 True statement * Distributive property

359-7 True statement * Distributive property

359-8 True statement * Distributive property

359-9 False statement

359-10 $\frac{4}{9}$

359-11 $\frac{11}{12}$

359-12 $\frac{4}{7}$

359-13 $\frac{3}{4}$

359-14 $\frac{6}{7}$

359-15 $\frac{2}{7}$

359-16 $\frac{3}{6}$

359-17 $\frac{3}{13}$

359-18 $13(2\frac{1}{2}) = 13(2 + \frac{1}{2})$.
$13(2 + \frac{1}{2}) = 13(2) + 13(\frac{1}{2})$.
$13(2) + 13(\frac{1}{2}) = 26 + 6\frac{1}{2}$.
$26 + 6\frac{1}{2} = 32\frac{1}{2}$.

359-19 $(28\frac{1}{3})5 = (28 + \frac{1}{3})5$.
$(28 + \frac{1}{3})5 = (28)5 + (\frac{1}{3})5$.
$(28)5 + (\frac{1}{3})5 = 140 + 1\frac{2}{3}$.
$140 + 1\frac{2}{3} = 141\frac{2}{3}$.

359-20 $16(41\frac{7}{8}) = 16(41 + \frac{7}{8})$.
$16(41 + \frac{7}{8}) \doteq$
$16(41) + 16(\frac{7}{8})$.
$16(41) + 16(\frac{7}{8}) = 656 + 14$.
$656 + 14 = 670$.

359-21 $(3\frac{1}{4})4 = (3 + \frac{1}{4})4$.
$(3 + \frac{1}{4})4 = (3)4 + (\frac{1}{4})4$.
$(3)4 + (\frac{1}{4})4 = 12 + 1$.
$12 + 1 = 13$.

359-22 $(20\frac{1}{5})7 = (20 + \frac{1}{5})7$.
$(20 + \frac{1}{5})7 = (20)7 + (\frac{1}{5})7$.
$(20)7 + (\frac{1}{5})7 = 140 + 1\frac{2}{5}$.
$140 + 1\frac{2}{5} = 141\frac{2}{5}$.

359-23 $2(6\frac{3}{5}) = 2(6 + \frac{3}{5})$.
$2(6 + \frac{3}{5}) = 2(6) + 2(\frac{3}{5})$.
$2(6) + 2(\frac{3}{5}) = 12 + 1\frac{1}{5}$.
$12 + 1\frac{1}{5} = 13\frac{1}{5}$.

359-24 $32(5\frac{1}{4}) = 32(5 + \frac{1}{4})$.
$32(5 + \frac{1}{4}) = 32(5) + 32(\frac{1}{4})$.
$32(5) + 32(\frac{1}{4}) = 160 + 8$.
$160 + 8 = 168$.

359-25 $(7\frac{2}{5})54 = (7 + \frac{2}{5})54$.
$(7 + \frac{2}{5})54 = (7)54 + (\frac{2}{5})54$.
$7(54) + (\frac{2}{5})54 = 378 + 36$.
$378 + 36 = 414$.

359-26 $15(12\frac{2}{3}) = 15(12 + \frac{2}{3})$.
$15(12 + \frac{2}{3}) =$
$15(12) + 15(\frac{2}{3})$.
$15(12) + 15(\frac{2}{3}) = 180 + 10$.
$180 + 10 = 190$.

One example is given for each of exercises 27 through 35.

359-27 $\frac{6}{12} + \frac{15}{16} = \frac{3}{4}(\frac{2}{3}) + \frac{3}{4}(\frac{5}{4})$.
$\frac{3}{4}(\frac{2}{3}) + \frac{3}{4}(\frac{5}{4}) = \frac{3}{4}(\frac{2}{3} + \frac{5}{4})$.
$\frac{3}{4}(\frac{2}{3} + \frac{5}{4}) = \frac{3}{4}(\frac{23}{12})$.

359-28 $\frac{2}{12} + \frac{16}{21} = \frac{2}{3}(\frac{1}{4}) + \frac{2}{3}(\frac{8}{7})$.
$\frac{2}{3}(\frac{1}{4}) + \frac{2}{3}(\frac{8}{7}) = \frac{2}{3}(\frac{1}{4} + \frac{8}{7})$.
$\frac{2}{3}(\frac{1}{4} + \frac{8}{7}) = \frac{2}{3}(\frac{39}{28})$.

359-29 $\frac{3}{7} + \frac{5}{14} = \frac{1}{7}(\frac{3}{1}) + \frac{1}{7}(\frac{5}{2})$.
$\frac{1}{7}(\frac{3}{1}) + \frac{1}{7}(\frac{5}{2}) = \frac{1}{7}(\frac{3}{1} + \frac{5}{2})$.
$\frac{1}{7}(\frac{3}{1} + \frac{5}{2}) = \frac{1}{7}(\frac{11}{2})$.

359-30 $\frac{8}{12} + \frac{8}{27} = \frac{8}{3}(\frac{1}{4}) + \frac{8}{3}(\frac{1}{9})$.
$\frac{8}{3}(\frac{1}{4}) + \frac{8}{3}(\frac{1}{9}) = \frac{8}{3}(\frac{1}{4} + \frac{1}{9})$.
$\frac{8}{3}(\frac{1}{4} + \frac{1}{9}) = \frac{8}{3}(\frac{13}{36})$.

359-31 $\frac{24}{5} + \frac{3}{10} = \frac{3}{5}(\frac{8}{1}) + \frac{3}{5}(\frac{1}{2})$.
$\frac{3}{5}(\frac{8}{1}) + \frac{3}{5}(\frac{1}{2}) = \frac{3}{5}(\frac{8}{1} + \frac{1}{2})$.
$\frac{3}{5}(\frac{8}{1} + \frac{1}{2}) = \frac{3}{5}(\frac{17}{2})$.

359-32 $\frac{5}{18} + \frac{20}{21} = \frac{5}{3}(\frac{1}{6}) + \frac{5}{3}(\frac{4}{7})$.
$\frac{5}{3}(\frac{1}{6}) + \frac{5}{3}(\frac{4}{7}) = \frac{5}{3}(\frac{1}{6} + \frac{4}{7})$.
$\frac{5}{3}(\frac{1}{6} + \frac{4}{7}) = \frac{5}{3}(\frac{31}{42})$.

359-33 $\frac{17}{2} + \frac{34}{3} = \frac{17}{1}(\frac{1}{2}) + \frac{17}{1}(\frac{2}{3})$.
$\frac{17}{1}(\frac{1}{2}) + \frac{17}{1}(\frac{2}{3}) = \frac{17}{1}(\frac{1}{2} + \frac{2}{3})$.
$\frac{17}{1}(\frac{1}{2} + \frac{2}{3}) = \frac{17}{1}(\frac{7}{6})$.

359-34 $\frac{8}{15} + \frac{1}{10} = \frac{1}{5}(\frac{8}{3}) + \frac{1}{5}(\frac{1}{2})$.
$\frac{1}{5}(\frac{8}{3}) + \frac{1}{5}(\frac{1}{2}) = \frac{1}{5}(\frac{8}{3} + \frac{1}{2})$.
$\frac{1}{5}(\frac{8}{3} + \frac{1}{2}) = \frac{1}{5}(\frac{19}{6})$.

359-35 $\frac{12}{32} + \frac{24}{56} = \frac{12}{8}(\frac{1}{4}) + \frac{12}{8}(\frac{2}{7})$.
$\frac{12}{8}(\frac{1}{4}) + \frac{12}{8}(\frac{2}{7}) = \frac{12}{8}(\frac{1}{4} + \frac{2}{7})$.
$\frac{12}{8}(\frac{1}{4} + \frac{2}{7}) = \frac{12}{8}(\frac{15}{28})$.

Keeping Skilful

359-1 $9\frac{8}{9}$

359-2 $\frac{1}{4}$

359-3 $4\frac{2}{3}$

359-4 $\frac{5}{8}$

359-5 $1\frac{1}{6}$

359-6 $17\frac{3}{7}$

359-7 $4\frac{2}{3}$

359-8 $1\frac{5}{18}$

359-9 $1\frac{1}{4}$

359-10 $\frac{20}{51}$

359-11 $6\frac{10}{11}$

359-12 $49\frac{3}{5}$

359-13 $8\frac{9}{10}$

359-14 $3\frac{7}{15}$

359-15 $8\frac{3}{4}$

359-16 $2\frac{3}{4}$

359-17 $\frac{9}{14}$

359-18 $14\frac{7}{15}$

Lesson 83

360-A Yes * $\frac{0}{5}$ * $\frac{2}{5}$ * Yes * The numeral $\frac{0}{5}$ names the same rational number as the numeral 0.

360-B Yes * $\frac{5}{7}$ * $2\frac{1}{2}$ * 8

360-C The universe for c is N. The universe for b and d is C. * Yes

360-D Yes * The sum property of rational numbers

360-E 0 * The natural number * Yes

360-F For each replacement of the variables:
$\frac{0}{b} + \frac{c}{d} = \frac{0 \cdot d + bc}{bd}$.
The sum property of addition of rational numbers.
$\frac{0 \cdot d + bc}{bd} = \frac{0 + bc}{bd}$.
The zero property of multiplication of natural numbers.
$\frac{0}{b} + \frac{c}{d} = \frac{0 + bc}{bd}$.
Transitive property of equality.

$\frac{0 + bc}{bd} = \frac{bc}{bd}$.
Identity-element property of addition of natural numbers.
$\frac{0}{b} + \frac{c}{d} = \frac{bc}{bd}$.
Transitive property of equality

360-G The reduction property of rational numbers

360-H For each replacement of the variables, you obtain a true statement from $\frac{0}{b} + \frac{c}{d} = \frac{bc}{bd}$ and $\frac{bc}{bd} = \frac{c}{d}$. Therefore, for each replacement of the variables, $\frac{0}{b} + \frac{c}{d} = \frac{c}{d}$ because of the transitive property of equality.

361-I For the statement expressed by sentence H, x was replaced by $\frac{4}{5}$. For the statement expressed by sentence I, x was replaced by $\frac{1}{4}$. For the statement expressed by sentence J, x was replaced by $8\frac{1}{3}$. * Yes * The identity-element property of addition of rational numbers

361-J Zero is the only number that can be added to a given rational number to obtain the given rational number.

361-K $x + 0 = x$. * Yes * $0 + x = x$ is the identity-element property of addition. Since $0 + x$ is the same as $x + 0$, $x + 0 = x$ is a property of addition.

361-L $x - x = 0$ is a property because $x - x = 0$ also means $x = x + 0$.
$x - 0 = x$ is a property because $x - 0 = x$ also means $x = 0 + x$.
$0 - x = x$ is not a property.

361-A Yes * Yes * Yes

361-B To obtain each of the products $\frac{9}{10}$, $3\frac{1}{2}$, and 6, you use the definition of the product of two rational numbers. * Yes

361-C The definition of the product of two rational numbers

361-D Yes * Yes * The identity-element property of multiplication

361-E Yes

361-F For each replacement of the variables, $\frac{1}{1} \times \frac{c}{d} = \frac{1 \times c}{1 \times d}$ and $\frac{1 \times c}{1 \times d} = \frac{c}{d}$. Therefore, for each replacement of the variables, $\frac{1}{1} \times \frac{c}{d} = \frac{c}{d}$ because of the transitive property of equality

361-G Yes * Yes * Yes * Yes * Yes

362-H For the statement expressed by sentence T, x was replaced by $\frac{5}{6}$. For the statement expressed by sentence U, x was replaced by $\frac{10}{13}$. For the statement expressed by sentence V, x was replaced by 1. * Yes * The identity-element property of multiplication of rational numbers

362-I Yes * Yes * Each of the numerals $\frac{2}{2}$ and $\frac{8}{8}$ names the same rational number as the numeral 1.

362-J $x \cdot 1 = x$. * Yes * $1 \cdot x = x$ is the identity-element property of multiplication. Since $1 \cdot x$ is the same as $x \cdot 1$, $x \cdot 1 = x$ is a property of multiplication.

362-K Yes * 0

362-L Yes * For each replacement of the variables: $\frac{0}{b} \cdot \frac{c}{d} = \frac{0 \cdot c}{bd}$.
The definition of product of two rational numbers.
$\frac{0 \cdot c}{bd} = \frac{0}{bd}$.
The zero property of multiplication of natural numbers.
$\frac{0}{b'} \cdot \frac{c}{d} = \frac{0}{bd}$.
The transitive property of equality

362-M Yes * Regardless of the replacements, if the universe for b and d is C, $\frac{0}{bd}$ is the same as the rational number 0.

362-N Yes * $0 \cdot x = 0$ is the zero property of multiplication of rational numbers. Since $0 \cdot x$ is the same as $x \cdot 0$, $x \cdot 0 = 0$ is a property of multiplication.

On your own

362-1 Produces a false statement except when x is replaced by 0

362-2 Produces a true statement

362-3 Produces a false statement except when x is replaced by 0

362-4 Produces a true statement

362-5 Produces a false statement

362-6 Produces a false statement except when x is replaced by 0

362-7 Produces a false statement except when x is replaced by 0

362-8 Produces a false statement except when x is replaced by 0

362-9 Produces a true statement

362-10 Produces a false statement except when x is replaced by 0

362-11 Produces a true statement

362-12 Produces a true statement

362-13 Produces a true statement

362-14 Produces a false statement

362-15 True statement

362-16 False statement

362-17 True statement

362-18 True statement

362-19 False statement

362-20 True statement

362-21 True statement

362-22 True statement

362-23 False statement

362-24 False statement

362-25 False statement

362-26 True statement

362-27 True statement

362-28 True statement

362-29 True statement

362-30 True statement

Lesson 84

363-A Yes * From the definition of the product of two rational numbers and the reduction property of rational numbers

363-B Yes * Yes

363-C The product of $\frac{12}{5}$ and $\frac{5}{12}$ is 1. * $\frac{5}{12}$

363-D Yes * The product of $4\frac{7}{2}$ and $\frac{2}{47}$ is 1. * Yes

363-E Yes * Yes * Since the product of $\frac{6}{10}$ and $\frac{10}{6}$ is 1, each of the numbers $\frac{6}{10}$ and $\frac{10}{6}$ is the reciprocal of the other.

363-F Yes * Yes

363-G 12 * Yes * 5

363-H $\frac{7}{4}$

363-I $\frac{9}{100}$

363-J $\frac{23}{16}$

363-K $\frac{20}{5}$

363-L $\frac{8}{3}$

363-M Yes. $2\frac{2}{3}$ names the same rational number as $\frac{8}{3}$. The product of $\frac{8}{3}$ and $\frac{3}{8}$ is the same as $\frac{24}{24}$, or 1. * Yes. The product of $\frac{8}{3}$ and $\frac{3}{8}$ is 1. * Yes. $2\frac{2}{3}$ is the same as $\frac{8}{3}$.

363-N Yes. The numeral 5 names the same rational number as the numeral $\frac{5}{1}$. The product of $\frac{5}{1}$ and $\frac{1}{5}$ is 1. * Yes. Since 5 and $\frac{1}{5}$ are reciprocals, each of the numbers 5 and $\frac{1}{5}$ is the reciprocal of the other. * Yes * Yes. $\frac{2}{10}$ is the same as $\frac{1}{5}$.

363-O 0 * Because of the zero property of multiplication of rational numbers

363-P No. Since the product of 0 and any rational number is 0, the rational number 0 does not have a reciprocal. * No. Since 0 does not have a reciprocal, 0 is not the reciprocal of any number.

364-A $\frac{a}{b} \cdot \frac{b}{a} = 1$. * Yes

364-B For the statement expressed by sentence H, a was replaced by 11 and b by 12. For the statement expressed by sentence I, a was replaced by 5 and b by 20. * Yes

364-C $\frac{5}{9} * \frac{12}{11} * \frac{5}{20}$, or $\frac{1}{4}$

364-D Yes. Because of the definition of the product of two rational numbers * Yes. Because of the reduction property of rational numbers * Yes. Because of the transitive property of equality

364-E $\frac{b}{a} * \frac{a}{b}$

364-F Yes * $\frac{6}{10}$, or $\frac{3}{5} * \frac{10}{6}$, or $\frac{5}{3}$

364-G Yes * The well-defined property of multiplication of rational numbers

364-H Yes * Yes. $\frac{3}{5}$ is the same rational number as $\frac{6}{10}$. The product of $\frac{3}{5}$ and $\frac{10}{6}$ is 1. * Yes. Since $\frac{3}{5}$ and $\frac{10}{6}$ are reciprocals, each of the numbers $\frac{3}{5}$ and $\frac{10}{6}$ is a reciprocal of the other.

364-I Yes * Because of the well-defined property of multiplication of rational numbers

364-J Yes * Yes. The product of $\frac{6}{10}$ and $\frac{5}{3}$ is 1.

364-K $\frac{5}{3}$, $\frac{10}{6}$, $\frac{15}{9}$, and $\frac{20}{12}$ * One. Equivalent fractions indicate the same rational number.

364-L $\frac{1}{1}$ * Because $\frac{1}{1} \times \frac{1}{1} = \frac{1}{1}$.

On your own

364-1 $\frac{5}{2}$ and $\frac{2}{5}$ are reciprocals of each other because $\frac{5}{2} \times \frac{2}{5} = 1$.

364-2 $\frac{1}{12}$ and 12 are reciprocals of each other because $\frac{1}{12} \times 12 = 1$.

364-3 $\frac{19}{2}$ and $9\frac{1}{2}$ are not reciprocals of each other because the product of $\frac{19}{2}$ and $9\frac{1}{2}$ is not 1.

364-4 $\frac{4}{17}$ and $\frac{71}{4}$ are not reciprocals of each other be-

cause the product of $\frac{4}{17}$ and $\frac{7}{4}$ is not 1.

364-5 $5\frac{3}{4}$ and $5\frac{4}{3}$ are not reciprocals of each other because the product of $5\frac{3}{4}$ and $5\frac{4}{3}$ is not 1.

364-6 $6\frac{2}{3}$ and $\frac{3}{20}$ are reciprocals of each other because $6\frac{2}{3} \times \frac{3}{20} = 1$.

364-7 $\frac{23}{12}$ and $\frac{21}{32}$ are not reciprocals of each other because the product of $\frac{23}{12}$ and $\frac{21}{32}$ is not 1.

364-8 $\frac{7}{3}$ and $\frac{21}{9}$ are not reciprocals of each other because the product of $\frac{7}{3}$ and $\frac{21}{9}$ is not 1.

364-9 $\frac{5}{5}$ is the reciprocal of itself because $\frac{5}{5} \times \frac{5}{5} = 1$.

364-10 $\frac{12}{10}$ and $\frac{5}{6}$ are reciprocals of each other because $\frac{12}{10} \times \frac{5}{6} = 1$.

364-11 $\frac{0}{3}$ and $\frac{0}{4}$ are not reciprocals of each other because the product of $\frac{0}{3}$ and $\frac{0}{4}$ is not 1.

364-12 $\frac{9}{10}$ and $1\frac{1}{9}$ are reciprocals of each other because $\frac{9}{10} \times 1\frac{1}{9} = 1$.

364-13 $\frac{3}{14}$

364-14 $\frac{8}{7}$

364-15 $\frac{2}{15}$

364-16 $\frac{1}{21}$

364-17 $\frac{1895}{652}$, or $2\frac{591}{652}$

364-18 $\frac{3}{14}$

364-19 $\frac{1}{327}$

364-20 $\frac{9}{64}$

364-21 $\frac{61}{27}$, or $2\frac{7}{27}$

364-22 100

364-23 $\frac{12}{6}$, or 2

364-24 $\frac{8}{25}$

364-25 $\frac{1}{5000}$

364-26 $\frac{2}{21}$

364-27 9

365-28 $1\frac{3}{10} * \frac{10}{13}$

365-29 $1\frac{239}{72} * \frac{72}{101}$

365-30 $8\frac{3}{10} * \frac{10}{83}$

365-31 $4\frac{10}{21} * \frac{21}{94}$

365-32 $\frac{3}{7} * 2\frac{1}{3}$

365-33 $10\frac{3}{4} * \frac{4}{43}$

365-34 $\frac{8}{15} * 1\frac{7}{8}$

365-35 $6\frac{3}{7} * \frac{7}{45}$

365-36 $1\frac{1}{27} * \frac{27}{28}$

365-37 $\frac{13}{27} * 2\frac{1}{13}$

365-38 $\frac{3}{250} * 83\frac{1}{3}$

365-39 $\frac{2}{17} * 8\frac{1}{2}$

365-40 $4\frac{8}{11} * \frac{11}{32}$

365-41 $7\frac{1}{2} * \frac{2}{15}$

365-42 $1 * 1$

365-43 $87 * \frac{1}{87}$

Keeping Skilful

365-1 62

365-2 4258

365-3 81,696

365-4 88,287

365-5 4999

365-6 118,052

365-7 134,460

365-8 51,158

365-9 47, remainder 25

365-10 94, remainder 0

365-11 403, remainder 0

365-12 816, remainder 61

365-13 49, remainder 10

365-14 81, remainder 0

365-15 862, remainder 110

365-16 603, remainder 0

365-17 {91}

365-18 {120}

365-19 {336}

365-20 {4150}

365-21 {46}

365-22 {93}

365-23 {1, 2, 3, ..., 431}

365-24 {6}

365-25 {438, 439, 440, ...}

365-26 {9}

365-27 $2\frac{1}{6}$

365-28 $1\frac{4}{5}$

365-29 $\frac{1}{3}$

365-30 $\frac{5}{12}$

365-31 $\frac{18}{35}$

365-32 $4\frac{1}{2}$

365-33 $\frac{5}{8}$

365-34 $14\frac{1}{4}$

365-35 $6\frac{8}{9}$

365-36 $4\frac{3}{8}$

365-37 $4\frac{4}{21}$

365-38 $4\frac{14}{15}$

365-39 $3\frac{7}{12}$

365-40 $7\frac{7}{12}$

365-41 $78\frac{1}{5}$

365-42 $18\frac{11}{24}$

365-43 $18\frac{11}{21}$

365-44 $1\frac{17}{18}$

Lesson 85

365-A 3 * Yes

365-B Yes * { }

365-C No * If the set of natural numbers is closed under division, then any two natural numbers have a quotient that is a natural number. You know that when you divide the natural number 80 by the natural number 17, you do not get a natural number.

365-D $\frac{3}{4}$

365-E Yes

366-F $\frac{4}{3}$

366-G Yes * The associative property

366-H Yes * 1 * Yes

366-I Yes * Because of the identity-element property of multiplication of rational numbers

366-J Yes * The transitive property of equality

366-K Yes * $\frac{2}{3} = y \cdot \frac{3}{4}$ has the same solution set as $\frac{2}{3} \div \frac{3}{4} = y$ because $\frac{2}{3} \div \frac{3}{4} = y$ also means $\frac{2}{3} = y \cdot \frac{3}{4}$.

367-L No * Since 0 does not have a reciprocal, there

is no quotient of a rational number and 0.

367-H $\frac{5}{6} \div \frac{1}{8} = x$.
$\frac{5}{6} = x \cdot \frac{1}{8}$.
$\frac{5}{6} \times \frac{8}{1} = (x \cdot \frac{1}{8})\frac{8}{1}$.
$\frac{5}{6} \times \frac{8}{1} = x(\frac{1}{8} \cdot \frac{8}{1})$.
$\frac{5}{6} \times \frac{8}{1} = x \cdot 1$.
$\frac{5}{6} \times \frac{8}{1} = x$.
Since the conditions expressed above have the same solution set,
$\frac{5}{6} \div \frac{1}{8} = \frac{5}{6} \times \frac{8}{1}$.

367-N $\frac{a}{b} \div \frac{c}{d} = y$.
$\frac{a}{b} = y \cdot \frac{c}{d}$.
$\frac{a}{b} \times \frac{d}{c} = (y \cdot \frac{c}{d})\frac{d}{c}$.
$\frac{a}{b} \times \frac{d}{c} = y(\frac{c}{d} \cdot \frac{d}{c})$.
$\frac{a}{b} \times \frac{d}{c} = y \cdot 1$.
$\frac{a}{b} \times \frac{d}{c} = y$.
Therefore,
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

367-O Yes * 2 x $\frac{5}{4}$, or $2\frac{1}{2}$

367-P $\frac{1}{4}$ * Yes * $\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$.

367-Q $6\frac{2}{3}$

367-R 5

367-S 0

367-T $8\frac{3}{4}$

367-U 1

367-V $\frac{1}{81}$

367-W 1

367-X $1\frac{1}{6}$

367-Y 2

367-A The quotient of any two non-zero rational numbers is always a rational number.

367-B You know that the quotient of any two non-zero rational numbers is the same as the product of the dividend and the reciprocal of the divisor. Since multiplication of rational numbers is closed, so is division of non-zero rational numbers.

367-C Yes

367-D Yes * Yes * Yes * Both $\frac{3}{8} \div \frac{1}{4}$ and $\frac{6}{16} \div \frac{3}{12}$ are the same as $\frac{3}{2}$.

367-E Yes

367-F $\frac{12}{32}$ * Yes

367-G The quotient of two rational numbers remains the same when either the dividend or the divisor, or both, are replaced by equal rational numbers.

367-H For each replacement of the variables:
$\frac{a}{b} \div 1 = \frac{a}{b} \times 1$.
The quotient property of rational numbers.
$\frac{a}{b} \times 1 = \frac{a}{b}$.
The identity-element property of multiplication of rational numbers.
$\frac{a}{b} \div 1 = \frac{a}{b}$.
The transitive property of equality * The quotient of any given rational number and 1 is the given rational number.

367-I For each replacement of the variables:
$0 \div \frac{a}{b} = 0 \times \frac{b}{a}$.
The quotient property of rational numbers.
$0 \times \frac{b}{a} = 0$.
The zero property of multiplication of rational numbers.
$0 \div \frac{a}{b} = 0$.
The transitive property of equality

368-J (Column 1) Division of rational numbers does not have the commutative property. The number that satisfies $\frac{7}{8} \div \frac{1}{3} = x$ is $2\frac{5}{8}$. The number that satisfies $\frac{1}{3} \div \frac{7}{8} = y$ is $\frac{8}{21}$. Therefore, $\frac{7}{8} \div \frac{1}{3}$ is not the same as $\frac{1}{3} \div \frac{7}{8}$.

368-K (Column 1) Division of rational numbers does not have the associative property. The number that satisfies $(\frac{4}{5} \div \frac{2}{3}) \div \frac{1}{8} = m$ is $9\frac{3}{5}$. The number that satisfies $\frac{4}{5} \div (\frac{2}{3} \div \frac{1}{8}) = n$ is $\frac{3}{20}$. Therefore, $(\frac{4}{5} \div \frac{2}{3}) \div \frac{1}{8}$ is not the same as $\frac{4}{5} \div (\frac{2}{3} \div \frac{1}{8})$.

368-A Yes * Yes

368-B For each replacement of the variables:
$\frac{a}{1} \div \frac{b}{1} = \frac{a}{1} \times \frac{1}{b}$.
The quotient property of rational numbers.
$\frac{a}{1} \times \frac{1}{b} = \frac{a \cdot 1}{1 \cdot b}$.
The definition of the product of two rational numbers.
$\frac{a}{1} \div \frac{b}{1} = \frac{a \cdot 1}{1 \cdot b}$.
The transitive property of equality.
$\frac{a \cdot 1}{1 \cdot b} = \frac{a}{b}$.
The identity-element property of multiplication of natural numbers.
$\frac{a}{1} \div \frac{b}{1} = \frac{a}{b}$.
The transitive property of equality

368-C Yes

368-D 17 is the same rational number as $\frac{17}{1}$, and 3 is the same rational number as $\frac{3}{1}$. Therefore, since sentence Q in D6 expresses a true statement,

so does sentence R.

368-E Yes

368-F Yes * The transitive property of equality and the true statements expressed by sentences R and S in D6

368-G $3\frac{3}{4}$

368-H $2\frac{2}{5}$

368-I $7\frac{2}{3}$

368-J (Column 2) $5\frac{1}{4}$

368-K (Column 2) $3\frac{4}{5}$

368-L $1\frac{1}{8}$

368-M $2\frac{7}{10}$

368-N $7\frac{1}{2}$

368-O $6\frac{5}{16}$

368-P $2\frac{9}{20}$

368-Q Yes * Yes

368-R Yes * $1/\frac{4}{5}$ means $1 \div \frac{4}{5}$ and $1 \div \frac{4}{5}$ is the same as $1 \times \frac{5}{4}$, or $\frac{5}{4}$.

368-S Yes * Yes

368-T Yes * Yes * Yes

On your own

369-1 $3\frac{8}{9}$

369-2 $3\frac{1}{2}$

369-3 $22\frac{1}{2}$

369-4 1

369-5 $2\frac{1}{12}$

369-6 0

369-7 $\frac{35}{36}$

369-8 $\frac{1}{5}$

369-9 $\frac{25}{36}$

369-10 $10\frac{1}{2}$

369-11 $7\frac{1}{3}$

369-12 1

369-13 $1\frac{1}{2}$

369-14 1

369-15 0

369-16 $5\frac{2}{5}$

369-17 $4\frac{2}{3}$

369-18 $1\frac{1}{2}$

369-19 $4\frac{2}{5}$

369-20 $1\frac{3}{5}$

369-21 $9\frac{3}{4}$

369-22 $1\frac{2}{11}$

369-23 $6\frac{3}{7}$

369-24 $1\frac{1}{2}$

369-25 $8\frac{1}{3}$

369-26 False statement

369-27 True statement

369-28 True statement

369-29 False statement

369-30 False statement

369-31 True statement

369-32 False statement

369-33 True statement

369-34 False statement

Checking up

Test 86

369-1 $\frac{11}{15}$

369-2 $\frac{9}{5}$

369-3 $\frac{5}{42}$

369-4 $\frac{1}{14}$

369-5 $\frac{70}{1}$, or 70

369-6 $\frac{10}{3}$

369-7 1

369-8 0

369-9 $\frac{2}{3}$

369-10 Identity-element property of addition

369-11 Commutative property of multiplication

369-12 Associative property of addition

369-13 Distributive property of multiplication over addition

369-14 Identity-element property of multiplication

369-15 Commutative property of addition

369-16 Associative property of multiplication

369-17 $m = n$.

Test 87

369-18 $8\frac{2}{3}$

369-19 $8\frac{11}{12}$

369-20 $1\frac{13}{14}$

369-21 $9\frac{5}{6}$

369-22 $1\frac{1}{5}$

369-23 $\frac{1}{60}$

369-24 $10\frac{1}{2}$

369-25 $15\frac{3}{7}$

Lesson 86

370-A Yes * The first power and second power

370-B $4(10^1) + 8(1)$ * $8(10^4) + 6(10^3) + 7(10^2) + 2(10^1) + 8(1)$

370-C Yes * 10

370-D $\frac{500}{1000}$, $\frac{60}{1000}$, and $\frac{6}{1000}$ * Yes

370-E Replace $\frac{500}{1000}$ by the equal rational number $\frac{5}{10}$ and $\frac{60}{1000}$ by the equal rational number $\frac{6}{100}$. * 10

370-F The well-defined property of addition of rational numbers

370-G The numeral 10^1 names the same number as the numeral 10. The numeral 10^2 names the same number as the numeral 100. The numeral 10^3 names the same number as the numeral 1000.

370-H Yes * The symbol $5(\frac{1}{10^1})$ * Yes * Yes

370-I Yes * Yes * The first power * The second power * The third power

370-J $\frac{1}{10^2}$ * $\frac{1}{10^3}$ * Yes

371-K The second power * The first power * Yes

371-L Yes * 1

371-M Yes * $\frac{1}{10^1}$

371-N Yes * 6

371-O Yes

371-P 9 and 7 * 10

371-Q 5, 6, and 4 * Each of these digits is associated with the reciprocal of a power of 10. Powers of 10, like 10^1, 10^2,

and 10^3, have reciprocals that are less than 1.

371-R The digit 5 is in the tenths' position. The digit 6 is in the hundredths' position. The digit 4 is in the thousandths' position.

371-S Nine hundred seventy-two and five hundred sixty-four thousandths

372-T Fifty-seven hundredths

372-U Ninety-two and six tenths

372-V Forty-two and five thousandths

372-W Thirty-nine hundredths

372-X Nine thousand seven hundred forty-two and eighty-one hundredths

372-Y Ten thousandths

372-Z Three thousand three hundred thirty and thirty-three hundredths

372-A $\frac{9}{10}$

372-B .9

372-C Yes * $\frac{1}{10^1}$ * $\frac{1}{10^4}$

372-D Yes

372-E The numeral 1000 * The numeral .1

372-F Yes * Yes * The numeral .01 * The numeral .001

372-G In a decimal, the fourth digit to the right of the decimal point is associated with $\frac{1}{10000}$, or $\frac{1}{10^4}$. Therefore, the decimal .0001 names the rational number $1(\frac{1}{10^4})$, or $\frac{1}{10000}$. * Yes

372-H 6 * 5 * Yes * Yes

372-I 3 * Yes * Yes

372-J 4 * Yes * Yes

372-K 2 * Yes

372-L 9 * Yes * Yes

373-M Thousands

373-N Immediately to the left of the thousands' period is the millions' period. Immediately to the left of the millions' period is the billions' period.

373-O Ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, ten millions, hundred millions, billions, ten billions, hundred billions

373-P Tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths

373-Q Five and three hundred forty-six thousand eight hundred ninety-two millionths

373-R Ten-millionths * Hundred millionths * Billionths

373-S Six hundred ninety-eight thousand five hundred ten and thirty-four thousand two hundred ninety-seven hundred-thousandths

373-T Ninety-two thousand eight hundred forty and sixty-two thousand eleven hundred-thousandths

373-U Two hundred forty-three and twenty-eight thousand four hundred fifty-one billionths

373-V Seventy-three thousand six and ten thousand one hundred one millionths

373-A $\frac{1}{10^1}$ * $\frac{1}{10^2}$ * Yes * Yes

373-B Yes * .425 = $\frac{425}{1000}$. * .0217 = $\frac{217}{10000}$. * .003 = $\frac{3}{1000}$. * .76511 = $\frac{76511}{100000}$.

373-C Yes * Yes * Yes

373-D Because of the definition of the sum of two rational numbers * Yes

373-E Yes * Yes * Yes

374-F Because of the definition of the sum of two rational numbers * Yes

374-G No * Yes. 100 hundredths is the same as 1. Since 106 hundredths is greater than 100 hundredths, 106 hundredths is greater than 1. * Because of the definition of the sum of two rational numbers * Yes

374-H Yes * Yes

374-I .035

374-J .99

374-K 2.058

374-L .172

374-M 6.48

374-N 9.0

374-O 27.3

374-P .0007

374-Q 45.0034

374-R $\frac{4}{10}$

374-S $\frac{500}{100}$

374-T $\frac{2}{10000}$

374-U $\frac{63}{10}$

374-V $\frac{45}{100000}$

On your own

374-1 .27

374-2 802.008

374-3 4001.0005

374-4 35.06

374-5 576.2054

374-6 Four and six tenths

374-7 Seven thousandths

374-8 Seventy and five hundred one ten-thousandths

374-9 Two thousand seven hundred eighty-nine and fifty-six thousand three hundred fifty-eight

hundred-thousandths

374-10 $3(10^1) + 3(1) + 0(\frac{1}{10^1}) + 0(\frac{1}{10^2}) + 6(\frac{1}{10^3}) + 5(\frac{1}{10^4}) + 4(\frac{1}{10^5})$

374-11 $9(10^2) + 8(10^1) + 2(1) + 0(\frac{1}{10^1}) + 8(\frac{1}{10^2}) + 0(\frac{1}{10^3}) + 8(\frac{1}{10^4}) + 1(\frac{1}{10^5}) + 2(\frac{1}{10^6})$

374-12 $6(10^3) + 0(10^2) + 0(10^1) + 5(1) + 4(\frac{1}{10^1}) + 7(\frac{1}{10^2}) + 0(\frac{1}{10^3}) + 8(\frac{1}{10^4})$

374-13 $\frac{47}{1000}$

374-14 $2\frac{36}{100}$

374-15 $\frac{721}{10000}$

374-16 $4\frac{9}{10000}$

374-17 $356\frac{7}{10}$

374-18 $\frac{6383}{10000}$

374-19 $\frac{4080}{10000}$

374-20 $34\frac{5}{100}$

374-21 $7\frac{1}{1000}$

374-22 $\frac{41}{100000}$

374-23 $\frac{9}{1000000}$

374-24 $32\frac{9710}{10000}$

374-25 .0015

374-26 .096

374-27 22.5

374-28 1.007

374-29 .00205

374-30 100.00

374-31 .0376

374-32 .4879

Keeping Skilful

374-1 {(1054,42)}

374-2 {21/9}

374-3 {(8,11)}

374-4 {3/2, 6/4, 9/6, 12/8, 15/10, 18/12, 21/14}

374-5 {(1,8), (2,7), (3,6)}

374-6 {(2808,736)}

374-7 {13/9, 26/18, 39/27}

374-8 {(123,1), (123,2), (123,3), (123,4), (123,5), (123,6), (123,7)}

374-9 {21/49}

374-10 {(31,1236)}

374-11 {(1,17), (2,17), (3,17), (4,17), (5,17)}

374-12 $1\frac{1}{57}$

374-13 $1\frac{1}{6}$

374-14 $6\frac{3}{5}$

374-15 $2\frac{2}{9}$

374-16 $40\frac{1}{2}$

374-17 $\frac{4}{7}$

374-18 $15\frac{1}{9}$

374-19 $\frac{2}{15}$

374-20 $2\frac{1}{30}$

374-21 $2\frac{1}{20}$

374-22 $8\frac{3}{4}$

374-23 $2\frac{13}{18}$

374-24 $\frac{10}{11}$

374-25 $\frac{16}{45}$

374-26 $4\frac{3}{4}$

374-27 $25\frac{2}{3}$

374-28 $\frac{8}{15}$

374-29 $1\frac{1}{2}$

Lesson 87

375-A 2.36 and .85 * Yes * Yes * Yes

375-B Yes * The definition of the sum of two rational numbers

375-C Yes * Find the sum of the numerators of $\frac{236}{100}$ and $\frac{85}{100}$.

375-D Yes * 3.21

375-E 5.604 and 3.972 * Yes * Yes * Yes

375-F Yes * The definition of the difference of two rational numbers

375-G Subtract the numerator of $\frac{3972}{1000}$ from the numerator of $\frac{5604}{1000}$.

375-H Yes * 1.632

375-I 321

375-J Yes * Yes * Yes

375-K 1632

376-L Yes * Numbers in thousandths * Yes

376-M 3.8 * .62

376-N Yes * 442 * Yes * Yes

376-O 4.320

376-P 2663 * Yes * Yes

376-Q 1.664 * .926

376-R 42.81 * 41.81

376-S 12.653 * 1.127

376-T 37.957 * 36.123

376-U 3.507 * .289

376-V 7.631 * 6.369

376-A 4.8 and .015 * Yes * Yes * Yes

376-B Yes * The definition of the product of two rational numbers

376-C Yes * Find the product of the numerators of $\frac{48}{10}$ and $\frac{15}{1000}$. * Find the product of the denominators of $\frac{48}{10}$ and $\frac{15}{1000}$.

376-D Yes * .0720

377-E 720

377-F Yes

377-G Yes * The product of the denominators of $\frac{48}{10}$ and $\frac{15}{1000}$ is ten thousand.

377-H 1 * 3

377-I 4 * 4

377-J 3 * 4 * 7 * The product of the denominators of $\frac{12784}{1000}$ and $\frac{509}{10000}$ is ten million. Therefore, the product will be in ten-millionths.

377-K .45

377-L .168

377-M 1.61424

377-N 30.7125

377-O 325.710

377-P .000315

377-Q 2.53 * Yes * 10^1 is the same as 10.

377-R No * None * 1

377-S 49800 * Yes

377-T Yes * Yes

377-U The decimal 49,800. has two more digits to the left of the decimal point

than the decimal 498.00.

377-V 3654.3 * 3654.3 * The decimals 3654.3 and 3.6543 differ only in the location of the decimal point. The decimal 3654.3 has three more digits to the left of the decimal point than the decimal 3.6543.

377-W 72,400 * 6000

377-X The fifth power * The fourth power * The third power

378-A Multiply .45 by 10^1. * Multiply 1.5 by 10^1.

378-B N * C * $\frac{d}{c}$ * Yes

378-C C * Yes * $\frac{mc}{nd}$ * Yes * The quotient property of rational numbers

378-D Yes * For each replacement of the variables, $\frac{ma}{nb} \cdot \frac{nd}{mc} = \frac{ad}{bc}$ because of the definition of the product of two rational numbers and the reduction property of rational numbers.

378-E Yes * $\frac{ad}{bc}$ * Yes

378-F Yes * The quotient of .45 and 1.5 is the same as the quotient of 4.5 and 15.

378-G $\frac{45}{10}$ * $\frac{15}{1}$ * Yes

378-H $\frac{1}{15}$ * Yes * Quotient property of rational numbers

378-I $\frac{3}{10}$ * .3 * Yes

378-J 3

379-K .3 * Yes

379-L Yes * In tenths

379-M 10^2, or 100 * Yes

379-N 153 * 15.3

379-O Yes * Yes

379-P The divisor, 42, is already expressed in ones.

379-Q

42)	98.28	200
	84 00	
	14 28	30
	12 60	
	1 68	4
	1 68	
	0	2.34

* 2.34

379-R 2 * 2 * Yes

379-A Both the dividend and the divisor can be multiplied by the same number without affecting the quotient.

379-B You want to find the quotient in hundredths.

380-C By subtracting 584 (the product of 8 and 73) from 650 * Yes

380-D $36\frac{1}{2}$ * Yes * 66 is greater than $\frac{1}{2}$ times the divisor, or $36\frac{1}{2}$.

380-E 9.59 * 9.59

380-F 10^1, or 10 * 6.3 x 10 = 63. The decimal 63 expresses the divisor in ones.

380-G You want to find the quotient in thousandths.

380-H 4 * 4 is less than $\frac{1}{2}$ times the divisor, or $31\frac{1}{2}$.

380-I 2.832 * 2.832

380-J 10^1, or 10 * Because 5.4 x 10 = 54 and the decimal 54 expresses the divisor in ones * 25.11

54)	25.11	40
	21 60	
	3 51	6
	3 24	
	27	.46

380-K Yes

380-L 10 * Yes

380-M 4.76

381-N Yes * Yes * $\frac{10}{25}$

381-O Yes * Yes * .4

381-P .4

381-Q Yes

381-R Yes * .125

381-S 5.28 * Yes * The decimals 5.28 and 52.8 differ only in the location of the decimal point. The decimal 5.28 has one fewer digits to the left of the decimal point than the decimal 52.8.

381-T .2104 * Yes * The decimal .2104 has two fewer digits to the left of the decimal point than the decimal 21.04.

381-U .000638 * .000638 * Neither of the decimals has digits to the left of the decimal point. However, the decimal .638 has no digits for zero immediately following the decimal point while the decimal .000638 has three digits for zero immediately following the decimal point.

381-V .0097

381-W 248.36 ÷ 10 = 24.836. * 248.36 ÷ 100 = 2.4836. * 248.36 ÷ 1000 = .24836. * 248.36 ÷ 10000 = .024836. * 248.36 ÷ 100000 = .0024836.

On your own

381-1 3312.05

381-2 .0011

381-3 329.607

381-4 12.0009

381-5 1.16

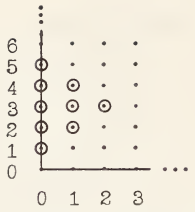
381-6 9.05

381-7 437.162

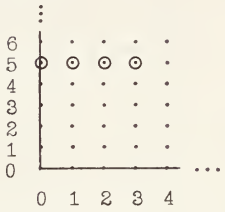
381-8 2.5207
381-9 118.92
381-10 281.2976
381-11 21.55
381-12 1.804
381-13 .534
381-14 .5
381-15 275.5324
381-16 .051
381-17 64.02
381-18 137.7063
381-19 5.32
381-20 32.498
381-21 2780
381-22 113
381-23 32.64
381-24 13.340
381-25 6.1828
381-26 .009063
381-27 4700
381-28 .9396
381-29 6.104
381-30 .660
382-31 2.49
382-32 1735.4
382-33 13
382-34 316.5
382-35 .82
382-36 8.41
382-37 .3333
382-38 .125
382-39 .048
382-40 .25
382-41 .2
382-42 1.25
382-43 .625
382-44 40
382-45 10

Keeping Skilful

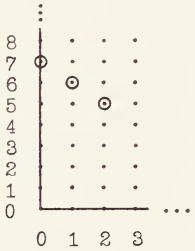
382-1 {(0,1), (0,2), (0,3), (0,4), (0,5), (1,2), (1,3), (1,4), (2,3)}



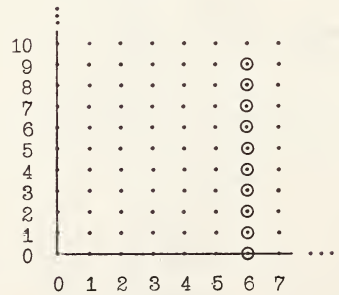
382-2 {(0,5), (1,5), (2,5), (3,5)}



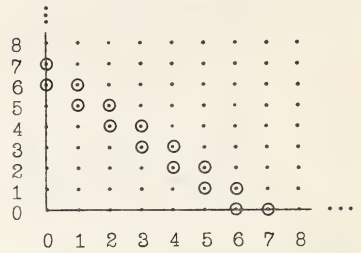
382-3 {(0,7), (1,6), (2,5)}



382-4 {(6,0), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (6,9)}



382-5 {(0,6), (0,7), (1,5), (1,6), (2,4), (2,5), (3,3), (3,4), (4,2), (4,3), (5,1), (5,2), (6,0), (6,1), (7,0)}



382-6 42 * 117
382-7 101 * 1290
382-8 98 * 2401
382-9 127 * 2472
382-10 419 * 18,768
382-11 957 * 59,630
382-12 756 * 35,300
382-13 333 * 6842
382-14 786 * 154,400
382-15 222 * 12,321
382-16 430 * 37,944
382-17 1135 * 303,150
382-18 940 * 172,500
382-19 913 * 87,636
382-20 1426 * 471,120

Lesson 88

382-A .376 is the same as $3(\frac{1}{10^1}) + 7(\frac{1}{10^2}) + 6(\frac{1}{10^3})$. This sum is the same as $\frac{3}{10} + \frac{7}{100} + \frac{6}{1000}$, or $\frac{376}{1000}$.

382-B $\frac{47}{125}$

382-C Yes

382-D Yes. 2.8 is the same as $2(1) + 8(\frac{1}{10^1})$. This sum is the same as $2 + \frac{8}{10}$, or $\frac{28}{10}$. * Yes. $\frac{28}{10} = \frac{14}{5}$. Reduction property of rational numbers. $2.8 = \frac{14}{5}$. Transitive property

382-E First find the fraction whose denominator is a power of 10 that indicates the rational number

named by the decimal. Then find the basic fraction.

382-F $\frac{77}{100}$ (.77 = $\frac{77}{100}$. The fraction $\frac{77}{100}$ is a basic fraction.)

382-G $\frac{11}{20}$ (.55 = $\frac{55}{100}$ and $\frac{55}{100} = \frac{11}{20}$.)

382-H $\frac{18}{5}$ (3.6 = $\frac{36}{10}$ and $\frac{36}{10} = \frac{18}{5}$.)

382-I $\frac{13}{1000}$

382-J $\frac{643}{25}$

382-K $\frac{1}{400}$

382-L $\frac{7}{8}$

382-M $\frac{191}{500}$

382-N 1.00

382-O .25 * Yes

382-P Yes

383-Q 3.000

383-R .375 * Yes

383-S .65

383-T 1.4

383-U .125

383-V .3125

383-W 6.25

383-X 2.5

383-A Yes

383-B 7.0 * 6 * 4

383-C 7.00 * 3 * 7

384-D The digit zero was inserted after the numerals 7.00, 660, 40, 33, 60, and 3.

384-E 70

384-F Yes

384-G Yes * In the second step, you divide 40 by 11 to find the second digit in the decimal for the quotient. In the fourth step, you also divide 40 by 11 to find the fourth digit in the decimal for the quotient. In both steps, you obtain the digit 3 in the quotient and a remainder of 7

384-H The fourth remainder is 7, not 0.

384-I The digits for the quotient that you will obtain in ten steps, starting with the first step, are 6, 3, 6, 3, 6, 3, 6, 3, 6, and 3. * The remainders that you will obtain in ten steps, starting with the first step, are 4, 7, 4, 7, 4, 7, 4, 7, 4, and 7.

384-J No * No * .6364

384-K When you use decimals to divide 7 by 11, the fourth remainder is 7, not 0. This means that the quotient of 7 and 11, or $\frac{7}{11}$, is greater than .6363.

384-L Yes

384-M Yes

384-N 1.00

384-O The digit 1 is obtained in the first step, and the digit 6 is obtained in the second step.

385-P The digits you will obtain in 5 steps, starting with the first step, are 1, 6, 6, 6, and 6.

385-Q Yes * In the second step, you divide 40 by 6 to find the second digit in the decimal for the quotient. In the third step, you also divide 40 by 6 to find the third digit in the decimal for the quotient. In both steps you obtain the digit 6 in the quotient and a remainder of 4.

385-R Since the third step is the same as the second step, the succeeding steps will be the same. This means that a remainder of 4 will be obtained in each step after the first.

385-S 6 * The digit 1 is not part of the repetend.

385-T No

385-U $.\overline{3}$

385-V $.\overline{5}$

385-W $.\overline{428571}$

385-X $.18\overline{3}$

385-Y $.8\overline{3}$

385-Z $.\overline{6}$

385-A Yes * Yes * Yes

385-B .2500000000

385-C Yes. No matter how many times you insert the digit for zero after the digit 5 in .25, you will always obtain a decimal that names the same rational number as .25, or $\frac{1}{4}$.

385-D Yes

385-E First find the basic fraction that indicates the rational number. If the denominator of this fraction has no prime factors other than 2 or 5, then the rational number can be expressed by a terminating decimal.

386-F (Column 1, top) Yes. 4 has no prime factors other than 2.

386-G (Column 1, top) No. 3 has prime factors other than 2 or 5.

386-H (Column 1, top) No. 7 has prime factors other than 2 or 5.

386-I (Column 1, top) Yes. 5 has no prime factors other than 5.

386-J (Column 1, top) No. The basic fraction that indicates $\frac{2}{6}$ is $\frac{1}{3}$. 3 has prime factors other than 2 or 5.

386-K (Column 1, top) Yes. 20 has no prime factors other than 2 and 5.

386-A Yes. .4 is the same as $4(\frac{1}{10})$, which is the same as $\frac{4}{10}$. * Yes. .5 is the same as $5(\frac{1}{10})$, which is the same as $\frac{5}{10}$. * Yes. From definition of "less than" for rational numbers, since 4(10) < 5(10), .4 is less than .5.

386-B Yes

386-C Yes * No

386-D Yes

386-E Yes. The repetend is 0. * Yes

386-F (Column 1, bottom) Yes * No

386-G (Column 1, bottom) $.\overline{5}$ is the same as $.555\overline{5}$. Since .55 is less than $.555\overline{5}$, .55 must be less than $.\overline{5}$.

386-H (Column 1, bottom) If the decimals do not have the same number of digits to the left of the decimal point, then the decimal with the most digits to the left of the decimal point expresses the greater number. The digit for zero is not considered unless it is necessary to write the decimal. In the decimals 0.58, 002.3, 008, and 7.5000, the digits for zero are not necessary. The digits for zero in the decimals 507, .009, 20, 40.08, and .0807, are necessary. If the decimals have the same number of digits to the left of the decimal point, compare corresponding digits in order beginning with those farthest to the left. The first corresponding digits that are different tell you which decimal expresses the greater number.

386-I (Column 1, bottom) True statement

386-J (Column 1, bottom) False statement

386-K (Column 1, bottom) True statement

386-L True statement

386-M True statement

386-N False statement

On your own

386-1 $\frac{12}{25}$

386-2 $\frac{7}{1000}$

386-3 $\frac{67}{50}$

386-4 $\frac{203}{100}$

386-5 $\frac{9}{100}$

386-6 $\frac{39}{5}$

386-7 $\frac{771}{50}$

386-8 $\frac{11}{1250}$

386-9 $\frac{113}{125}$

386-10 $\frac{33}{2500}$

386-11 $\frac{403}{50}$

386-12 $\frac{1}{100000}$

386-13 .5

386-14 .6

386-15 $.\overline{571428}$

386-16 $.\overline{4}$

386-17 .625

386-18 $1.1\overline{6}$

386-19 $1.\overline{09}$

386-20 $.\overline{076923}$

386-21 $.4\overline{6}$

386-22 .04

386-23 $1.\overline{6}$

386-24 .75

386-25 .037, .073, .307, .357, .370, .375

386-26 .4, .406, .4068, .4086, .46, .468

386-27 $.1\overline{6}$, .6, .601, .61, .63, $.\overline{6}$

386-28 .012, .12, $.\overline{12}$, .122, $.1\overline{2}$, .21

386-29 .8 < .75. * False statement

386-30 .875 > $.8\overline{3}$. * True statement

386-31 .36 > .35. * True statement

386-32 $.1\overline{6} < .\overline{27}$. * True statement

386-33 .68 > .6875. * False statement

386-34 $.\overline{5} < .\overline{571428}$. * True statement

386-35 $.\overline{538461} < .\overline{54}$. * True statement

386-36 1.25 > 1.05. * True statement

386-37 .6 > .625. * False statement

386-38 $.\overline{3} < .375$. * True statement

386-39 $.\overline{6} > .6$. * True statement

386-40 .08 < .15. * True statement

Keeping Skilful

387-1 24.05

387-2 .490

387-3 60.96

387-4 .7885

387-5 1.48

387-6 .834

387-7 11.419

387-8 2.785

387-9 39.552
387-10 .0325
387-11 25.292
387-12 .27600
387-13 8.95044
387-14 3.7415
387-15 12.2388
387-16 1.1172
387-17 .86
387-18 7.9
387-19 75
387-20 .713
387-21 .1429
387-22 492.31
387-23 1.262

Checking up

Test 88

388-1 (c) $2(1) + 0(\frac{1}{10^1}) + 3(\frac{1}{10^2}) + 0(\frac{1}{10^3}) + 4(\frac{1}{10^4})$

388-2 (c) 47,832

388-3 (b) 0.52

388-4 (d) $\frac{3}{8}$

388-5 (b) $\frac{1}{10^3}$

388-6 (b) $.\overline{09}$

388-7 (c) $\frac{1}{12500}$

388-8 (c) .000586

Test 89

388-9 3.3109

388-10 38.855

388-11 .65

388-12 84.391

388-13 423.335

388-14 .00528

388-15 .4

388-16 .57

Lesson 89

388-A 0, 1, 2, 3, 4, 5, and 6 * The three small dots tell you that the points in the line associated with natural numbers go on and on.

388-B { } * Yes * Yes

388-C {1, 2, 3, 4} * Yes * Yes

388-D Yes * Yes * Yes

388-E Yes * Yes * 26 * 88

388-F It tells you that the points in the line associated with rational numbers go on and on.

389-G $\frac{1}{2}$ * Yes

389-H No * $\frac{1}{4}$ is greater than 0 and less than $\frac{1}{2}$. * Yes

389-I Yes * Yes * Examples are: $\frac{1}{17}$, $\frac{1}{32}$, and $\frac{1}{100}$.

389-J $4 \cdot 3 < 1 \cdot 15$.

389-K Yes * Since $\frac{5}{8} < \frac{3}{4}$ and $\frac{3}{4} < 1$, you know that $\frac{3}{4}$ is between $\frac{5}{8}$ and 1.

389-L Yes * Yes * $(5 + 9) \div 2$ is the same as 7. Since $5 < 7$ and $7 < 9$ you know that $(5 + 9) \div 2$ is between 5 and 9.

389-M Yes

389-N Yes * Yes * Because $\frac{13}{2} < \frac{32}{4}$ and $\frac{32}{4} < \frac{19}{2}$

389-O $(\frac{13}{2} + \frac{32}{4}) \div 2$ is the same as $\frac{58}{8}$. $\frac{13}{2} < \frac{58}{8}$ and $\frac{58}{8} < \frac{32}{4}$.

389-P $(\frac{32}{4} + \frac{58}{8}) \div 2$ is the same as $\frac{122}{16}$. $\frac{32}{4} < \frac{122}{16}$ and $\frac{122}{16} < \frac{58}{8}$.

389-Q For each replacement of the variables, if $x < y$, then $(x + y) \div 2$ is between x and y.

389-R Divide the sum of $\frac{4}{5}$ and $\frac{9}{7}$ by 2. * $\frac{73}{70}$

389-S No * No

389-T Yes * $0 < \frac{3}{4} \wedge \frac{3}{4} < 1$.

389-U Points $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{7}{8}$ * Yes * Examples are: $\frac{3}{16}$, $\frac{11}{20}$, and $\frac{73}{100}$.

389-V From the density property, there is a rational number r between x and y. From the density property there is a rational number between x and r and another between r and y. These last two numbers are also between x and y. No matter how many numbers you find between x and y, the density property tells you that there are more and more.

389-W Yes. There is an infinite set of numbers between x and y. Each of these numbers is associated with a point that is between point x and point y.

390-A { } * Yes * Yes

390-B No

390-C Yes * Yes * From the density property you know that there is an infinite set of rational numbers between any two given numbers. You also know that no rational number of arithmetic is less than 0.

390-D Yes * No. $\frac{1}{2}$ does not immediately follow 0 because $\frac{1}{4}$ is between 0 and $\frac{1}{2}$.

390-E Yes * $\frac{1}{4}$ does not immediately follow 0 because $\frac{1}{8}$ is between 0 and $\frac{1}{4}$. * $\frac{1}{16} < \frac{1}{8}$ and $\frac{1}{16} > 0$. Therefore, $\frac{1}{8}$ does not immediately follow 0 because $\frac{1}{16}$ is between 0 and $\frac{1}{8}$.

390-F For any two given rational numbers, one number cannot immediately follow the other because the density property tells you that there is another number between the two given rational numbers.

390-G Yes * Yes

390-H No. There is no rational number that immediately follows another rational number. * No. You cannot name the first three members of the set.

390-I 20 * {0, 1, 2, ..., 20}

390-J No * For any rational number you name, the density property tells you that there is another rational number between the number named and $\frac{1}{3}$.

On your own

390-1 $2\frac{1}{2}$, $\frac{7}{4}$, $\frac{23}{52}$

390-2 5, $3\frac{4}{5}$, $\frac{7}{9}$, $\frac{20}{3}$

390-3 $\frac{7}{4}$, 2, $\frac{20}{8}$, $1\frac{2}{3}$

390-4 $\frac{7}{48}$, 1, $\frac{10}{24}$

390-5 $\frac{2}{2}$, $\frac{3}{1}$, $\frac{7}{8}$

390-6 $\frac{2}{5}$, $\frac{1}{2}$, $\frac{12}{21}$

390-7 $6\frac{1}{2}$, 7, $7\frac{1}{2}$

390-8 $3\frac{1}{2}$, 4, $4\frac{1}{2}$

390-9 $\frac{8}{14}$, $\frac{9}{16}$, $\frac{5}{9}$

390-10 0, $\frac{1}{20}$, $\frac{1}{15}$, $\frac{1}{9}$, $\frac{1}{8}$

390-11 $\frac{0}{8}$, $\frac{12}{12}$, $\frac{67}{10}$, $14\frac{1}{2}$, 16

390-12 $\frac{1}{6}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{7}{8}$, $\frac{19}{20}$

390-13 $\frac{7}{3}$, 6, $\frac{3}{2}$, 7, $\frac{22}{3}$

390-14 Finite

390-15 Infinite

390-16 Finite. The empty set is a finite set.

391-17 Finite

391-18 Finite

391-19 Infinite

391-20 Finite

391-21 Infinite

391-22 The least member is 0. * The greatest member is 2.

391-23 The least member is 0. * There is no greatest member.

391-24 The least member is 16. * There is no greatest member.

391-25 The least member is 0. * There is no greatest member.

391-26 There is no least member. * There is no greatest member.

391-27 There is no least member. * There is no greatest member.

391-28 The least member is 3. * The greatest member is 4.

391-29 There is no least member. * There is no greatest member.

391-30 The least member is 4. * The greatest member is 8.

Keeping Skilful

391-1 $9\frac{2}{3}$

391-2 $13\frac{5}{11}$

391-3 $8\frac{4}{9}$

391-4 $34\frac{1}{3}$

391-5 $32\frac{1}{4}$

391-6 $6\frac{11}{13}$

391-7 $26\frac{1}{2}$

391-8 $13\frac{5}{7}$

391-9 $1.\overline{6}$

391-10 3.125

391-11 $.1\overline{6}$

391-12 2.2

391-13 $.\overline{7}$

391-14 .52

391-15 11.75

391-16 $.\overline{09}$

391-17 .625

391-18 .8

391-19 $.\overline{428571}$

391-20 .25

391-21 .0734

391-22 .975

391-23 36.02

391-24 11.31

391-25 3.185

391-26 .787

391-27 13.27

391-28 .2982

391-29 3.8442

391-30 1.95

391-31 .57436

391-32 .3028

391-33 2.363

391-34 1.0314

Lesson 90

391-A Yes

391-B Yes

391-C R_a

391-D Yes

391-E $\{\frac{3}{4}\}$ * Add $\frac{1}{4}$ to $\frac{1}{2}$.

392-F Yes

392-G Yes

392-H R_a

392-I Yes

392-J $\{\frac{1}{6}\}$ * Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

392-K Yes

392-L .31 + .18 = .49 also means .31 = .49 - .18.

392-M R_a

392-N x + .18 = .49 also means x = .49 - .18.

392-O {.31} * Subtract .18 from .49.

392-P Yes

392-Q $\frac{3}{5} - \frac{1}{2} = \frac{1}{10}$ also means $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$.

392-R $\frac{1}{2} + \frac{1}{10} = \frac{3}{5}$ also means $\frac{1}{2} = \frac{3}{5} - \frac{1}{10}$.

392-S R_a

392-T $\frac{3}{5} - x = \frac{1}{10}$ also means $\frac{3}{5} = x + \frac{1}{10}$.

392-U $x + \frac{1}{10} = \frac{3}{5}$ also means $x = \frac{3}{5} - \frac{1}{10}$.

392-V $\{\frac{1}{2}\}$ * Subtract $\frac{1}{10}$ from $\frac{3}{5}$.

392-W $\{\frac{11}{40}\}$ * Subtract $\frac{3}{5}$ from $\frac{7}{8}$.

392-X $\{2\frac{5}{8}\}$ * Subtract $1\frac{3}{8}$ from 4.

392-Y {2.4} * Subtract 2.6 from 5.

392-A $\frac{1}{4} + \frac{3}{8} = \frac{5}{8}$ is a true statement.

393-B No * No

393-C Yes * Yes * No * No

393-D No

393-E Yes * Yes * Yes

393-F Yes

393-G Yes * Only the numbers that are greater than $\frac{1}{4}$ satisfy $x > \frac{1}{4}$. Only the numbers that are greater than $\frac{1}{4}$ satisfy $x + \frac{3}{8} > \frac{5}{8}$. Therefore, $\{x \mid x > \frac{1}{4}\}$ has exactly the same members as $\{x \mid x + \frac{3}{8} > \frac{5}{8}\}$.

393-H Yes

393-I No

393-J No

393-K No * No * No * No

393-L Yes * Yes * Yes * Yes

393-M Yes

393-N Yes

393-O You cannot name the first three members of the set.

393-A R_a

394-B Point $\frac{1}{4}$ * Yes * Yes

394-C Yes

394-D Yes

394-E No

394-F Yes

394-G N * Yes

394-H {0, 1, 2, 3} * Yes * Yes * Yes

394-I Yes * R_a

394-J By the bar below some of the dots in the graph of R_a

394-K No * No

395-A $\frac{2}{3} + x > 1 \wedge \frac{1}{2} + x < 2$. * R_a

395-B Yes

395-C $\{x \mid x < \frac{3}{2}\}$

395-D Yes

395-E Yes * Each set contains those numbers, and only those numbers, that satisfy both $x > \frac{1}{3}$ and $x < \frac{3}{2}$.

395-F No * You cannot name the first or the last member of the set.

395-G Yes

395-H By the bar above some of the dots in the graph of R_a

395-I By the portion of the bars that overlap each other * Yes. There are infinitely many rational numbers between $\frac{1}{3}$ and $\frac{3}{2}$. Each of these numbers is a member of $\{x \mid x > \frac{1}{3} \wedge x < \frac{3}{2}\}$.

395-J R_a

395-K Yes * $\{x \mid x > 4\}$

395-L Yes

395-M Yes * All the rational numbers, and only those rational numbers, that are greater than 4 satisfy both $2 + x > 4$ and $x - 1 > 3$.

395-N All the dots that represent points in the rational-number line associated with numbers greater than 4 * By the portion of the bars that overlap each other

395-O Yes * These solution sets contain the same members and, therefore, their graphs are the same.

396-P $3 - x > 2 \wedge .5 + x > 1.5$. * Z

396-Q Yes * No. 0 is not a member of the universe, which is Z.

396-R $\{x \mid x > 1\}$ * No. A rational number cannot be both greater than 1 and less than 1.

396-S Yes

396-T No. The curved strokes indicate that the dot for point 1 is not in either graph.

396-U { }

On your own

396-1 False statement

396-2 True statement

396-3 True statement

396-4 True statement

396-5 False statement

396-6 False statement

396-7 True statement

396-8 False statement

396-9 True statement

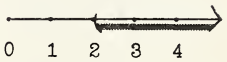
396-10 True statement

396-11 False statement

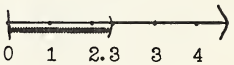
396-12 False statement

396-13 False statement

396-14



396-15



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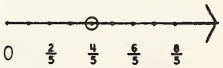
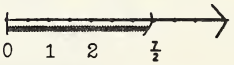
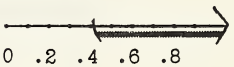


396-17



396-18



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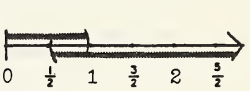

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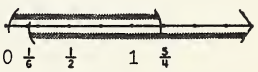

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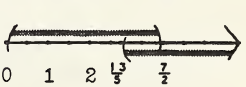

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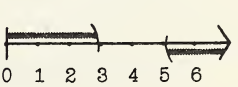
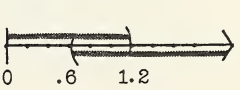
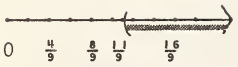


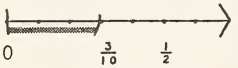
396-23



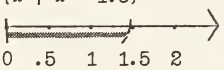
396-24 $\{x \mid x > \frac{11}{9}\}$



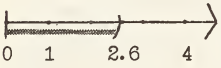
396-25 $\{x \mid x < \frac{3}{10}\}$



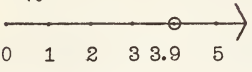
396-26 $\{x \mid x < 1.5\}$



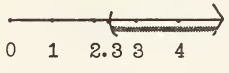
396-27 $\{x \mid x < 2.6\}$



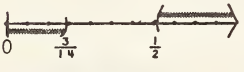
396-28 $\{3\frac{9}{10}\}$



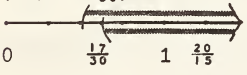
396-29 $\{x \mid x > 2.3\}$



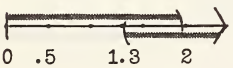
396-30 $\{\ \}$



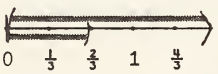
396-31 $\{x \mid x > \frac{17}{30}\}$



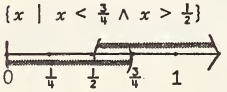
396-32 $\{x \mid x > 1.3 \wedge x < 2\}$



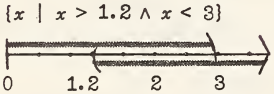
396-33 $\{x \mid x < \frac{4}{3} \wedge x > 0\}$



396-34 $\{x \mid x < \frac{3}{4} \wedge x > \frac{1}{2}\}$



396-35 $\{x \mid x > 1.2 \wedge x < 3\}$



Lesson 91

397-A Yes * The number of yards of oilcloth Mrs. Jones had before she bought the $\frac{1}{2}$ yd.

397-B You must imagine that the $\frac{1}{2}$ yd. of oilcloth Mrs. Jones bought is put with, not taken away from, the oilcloth she already had.

397-C Both $y + \frac{1}{2}$ and $2\frac{1}{4}$ refer to the number of yards of oilcloth Mrs. Jones had in all. Therefore, after replacement of the variable, $y + \frac{1}{2}$ is to be the same number as $2\frac{1}{4}$.

397-D Subtract $\frac{1}{2}$ from $2\frac{1}{4}$. * $1\frac{3}{4}$

397-E $\{1\frac{3}{4}\}$

397-F You must imagine that the money Jerry spent for the present is taken from, not put with, the \$13.50 he had before he bought the present. * Both $13.50 - n$ and 10.91 refer to the amount of money Jerry had left after buying the present. * For the amount of money Jerry spent for the present

397-G Subtract 10.91 from 13.50.

397-H $\{2.59\}$ * Jerry spent \$2.59 for the present.

397-I $1\frac{1}{4} + x$ refers to the number of yards of lace Sue had after she bought some more. From the problem, you know that this number is less than $2\frac{3}{4}$.

397-J You must imagine that the lace Sue bought is put with, not taken away from, the $1\frac{1}{4}$ yd. of lace she already had.

397-K You know that after Sue bought some more lace, she still had less than $2\frac{3}{4}$ yd.

397-L $1\frac{1}{2}$

397-M No

398-N $1\frac{1}{2}$

398-O All those rational numbers, and only those rational numbers, that satisfy $x < 1\frac{1}{2}$ also satisfy $1\frac{1}{4} + x < 2\frac{3}{4}$.

398-P No * You cannot name either the first or the last member of $\{x \mid x < 1\frac{1}{2}\}$.

398-Q No * The universe is Z and 0 is not a member of Z.

398-A s is a variable for the amount of money Ken had before he earned the \$2.75. From the problem, you know this number is greater than 9.10.

398-B You know that when the \$2.75 Ken earned was put with the money he already had, he still had less than \$14.00.

398-C Yes

398-D $s + 2.75 = 14.00$.

398-E Since you know that 11.25 satisfies $s + 2.75 = 14.00$, you also know that each number that satisfies $s + 2.75 < 14.00$ must be less than 11.25.

398-F No * Some members of $\{s \mid s < 11.25\}$ do not satisfy $s > 9.10$.

398-G Yes

398-H $\{s \mid s > 9.10 \wedge s < 11.25\}$ * It is easier to tell from this description just what numbers are members of the solution set.

398-I 10.256 is between 9.10 and 11.25. * 11.0012 is between 9.10 and 11.25.

398-J Yes * 11.24

399-K Yes * Yes

399-L 10.013 and 11.115 do not give sensible answers to the problem.

399-M Yes * For the amount of paint Ralph used

399-N No * Since Ralph did not use all the paint, you know that he used less than $3\frac{1}{4}$ gal.

399-O $3\frac{1}{4} - m < 1 \wedge m < 3\frac{1}{4}$. * $\{m \mid m > 2\frac{1}{4}\}$

399-P No * Some members of $\{m \mid m > 2\frac{1}{4}\}$ do not satisfy $m < 3\frac{1}{4}$.

399-Q Only these numbers satisfy $m < 3\frac{1}{4}$.

399-R $\{m \mid m > 2\frac{1}{4} \wedge m < 3\frac{1}{4}\}$ * Ralph used between $2\frac{1}{4}$ and $3\frac{1}{4}$ gal. of paint.

399-A Yes * Yes

399-B $x > 1\frac{1}{2} \wedge 2\frac{1}{3} + x < 5\frac{2}{3}$.

399-C Yes * $x < 3\frac{1}{3}$ has the same solution set as $2\frac{1}{3} + x < 5\frac{2}{3}$.

399-D Yes

399-E No * $\frac{6}{5} + s > \frac{23}{10}$.

399-F $\{s \mid s > \frac{11}{10}\}$ * Yes

399-G $n < .8 \wedge 1.5 + n > 2.3$.

399-H $\{n \mid n < .8 \wedge n > .8\}$

399-I No

399-J $\{\ \}$

On your own

400-1 x, number of pounds of salted nuts Jill bought.
$\frac{7}{8} + \frac{5}{8} = x$.
$\{1\frac{1}{2}\}$. $1\frac{1}{2}$ lb.

400-2 x, number of pounds Tom weighed before he lost the $9\frac{1}{2}$ lb.
$x - 9\frac{1}{2} = 145\frac{3}{4}$.
$\{155\frac{1}{4}\}$. $155\frac{1}{4}$ lb.

400-3 x, number of gallons of mint ice cream Mrs. Harris bought.
$x + \frac{1}{4} < 1\frac{3}{8}$.
$\{x \mid x < 1\frac{1}{8}\}$.
Less than $1\frac{1}{8}$ gal. of mint ice cream

400-4 x, number of feet of the board Henry had left after building the shelf.
$5\frac{1}{2} - 2\frac{3}{4} = x$.
$\{2\frac{3}{4}\}$. $2\frac{3}{4}$ ft.

400-5 x, number of miles Mr. Larson's home is from Charlottetown.
$13648.8 + x = 13824.5$.
$\{175.7\}$. 175.7 mi.

400-6 x, number of cups of vegetables Mrs. Allen used for the pie.
$x = \frac{2}{3} + \frac{2}{3} + \frac{1}{4}$.
$\{1\frac{7}{12}\}$. $1\frac{7}{12}$ cups

400-7 x, number of pints of milk the supervisor ordered on Tuesday.
$48\frac{1}{2} + x < 97\frac{1}{2}$.
$\{x \mid x < 49\}$.
Less than 49 pt. of milk

400-8 x, number of inches of precipitation more than Halifax receives than Calgary.
$54.26 - 17.47 = x$.
$\{36.79\}$. 36.79 in.

400-9 x, number of hours Kurt worked in the store on the three days.
$x = 1\frac{1}{2} + 1\frac{3}{4} + 4\frac{3}{4}$.
$\{8\}$. 8 hr.

400-10 x, amount of money Ruth had before she spent the $1.59.
$x > 1.75 \wedge x - 1.59 < .43$.
$\{x \mid x > 1.75 \wedge x < 2.02\}$.
Between $1.75 and $2.02

400-11 x, amount of money Mrs. Murphy spent in all for the three appliances.
$x = 134.95 + 199.50 + 239.75$.
$\{574.20\}$. $574.20

400-12 x, number of feet that is the length of the "United States."
$1035\frac{1}{6} - x = 45\frac{1}{6}$.
$\{990\}$. 990 ft.

400-13 x, number of feet that is the elevation of Lake Erie.
$x - 245.88 = 326.52$.
$\{572.40\}$. 572.40 ft.

400-14 x, number of feet of copper pipe the plumber used.
$x < 18\frac{1}{3} \wedge 18\frac{1}{3} - x < 6\frac{1}{4}$.
$\{x \mid x < 18\frac{1}{3} \wedge x > 12\frac{1}{12}\}$.
Between $12\frac{1}{12}$ ft. and $18\frac{1}{3}$ ft.

400-15 x, number of gallons of punch the Drama Club members made.
$x > 5\frac{1}{2} \wedge x - 4\frac{3}{4} < 1\frac{1}{2}$.
$\{x \mid x > 5\frac{1}{2} \wedge x < 6\frac{1}{4}\}$.
Between $5\frac{1}{2}$ and $6\frac{1}{4}$ gal.

400-16 x, number of miles Mr. Roberts drove last week.
$x < 759.3 \wedge x + 49.9 > 800.0$.
$\{x \mid x < 759.3 \wedge x > 750.1\}$. Between 750.1 and 759.3 mi.

401-17 $x = 6\frac{3}{5} \div 2\frac{1}{2}$.
$\{2\frac{16}{25}\}$

401-18 $3.024 + x = 5.112$.
$\{2.088\}$

401-19 $x > \frac{1}{6} \wedge 5\frac{1}{6} + x < 5\frac{1}{3}$.

$\{x \mid x > \frac{1}{6} \wedge x < \frac{1}{6}\}$.
Since the solution set is the empty set, it can be tabulated as { }.

401-20 $x < 1\frac{1}{3} \times 2\frac{1}{7}$.
$\{x \mid x < 2\frac{6}{7}\}$

401-21 $x > 7.136 \wedge 1.084 + x > 8.429$.
$\{x \mid x > 7.345\}$

401-22 $x < 3\frac{1}{8} \wedge 9\frac{1}{4} - x > 4$.
$\{x \mid x < 3\frac{1}{8}\}$

Checking up

Test 90

401-1 Between $\frac{3}{7}$ and $\frac{8}{15}$

401-2 Not between $\frac{3}{7}$ and $\frac{8}{15}$

401-3 Not between $\frac{3}{7}$ and $\frac{8}{15}$

401-4 Between $\frac{3}{7}$ and $\frac{8}{15}$

401-5 Not between $\frac{3}{7}$ and $\frac{8}{15}$

401-6 Not a solution

401-7 A solution

401-8 A solution

401-9 Not a solution

401-10 A solution

401-11 Not a member

401-12 A member

401-13 Not a member

401-14 A member

401-15 Not a member

401-16 $\{7\frac{2}{22}\}$

401-17 $\{3\frac{1}{5}\}$

401-18 $\{6\frac{7}{9}\}$

401-19 $\{\frac{17}{21}\}$

401-20 A graph

401-21 Not a graph

401-22 Not a graph

401-23 A graph

401-24 Not a graph

401-25 A graph

Test 91

401-26 x, number of hours Russell spent on his homework on Thursday.
$1\frac{2}{3} + x = 3\frac{1}{4}$.
$\{1\frac{7}{12}\}$. $1\frac{7}{12}$ hr.

401-27 x, amount of money Carol paid for her lunch.
$.75 - x > .40$.
$\{x \mid x < .35\}$.
Less than \$.35

401-28 x, number of inches of rain Edmonton had on Sunday.
$x > .3 \wedge x + 1.1 < 2$.
$\{x \mid x > .3 \wedge x < .9\}$.
Between .3 and .9 in.

401-29 $7\frac{1}{5} - x > 3\frac{1}{7}$.
$\{x \mid x > 4\frac{2}{35}\}$

401-30 $x < 9\frac{1}{2} \wedge x - 2\frac{3}{4} > 5$.
$\{x \mid x < 9\frac{1}{2} \wedge x > 7\frac{3}{4}\}$

Lesson 92

402-A The amount of peanuts * The amount of cashew nuts

402-B $7\frac{1}{2}$ refers to an amount of peanuts, and the first component of $1\frac{1}{4}/\frac{1}{2}$ refers to an amount of peanuts.

402-C Yes * The amount of cashew nuts in the mixture that has $7\frac{1}{2}$ lb. of peanuts

402-D Yes * Z

402-E That the first component of $1\frac{1}{4}/\frac{1}{2}$ is multiplied by the second component of $7\frac{1}{2}/y$ and that the first component of $7\frac{1}{2}/y$ is multiplied by the second component of $1\frac{1}{4}/\frac{1}{2}$. * By using the definition of equivalent ordered pairs

402-F Yes

402-G Yes. $1\frac{1}{4}$ is the same as $\frac{5}{4}$. * Z

403-H $\frac{4}{5}$

403-I Yes * Associative property of multiplication

403-J Yes * The product of a number and its reciprocal is 1.

403-K Yes. 1 is the identity element for multiplication. * Yes. Using the definition of the product of two rational numbers, $\frac{4}{5}(\frac{15}{4}) = \frac{3}{1}$, or 3.

403-L Yes * Yes

403-M {3}

403-N Yes * Z

403-O By using the definition of equivalent ordered pairs * Yes

403-P Yes. $1\frac{3}{2}$ is the same as $6\frac{1}{2}$.

403-Q Well-defined property of multiplication of rational numbers

403-R $\frac{2}{13}(\frac{20}{4}) = \frac{2}{13}(1\frac{3}{2}x)$.
$\frac{2}{13}(1\frac{3}{2}x) = (\frac{2}{13} \cdot 1\frac{3}{2})x$.
Associative property of multiplication.
$\frac{2}{13}(\frac{20}{4}) = (\frac{2}{13} \cdot 1\frac{3}{2})x$.
Transitive property

403-S $\frac{2}{13}(\frac{20}{4}) = (\frac{2}{13} \cdot 1\frac{3}{2})x$.
$(\frac{2}{13} \cdot 1\frac{3}{2})x = 1x$.
Definition of the product of two rational numbers.
$\frac{2}{13}(\frac{20}{4}) = 1x$.
Transitive property

403-T $\frac{2}{13}(\frac{20}{4}) = 1x$.
$1x = x$.
Identity-element property of multiplication.
$\frac{2}{13}(\frac{20}{4}) = x$.
Transitive property * $8\frac{1}{26}$

403-U $8\frac{1}{26}$ * $8\frac{1}{26}$

403-V $.2/10 \sim x/75$. * Z

403-W By using the definition of equivalent ordered pairs

403-X $\frac{1}{10}$ * Find the product of the reciprocal of 10 and 15.

403-Y {1.5}

404-Z Yes * Find the product of $\frac{c}{d}$ and the reciprocal of $\frac{a}{b}$. This is $\frac{cb}{da}$.

On your own

404-1 Equivalent

404-2 Equivalent

404-3 Not equivalent

404-4 Not equivalent

404-5 Equivalent

404-6 Equivalent

404-7 Equivalent

404-8 Not equivalent

404-9 Equivalent

404-10 Equivalent

404-11 Not equivalent

404-12 Equivalent

404-13 False statement

404-14 False statement

404-15 True statement

404-16 False statement

404-17 False statement

404-18 False statement

404-19 False statement

404-20 True statement

404-21 $\{2\frac{2}{9}\}$

404-22 $\{4\}$

404-23 $\{1\frac{1}{24}\}$

404-24 $\{\frac{39}{88}\}$

404-25 $\{.4083\}$, or $\{\frac{49}{120}\}$

404-26 $\{2\frac{14}{43}\}$

404-27 $\{1\frac{7}{9}\}$

404-28 $\{12\}$

404-29 $\{2\frac{25}{46}\}$

404-30 $\{5\frac{1}{11}\}$

404-31 $\{.125\}$

404-32 $\{3\frac{3}{5}\}$

404-33 $\{\frac{7}{24}\}$

404-34 $\{.8\}$

404-35 $\{\frac{4}{21}\}$

404-36 $\{80000\}$

404-37 $\{24\frac{1}{2}\}$

404-38 $\{13\frac{4}{9}\}$

404-39 $\{.0007\}$

404-40 $\{5\frac{1}{3}\}$

Lesson 93

404-A Yes * The amount of candy that Kay sold * The amount of candy that Agnes sold

404-B The amount of candy that Agnes sold when Kay sold 6 lb. * 6 refers to the amount of candy that Kay sold, and the first component of $\frac{3}{4}/\frac{1}{2}$ also refers to the amount of candy that Kay sold. * The amount of candy that Agnes sold

404-C Yes * $6/n$ is equivalent to $\frac{3}{4}/\frac{1}{2}$.

404-D Use the definition of equivalent ordered pairs.

404-E 4

404-F Yes * From the definition of equivalent ordered pairs, if $\frac{3}{4}n = 6 \cdot \frac{1}{2}$, then $\frac{3}{4}/\frac{1}{2} = 6/n$.

404-G $\{4\}$

405-H The amount of money that is the total cost of the four books * The number of books

405-I m refers to the amount of money that is the cost, and the first component of 6.36/4 also refers to the amount of money that the books cost. Therefore, m is the first component of the rate pair that you are to find.

405-J Yes * $m/1$ is equivalent to 6.36/4.

405-K $\{1.59\}$

405-L The average cost per book is \$1.59.

405-M Yes * Yes * Yes

405-N Yes

405-O The average number of hours that Lorraine worked per day * A number of days

405-P Yes * Number of hours Lorraine worked in all during the 4 days * Number of days Lorraine worked

405-Q n refers to a number of hours Lorraine worked, and the first component of $1\frac{1}{2}/1$ also refers to a number of hours Lorraine worked. Therefore, n is the first component of the rate pair you are to find. * $\{6\}$

406-A Z * Z

406-B Yes * Infinitely many * Infinitely many

406-C One

406-D Number that is the average weight per package

406-E No * y refers to a number of pounds that is the weight, and the first component of $x/1$ also refers to a number of pounds that is the weight. Therefore, y is the first component of the rate pair you are to find.

406-F Yes

406-G Yes * y, and not x, refers to the total number of pounds that is the weight of the three packages and so does $1\frac{3}{4} + 1\frac{1}{2} + 1\frac{1}{4}$.

406-H $4\frac{1}{2}$

406-I Yes

406-J $1\frac{1}{2}$

406-K $\{(1\frac{1}{2}, 4\frac{1}{2})\}$

406-L x is the variable for the number that is average weight per package. So the replacement for x is used to obtain the answer to the problem.

Since the universe is for (x, y), the first component of $(1\frac{1}{2}, 4\frac{1}{2})$ is used to obtain the answer.

407-A Number of cents each pencil costs * A number of pencils

407-B Number of pencils Mark can buy for 25 cents * x is a variable for the number of pencils Mark can buy for 25 cents. The second component of 8/1 also refers to a number of pencils. Therefore, x is the second component of the rate pair you are to find.

407-C $\{3\frac{1}{8}\}$

407-D No * You do not buy $\frac{1}{8}$ of a pencil.

407-E No * 25 cents is not enough to buy 4 pencils that cost 8 cents a pencil.

407-F Things that are sold by the pound (or the foot) can be sold by a fraction of a pound (or a foot).

407-G Number of cans of frozen lemonade * Amount of money that is the cost of 3 cans of frozen lemonade

407-H The amount of money that is the cost of 1 can of frozen lemonade * 1 refers to a number of cans of frozen lemonade. The first component of 3/34 also refers to a number of cans of frozen lemonade. Therefore, 1 is the first component of the rate pair you are to find.

407-I $\{11\frac{1}{3}\}$

407-J No * There is no coin whose value is $\frac{1}{3}¢$.

407-K You would pay 12¢ if you bought 1 can of lemonade.

407-L Total number of persons who attended the play * Number of nights on which the play was given

407-M Number that is the average attendance per night * s refers to a number of persons. The first component of 1715/6 also refers to a number of persons. Therefore, s is the first component of the rate pair you are to find.

407-N $\{285\frac{5}{6}\}$

407-O Since the problem tells you to give your answer as hundredths, the decimal 285.83 expresses $285\frac{5}{6}$ to the nearer hundredth.

407-P The average attendance per night was 285.83 persons.

On your own

408-1 x, number of blocks Martha lives from school.
$2\frac{1}{4}/1 \sim x/6\frac{1}{2}$.
$\{14\frac{5}{8}\}$. $14\frac{5}{8}$ blocks

408-2 x, number that is the average points scored per game.
$311/12 \sim x/1$.
$\{25\frac{11}{12}\}$. 25.9 points

408-3 x, amount of money that was the cost of transporting 1350 barrels of oil.
$4.50/100 \sim x/1350$.
$\{60.75\}$. \$60.75

408-4 x, number of cents that was the cost of 3 bars of soap.
$2/29 \sim 3/x$.
$\{43\frac{1}{2}\}$. 44¢

408-5 x, number of times larger Lake Nipissing is than Lake Dauphin.
$350/200 \sim x/1$.
$\{1\frac{3}{4}\}$. Lake Nipissing is $1\frac{3}{4}$ times larger than Lake Dauphin.

408-6 x, number of hours it took Mr. Russell to drive 327 mi.
$34.5/1 \sim 327/x$.
$\{9\frac{11}{23}\}$. $9\frac{11}{23}$ hr.

408-7 x, number that is the average score per test.
y, number that is the total of the 5 scores.
$x/1 \sim y/5 \wedge y = 94 + 98 + 93 + 85 + 91$.
$\{(92\frac{1}{5}, 461)\}$. Marge's average score was 92.2.

408-8 x, number of aprons Mary made.
$1/1\frac{1}{2} \sim x/5\frac{1}{2}$.
$\{3\frac{2}{3}\}$. Mary made 3 aprons.

408-9 x, amount of money Harold earned last week.
$1.40/1 \sim x/20\frac{1}{5}$.
$\{28.28\}$. \$28.28

408-10 x, number that is the average rate in miles per hour that Ken and his father hiked.
y, number of miles that Ken and his father hiked in 3 hours.
$x/1 \sim y/3 \wedge y = 3\frac{1}{4} + 2\frac{1}{2} + 3$.
$\{(2\frac{11}{12}, 8\frac{3}{4})\}$. Ken and his father hikes $2\frac{11}{12}$ miles per hour.

Lesson 94

408-A The Wilsons' income was \$5000 and they put \$325 into a savings account. The Powells' income was \$10,000 and they put \$75

into a savings account. The Hansons' income was $6000 and they put $700 into a savings account. The Benders' income was $6000 and they put $375 into a savings account. The Abrams' income was $8500 and they put $850 into a savings account.

408-B Number of dollars put into a savings account * Number of dollars that was the income

409-C Yes * Number of dollars put into a savings account for each $100 of income

409-D Use the definition of equivalent ordered pairs.

409-E Yes * You know that $325/5000 \sim 6\frac{1}{2}/100$ is a true statement because $325(100) = 6\frac{1}{2}(5000)$. Similarly, you know the other statements are true because $75(100) = \frac{3}{4}(10000)$; $700(100) = 11\frac{2}{3}(6000)$; $375(100) = 6\frac{1}{4}(6000)$; and $850(100) = 10(8500)$.

409-F The comparison of the number of dollars saved with each $100 of yearly income

409-G Yes

409-H Yes

409-I Yes

409-J The Hansons * The Abrams

409-K Yes * The Wilsons

409-L The Powells

409-M The Hansons, $\$11\frac{2}{3}$
The Abrams, $10
The Wilsons, $\$6\frac{1}{2}$
The Benders, $\$6\frac{1}{4}$
The Powells, $\$\frac{3}{4}$

410-N No

410-O The recorded precipitation in 1960 was 43.18 in. for Gander, 37.06 in. for Montreal, 31.82 in. for Toronto, 15.59 in. for Winnipeg, 19.64 in. for Edmonton, and 13.70 in. for Regina. * The average annual precipitation is 40 in. for Gander, 42 in. for Montreal, 31 in. for Toronto, 20 in. for Winnipeg, 18 in. for Edmonton, and 15 in. for Regina.

410-P Gander, 43.18/40.
Montreal, 37.06/42.
Toronto, 31.82/31.
Winnipeg, 15.59/20.
Edmonton, 19.64/18.
Regina, 13.70/15.

410-Q $43.18/40 \sim 107.95/100$.
$37.06/42 \sim 88.24/100$.
$31.82/31 \sim 102.65/100$.
$15.59/20 \sim 77.95/100$.
$19.64/18 \sim 109.11/100$.
$13.70/15 \sim 91.33/100$.

410-R $107.95/100 = 107.95\%$.
$88.24/100 = 88.24\%$.
$102.65/100 = 102.65\%$.
$77.95/100 = 77.95\%$.
$109.11/100 = 109.11\%$.
$91.33/100 = 91.33\%$.

410-S 77.95%, 88.24%, 91.33%, 102.65%, 107.95%, 109.11%

410-T Gander, Toronto, and Edmonton

On your own

410-1 $25\frac{1}{2}\%$

410-2 1.72%

410-3 $\frac{1}{10}\%$

410-4 100.6%

410-5 .06%

410-6 $133\frac{1}{3}\%$

410-7 10/100, or 10% * No

410-8 75/100

410-9 No

410-10 20%

410-11 $16\frac{2}{3}/100 * 16\frac{2}{3}\%$

410-12 $37\frac{1}{2}/100 * 37\frac{1}{2}\%$

410-13 $66\frac{2}{3}/100 * 66\frac{2}{3}\%$

410-14 $3\frac{1}{3}/100 * 3\frac{1}{3}\%$

410-15 $258\frac{1}{3}/100 * 258\frac{1}{3}\%$

410-16 $87\frac{1}{2}/100 * 87\frac{1}{2}\%$

410-17 15.20/100 * 15.20%

410-18 45.00/100 * 45.00%

410-19 45.45/100 * 45.45%

410-20 116.33/100 * 116.33%

410-21 140.00/100 * 140.00%

410-22 82.00/100 * 82.00%

410-23 87.50/100 * 87.50%

410-24 201.92/100 * 201.92%

410-25 70.39/100 * 70.39%

410-26 476.17/100 * 476.17%

410-27 6.54/100 * 6.54%

410-28 666.67/100 * 666.67%

Lesson 95

411-A Number of members who paid their dues * Number of members in the club

411-B The first component of the per cent of members who paid their dues

411-C It is equivalent to 5/15.

411-D Use the definition of equivalent ordered pairs.

411-E Yes * 500 is the same as 5(100). The definition of equivalent ordered pairs tells you that, if $5(100Q) = 15n$, then $5/15 \sim n/100$.

411-F $\{33\frac{1}{3}\}$

411-G The amount of Mr. Brown's commission for May

411-H Yes

411-I 6000 refers to the number of dollars that was Mr. Brown's total sales for May. The second component of 6.5/100 also

refers to a number of dollars of Mr. Brown's sales. Therefore, 6000 is the second component of the rate pair you are to find. * The number of dollars that is Mr. Brown's commission for May

411-J Yes * It is equivalent to 6.5/100.

411-K {390}

412-L (Column 1) The number 4 is $\frac{1}{2}$% of

412-M (Column 1) Yes

412-N The first component of $\frac{1}{2}$/100 is $\frac{1}{2}$% of 100. 4 is also $\frac{1}{2}$% of the number you are to find. Therefore, 4 is the first component of the rate pair whose second component you are to find. * The number 4 is $\frac{1}{2}$% of

412-O It is equivalent to $\frac{1}{2}$/100.

412-P {800}

412-Q Yes

412-A What per cent the increase is of last season's price

412-B 40¢ * 25¢

412-C The number of cents that is the increase with the number of cents that was the price of a ticket last season

412-D The number of cents that was the increase * The number of cents that was the price of a ticket last season

412-E The second component of a per cent is 100.

412-F Yes * The first component of a per cent that the increase is of last season's price

412-G {62$\frac{1}{2}$}

412-H What per cent the decrease is of the original value of Mr. Hill's car

412-I $2250 * $750

412-J The number of dollars that was the decrease with the number of dollars that was the original value * The number of dollars that was the decrease * The number of dollars that was the original value

412-K The second component of a per cent is 100.

412-L (Column 2) Yes * The first component of a per cent that the decrease is of the original value

412-M (Column 2) {33$\frac{1}{3}$}

413-A The second component of a per cent is 100. * The first component of the per cent of increase in enrollment

413-B No

413-C You want to compare the increase in enrollment with last year's enrollment. This comparison is represented by the rate pair c/240, not 240/c. * The comparison is with last year's enrollment, which was 240, not with this year's enrollment, which is 500.

413-D Yes

413-E Yes * 500 − 240 refers to the number that is the increase in enrollment. c, not b, is the variable for this number.

413-F 260

413-G Yes

413-H 108$\frac{1}{3}$

413-I The universe is a set of ordered pairs. This universe is for (b, c).

413-J {(108$\frac{1}{3}$, 260)}

413-K The first component of the per cent of increase in enrollment * The number that is the increase in enrollment

413-L Because you are asked to find the per cent of increase in enrollment

On your own

413-1 x, amount of money the sales tax is on a car priced at $2200.
3$\frac{1}{4}$/100 ~ x/2200.
{71.50}. $71.50

413-2 x, first component of the per cent of decrease in the number of hours Frank worked.
2$\frac{1}{2}$/12$\frac{1}{2}$ ~ x/100.
{20}. 20%

413-3 x, amount of money the house is worth now.
325/100 ~ x/2976.
{9672}. $9672

414-4 x, first component of the per cent of increase in Bill's hourly wages.
.30/1.00 ~ x/100.
{30}. 30%

414-5 x, first component of the per cent of decrease in the price of the jacket.
7/21 ~ x/100.
{33$\frac{1}{3}$}. 33$\frac{1}{3}$%

414-6 x, amount of money Mr. Keys had borrowed.
6/100 ~ 46.50/x.
{775}. $775

414-7 x, first component of the per cent of increase in the population.
y, number that is the

increase in the population.
$x/100 \sim y/4000 \wedge$
$9{,}000 - 4{,}000 = y.$
$\{(125, 5000)\}$. 125%

414-8 x, first component of a per cent that the sales tax was of the selling price.
$.63/14.00 \sim x/100.$
$\{4\frac{1}{2}\}$. $4\frac{1}{2}\%$

414-9 x, number of telephones in Africa.
$1.4/100 \sim x/124800000.$
$\{1747200\}$.
1,747,200 telephones

414-10 x, amount of money that is the selling price of the book.
$2/100 \sim .07/x.$
$\{3.50\}$. \$3.50

414-11 x, first component of the per cent of decrease in the number of hogs raised.
y, number of hogs that was the decrease.
$x/100 \sim y/96 \wedge$
$96 - 84 = y.$
$\{(12\frac{1}{2}, 12)\}$. $12\frac{1}{2}\%$

414-12 x, first component of the per cent of increase in the price per pound.
y, amount of money that was the increase.
$x/100 \sim y/.50 \wedge$
$.56 - .50 = y.$
$\{(12, .06)\}$. 12%

414-13 x, first component of the per cent of decrease in the amount of money Carl saved.
y, amount of money that was the decrease in the amount of money Carl saved.
$x/100 \sim y/26.55 \wedge$
$26.55 - 17.70 = y.$
$\{(33\frac{1}{3}, 8.85)\}$. $33\frac{1}{3}\%$

414-14 x, first component of the per cent that the weight of fresh water is of the weight of salt water.
$62.4/64.0 \sim x/100.$
$\{97.5\}$. 97.5%

414-15 $34/51 \sim x/100.$
$\{66\frac{2}{3}\}$

414-16 $50/100 \sim 1.5/x.$
$\{3\}$

414-17 $75/100 \sim x/5\frac{1}{3}.$
$\{4\}$

414-18 $.4/100 \sim x/20.$
$\{.08\}$

414-19 $5.2/16 \sim x/100.$
$\{32.5\}$

414-20 $125/100 \sim x/\frac{3}{4}.$
$\{1\frac{5}{16}\}$

414-21 $\frac{1}{4}/100 \sim 72/x.$
$\{28800\}$

414-22 $x/100 \sim 40/8000.$
$\{\frac{1}{2}\}$

414-23 $\frac{3}{5}/x \sim 45/100.$
$\{1\frac{1}{3}\}$

414-24 $x/80 \sim 22\frac{1}{2}/100.$
$\{18\}$

414-25 $42/560 \sim x/100.$
$\{7.5\}$

414-26 $15/x \sim 1\frac{1}{2}/100.$
$\{1000\}$

414-27 $5000/10 \sim x/100.$
$\{50000\}$

Keeping skilful

414-1 $\{33\frac{1}{3}\}$

414-2 $\{87\frac{1}{2}\}$

414-3 $\{\frac{5}{6}\}$

414-4 $\{16\frac{2}{3}\}$

414-5 $\{\frac{3}{8}\}$

414-6 $\{\frac{2}{3}\}$

414-7 $\{14\frac{2}{7}\}$

414-8 $\{\frac{1}{6}\}$

414-9 $\{62\frac{1}{2}\}$

414-10 $\{\frac{1}{12}\}$

414-11 $\{\frac{1}{3}\}$

414-12 $\{6\frac{1}{4}\}$

414-13 $\{12\frac{1}{2}\}$

414-14 $\{\frac{1}{6}\}$

414-15 5.577

414-16 27.318

414-17 1.6794

414-18 .11176

414-19 .8731

414-20 6.08

414-21 26.196

414-22 20.81

414-23 .8377

414-24 3357.90

414-25 .0038

414-26 1.690

414-27 .1080

414-28 .801

Applying mathematics

415-1 x, amount of fruit Joan bought.
$x = 3\frac{1}{2} + 1 + \frac{3}{4}.$
$\{5\frac{1}{4}\}$. $5\frac{1}{4}$ lb.

415-2 x, number of pickets Mr. Smith will need.
$9/4 \sim x/196.$
$\{441\}$. 441 pickets

415-3 x, number of pounds of white popcorn Ed bought.
$x + \frac{3}{4} < 1\frac{1}{8}.$
$\{x \mid x < \frac{3}{8}\}$. Ed bought less than $\frac{3}{8}$ lb. of white popcorn.

415-4 x, amount of money Marie paid for the school supplies.
$x > 3.50 \wedge 5.00 - x > 1.25.$
$\{x \mid x > 3.50 \wedge x < 3.75\}$.
Between \$3.50 and \$3.75

415-5 x, number that is the weight of silver.
$x - 12.011 = 95.869.$
$\{107.880\}$.
107.880 a.m.u.

415-6 x, number that is the weight of nickel.
$126.91 - x = 68.20.$
$\{58.71\}$. 58.71 a.m.u.

415-7 x, first component of the per cent of decrease in Marilyn's savings.
$1.30/5.20 \sim x/100$.
{25}. 25%

415-8 x, number that is the average distance per jump.
y, total number of feet in two jumps.
$x/1 \sim y/2 \wedge y = 290 + 272$.
{(281,562)}. 281 ft.

415-9 x, amount of money that was Mrs. Moore's telephone bill for April.
$2/100 \sim .17/x$.
{8.50}. $8.50

415-10 x, first component of the per cent of increase in the amount of snowfall.
y, difference in inches in the amount of snow in December and November.
$x/100 \sim y/.5 \wedge$
$8.0 - .5 = y$.
{(1500,7.5)}. 1500%

415-11 x, number of miles that Mr. Lyons drove last week.
$1\frac{1}{2}/1 \sim x/135.4$.
{203.1}. 203.1 mi.

415-12 x, number that is the average daily high temperature that week.
y, number that was the total of the daily high temperatures.
$x/1 \sim y/7 \wedge y = 75 + 83 + 75 + 78 + 73 + 73 + 72$.
{($75\frac{4}{7}$,529)}. 75.57 degrees

415-13 x, number of students in Sue's history class.
y, number of students who do not wear glasses.
$25/100 \sim 8/x \wedge 8 + y = x$.
{(32,24)}. 24 students

415-14 x, number of persons who lived in Lachine.
$.7/100 \sim x/4055000$.
{28385}. 28,385 persons

415-15 x, first component of the per cent of increase in the amount of ice cream Mr. Wilson sold.
y, difference in the number of gallons of ice cream sold on Tuesday and Monday.
$x/100 \sim y/24\frac{1}{2} \wedge$
$30\frac{5}{8} - 24\frac{1}{2} = y$.
{(25,$6\frac{1}{8}$)}. 25%

415-16 x, first component of the per cent of decrease in the enrollment.
y, number that was the decrease in the students enrolled.
$x/100 \sim y/700 \wedge$
$700 - 686 = y$.
{(2,14)}. 2%

415-17 $37\frac{4}{5} - 19\frac{3}{7} = x$.
{$18\frac{13}{35}$}

415-18 $175/100 \sim x/72$.
{126}

415-19 $4.62 + x < 5.01$.
$\{x \mid x < .39\}$

415-20 $x + y < 9 \wedge y - x = 3$.
{(0,3), (1,4), (2,5)}

415-21 $x > 5\frac{3}{4} \wedge 12 - x > 5\frac{1}{2}$.
$\{x \mid x > 5\frac{3}{4} \wedge x < 6\frac{1}{2}\}$

Checking up

Test 92

416-1 True statement

416-2 False statement

416-3 False statement

416-4 True statement

416-5 {.14}

416-6 $\{\frac{1}{18}\}$

416-7 $\{\frac{3}{6}\}$

416-8 {100}

416-9 2.5%, or 2.5/100

416-10 1.5%, or 1.5/100

416-11 .7%, or .7/100

416-12 .4%, or .4/100

416-13 2.5%, 1.5%, .7%, .4%

Test 93

416-14 x, amount of money the set of 8 volumes cost.
$1/4.98 \sim 8/x$.
{39.84}. $39.84

416-15 x, number of minutes it took Lynn to read the assignment.
$1\frac{1}{2}/1 \sim x/30$.
{45}. 45 minutes

416-16 x, number that was the average of Jack's three scores.
y, number that is the total of the three scores.
$x/1 \sim y/3 \wedge y = 133 + 115 + 140$.
{($129\frac{1}{3}$,388)}. Jack's average score was 129.

416-17 x, first component of the per cent of increase in John's weight.
y, number of pounds increase in John's weight.
$x/100 \sim y/150 \wedge$
$157\frac{1}{2} - 150 = y$.
{(5,$7\frac{1}{2}$)}. 5%

416-18 $5/100 \sim x/250$.
{12.5}

416-19 $1.25/250 \sim x/100$.
{.5}

416-20 $.1250/x \sim .05/100$.
{250}

Lesson 96

416-A Yes * The first component of 11/33 refers to the number of years that is the normal life expectancy of a dog. The second component of 11/33 refers to the number of years that is the normal life expectancy of the horse.

416-B The part of a year a dog may be expected to live for each year that a horse may be expected to live

416-C Yes * $\{\frac{1}{3}\}$

416-D It is equivalent to 11/33.

417-E $11/33 \sim x/1$. * $\{\frac{1}{3}\}$

417-F .5/1

417-G Yes

417-H Yes * Yes

417-I The number of calories in a glass of milk * The number that is .5 of 330

417-J {165}

417-K Since the common fraction numeral $\frac{1}{2}$ names the same rational number as the decimal .5, the sentence $\frac{1}{2}/1 \sim m/330$ expresses the same condition as the sentence $.5/1 \sim m/330$.

417-L 165 * 165

417-M $\frac{3}{4}/1$

417-N Yes

417-O Yes * Yes

417-P The number of satellites Jupiter has * The number of which 9 is $\frac{3}{4}$

417-Q {12}

417-R 12 * 12

On your own

417-1 x, number of bushels of corn Mr. Nichols sold.
$\frac{3}{5}/1 \sim x/10000$.
{6000}. 6000 bushels

418-2 x, amount of time Ed spent watching television on Saturday.
$\frac{1}{2}/1 \sim 1\frac{1}{4}/x$.
$\{2\frac{1}{2}\}$. $2\frac{1}{2}$ hr.

418-3 x, fraction of her weekly allowance Sally spent for the ticket.
$.60/.75 \sim x/1$.
$\{\frac{4}{5}\}$. Sally spent .8 of her weekly allowance for the ticket.

418-4 x, number of students who went to the picnic.
$.9/1 \sim x/30$.
{27}. 27 students

418-5 x, fraction of the games played that Wilson High School won.
$14/21 \sim x/1$.
$\{\frac{2}{3}\}$. The Wilson High School basketball team won $\frac{2}{3}$ of the games they played this year.

418-6 x, number of cupcakes Mrs. Taylor made.
$\frac{3}{4}/1 \sim 36/x$.
{48}. 48 cupcakes

418-7 x, amount of drapery material Mrs. Thomas had to begin with.
$\frac{7}{8}/1 \sim 4\frac{2}{3}/x$.
$\{5\frac{1}{3}\}$. $5\frac{1}{3}$ yd.

418-8 x, number of feet per second that is the speed of sound in the wood.
$\frac{2}{3}/1 \sim x/15000$.
{10000}. 10,000 ft. per second

418-9 x, fraction the speed at 20 mi. above the earth was of the speed at 250 mi. above the earth.
$3500/5000 \sim x/1$.
{.7}. The speed of the rocket at 20 mi. above the earth was .7 of the speed at 250 mi. above the earth.

418-10 x, amount of flour that Jean should have used.
$.5/1 \sim x/2\frac{1}{2}$.
$\{1\frac{1}{4}\}$. $1\frac{1}{4}$ cups

418-11 x, fraction the amount of milk the Miller family uses is of the amount of milk the King family uses.
$\frac{3}{4}/1\frac{1}{4} \sim x/1$.
$\{\frac{3}{5}\}$. The Miller family uses $\frac{3}{5}$ the amount of milk that the King family uses.

418-12 x, number of questions on the test.
$.85/1 \sim 102/x$.
{120}. 120 questions

418-13 $6.01/30.05 \sim x/1$.
{.2}

418-14 $\frac{2}{3}/1 \sim 12/x$.
{18}

418-15 $.35/1 \sim x/60$.
{21}

418-16 $1\frac{1}{2}/1 \sim x/\frac{1}{3}$.
$\{\frac{1}{36}\}$

418-17 $30/48 \sim x/1$.
$\{\frac{5}{8}\}$

418-18 $.4/1 \sim 3.6/x$.
{9}

418-19 $\frac{3}{8}/1 \sim 10\frac{1}{5}/x$.
$\{27\frac{1}{5}\}$

418-20 $750/1000 \sim x/1$.
$\{\frac{3}{4}\}$

418-21 $.67/1 \sim x/125$.
{83.75}

Keeping Skilful

418-1 15

418-2 $102\frac{1}{2}$

418-3 2.8

418-4 8.874

418-5 $10\frac{2}{9}$

418-6 .2035

418-7 $\frac{7}{10}$

418-8 $6\frac{27}{40}$

418-9 $4\frac{3}{7}$

418-10 6.825

418-11 $5\frac{5}{24}$

418-12 $1\frac{4}{9}$

418-13 $1\frac{3}{7}$

418-14 6.7032

418-15 3.6035

418-16 19

418-17 .018468

418-18 $\frac{13}{20}$

418-19 $\frac{42}{55}$

418-20 $3\frac{23}{44}$

418-21 2.344

418-22 $\frac{72}{365}$

418-23 19.8

418-24 1.67

418-25 125

418-26 .276

418-27 30.09

Lesson 97

419-A The second component of $37\frac{1}{2}/100$ refers to a price of \$100. The first component of $37\frac{1}{2}/100$ refers to the number of dollars discount on a price of \$100. * The amount of money that is the discount on a price of \$14 * Z

419-B Because of the definition of equivalent ordered pairs. $.375(100) = 37\frac{1}{2}(1)$. Thus, $.375/1 \sim 37\frac{1}{2}/100$. * Divide each component of $37\frac{1}{2}/100$ by 100.

419-C $\frac{3}{8}$ * The common fraction numeral $\frac{3}{8}$ names the same rational number as the decimal .375 * Since $n/14$ is equivalent to $.375/1$, it is also equivalent to $\frac{3}{8}/1$.

419-D Use the definition of equivalent ordered pairs. * Find the product of $37\frac{1}{2}$ and 14.

419-E Yes * Multiply 525 by $\frac{1}{100}$.

419-F {5.25}

419-G Use the definition of equivalent ordered pairs. $n(1)$ is the same as n because of the identity-element property of multiplication.

419-H Find the product of .375 and 14.

419-I {5.25}

419-J Use the definition of equivalent ordered pairs.

419-K $\frac{3}{8}$ and 14 * 5.25

419-L {5.25}

419-M Yes

419-N Joe received a discount of \$5.25.

420-O The second component of $33\frac{1}{3}/100$ refers to each 100 students enrolled at Lakeview Junior High School. The first component of $33\frac{1}{3}/100$ refers to the number of students out of each 100 students enrolled who are in the ninth grade. * Number of students who are enrolled at Lakeview

420-P $\frac{1}{3}/1$ * Divide each component of $33\frac{1}{3}/100$ by 100.

420-Q Yes

420-R {396}

420-S To find the number that satisfies $\frac{1}{3}y = 132$, you should multiply 132 by the reciprocal of $\frac{1}{3}$. The reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$, or 3.

420-T {396}

420-U Yes * Yes

420-V 396 students are enrolled at Lakeview Junior High School.

420-A The first component of 1000/2500 refers to the number of dollars of his savings Mr. Harris used to buy the car. The second component of 1000/2500 refers to the number of dollars Mr. Harris had saved. * The number of dollars Mr. Harris spent for each \$100 he had saved

420-B {40} * Mr. Harris used 40% of his savings to buy the car.

421-C The fraction of his savings Mr. Harris used to buy the car

421-D {.4} * $\{\frac{2}{5}\}$

421-E Yes * Yes

421-F Yes * Yes

421-G Yes * Yes

421-H The first component

421-I The number which 48 is $66\frac{2}{3}\%$ of

421-J $\frac{2}{3}/1 \sim 48/y$. * {72}

421-K 72

On your own

For the following responses, only one condition is expressed. If you use another condition, and you do not know whether or not it is correct, ask your teacher for help.

421-1 x, amount of money Ed earns each week.
$\frac{1}{2}/1 \sim 4.80/x$.
{9.60}. \$9.60

421-2 x, number of stamps Cal sold.
$\frac{1}{6}/1 \sim x/540$.
{90}. 90 stamps

421-3 x, first component of the per cent the distance from Murdock to Drayton is of the distance from Murdock to Clark City.
$69.6/92.8 \sim x/100$.
{75}. 75%

421-4 x, amount of discount Mrs. Thomas received.
$\frac{3}{8}/1 \sim x/120$.
{45}. \$45

421-5 x, amount of money that was the regular price of the book.

$\frac{1}{10}/1 \sim .65/x$.
{6.50}. $6.50

421-6 x, fraction that the charge for the picture tube was of the total bill.
$25/35 \sim x/1$.
$\{\frac{5}{7}\}$. The charge for the picture tube was $\frac{5}{7}$ of the total bill.

421-7 x, first component of the per cent of the staff that are boys.
$10/16 \sim x/100$.
{62.5}. 62.5%

421-8 x, number of persons who live in Hill City.
$\frac{3}{5}/1 \sim 10600/x$.
{15900}. 15,900 persons

421-9 x, first component of the per cent of his papers Archie sold.
$27/30 \sim x/100$.
{90}. 90%

422-10 x, number of questions the test contained.
$\frac{4}{5}/1 \sim 40/x$.
{50}. 50 questions

422-11 x, number of dollars interest Mr. Andrews paid.
$6\frac{1}{4}/100 \sim x/1600$.
{100}. $100

422-12 x, fraction of the total amount of tomato juice Mrs. Brown used.
$2\frac{1}{4}/3\frac{3}{4} \sim x/1$.
$\{\frac{3}{5}\}$. Mrs. Brown used $\frac{3}{5}$ of the tomato juice that was in the can.

422-13 $25/75 \sim x/100$.
$\{33\frac{1}{3}\}$

422-14 $\frac{1}{8}/1 \sim x/40$.
{5}

422-15 $.25/1 \sim 3.5/x$.
{14}

422-16 $5\frac{1}{4}/10\frac{1}{2} \sim x/100$.
{50}

422-17 $55/100 \sim 11/x$
{20}

422-18 $.9/1 \sim x/85.6$.
{77.04}

Keeping Skilful

422-1 $\{2\frac{3}{10}\}$

422-2 $\{x \mid x > 8.05\}$

422-3 {.69}

422-4 $\{10\frac{3}{40}\}$

422-5 $\{1\frac{62}{91}\}$

422-6 $\{g \mid g < 2\frac{4}{15}\}$

422-7 $\{5\frac{5}{9}\}$

422-8 $\{5\frac{9}{20}\}$

422-9 $\{d \mid d > 5\frac{15}{56}\}$

422-10 {8.58}

422-11 {2.075}

422-12 $\{k \mid k < 23\frac{1}{42}\}$

422-13 $\{4\frac{8}{33}\}$

422-14 $\{c \mid c < 1.96\}$

422-15 $\{8\frac{7}{15}\}$

422-16 $\{14\frac{29}{36}\}$

422-17 $\{x \mid x > 1\frac{1}{28} \wedge x < 1\frac{9}{28}\}$

422-18 $\{x \mid x > \frac{5}{8} \wedge x < \frac{7}{8}\}$

422-19 { }

422-20 $\{c \mid c < \frac{33}{40} \wedge c > \frac{11}{20}\}$

Applying mathematics

422-1 x, number of inches of green ribbon Marion bought.
$27 + x < 54$.
{1, 2, 3, ..., 26}.
From 1 through 26 inches of green ribbon

422-2 x, number of pounds Carl weighed when school started.
$x - 11\frac{1}{2} = 125\frac{1}{4}$.
$\{136\frac{3}{4}\}$. $136\frac{3}{4}$ lb.

422-3 x, number of men the Harvey Corporation employs.
$\frac{2}{3}/1 \sim 118/x$.
{177}. 177 men

422-4 x, number of degrees the temperature was at 6:00 A.M.
$x < 24 \wedge x + 17 > 36$.
{20, 21, 22, 23}. From 20 through 23 degrees

422-5 x, number of times as much Mrs. Fenton spent for the sofa as for the chair.
$266/42 \sim x/1$.
$\{6\frac{1}{3}\}$. Mrs. Fenton spent $6\frac{1}{3}$ times as much for the sofa as for the chair.

422-6 x, number of cents Sue paid for 9 oranges.
$6/49 \sim 9/x$.
$\{73\frac{1}{2}\}$. 74 cents

422-7 x, number of quarts of ice cream Mrs. Hunter has now.
$x + 2\frac{1}{2} > 3 \wedge x + \frac{1}{2} < 2$.
$\{x \mid x > \frac{1}{2} \wedge x < 1\frac{1}{2}\}$.
Between $\frac{1}{2}$ and $1\frac{1}{2}$ qt.

422-8 x, number of cents Lois spent for the 2 fish.
y, number of cents Evelyn spent for fish.
$x + y = 95 \wedge y = 21 + 25$.
{(49,46)}. 49 cents

423-9 x, number of pounds John weighs.
$\frac{7}{8}/1 \sim 112/x$.
{128}. 128 lb.

423-10 x, number of pairs of socks Janet can buy.
$3/1.60 \sim x/2.50$.
{4.6875}. 4 pairs of socks

423-11 x, number of tickets Tom sold.
y, number of tickets Harvey sold.
$x + y < 24 \wedge 3/1 \sim x/y$.
{(3,1), (6,2), (9,3), (12,4), (15,5)}.
Each member of the solution set may be used to obtain an answer to the problem. The answer obtained from (6,2) is:

Tom sold 6 tickets and Harvey sold 2 tickets.

423-12 x, number of trees Mr. Carey sold.
$\frac{2}{3}/1 \sim x/126$.
{84}. 84 trees

423-13 x, number that is the average amount Dave saved per month.
y, total number of dollars Dave saved in the four months.
$x/1 \sim y/4 \wedge y = 5.00 + 11.56 + 9.70 + 6.10$.
{(8.09, 32.36)}. $8.09

423-14 x, first component of the per cent of increase.
y, number of motor vehicles that was the increase.
$x/100 \sim y/2000000 \wedge 5200000 - 2000000 = y$.
{(160, 3200000)}. 160%

423-15 x, first component of the per cent of decrease in attendance.
y, number of persons that was the decrease.
$x/100 \sim y/600 \wedge 600 - 542 = y$.
{($9\frac{2}{3}$, 58)}. $9\frac{2}{3}$%

423-16 $x > 716 \wedge 989 - x > 178$.
{717, 718, 719, ..., 815}

423-17 $x + y = 89 \wedge y > 12(7)$.
{(4,85), (3,86), (2,87), (1,88), (0,89)}

423-18 $x/y \sim 5/15 \wedge x + y < 25$.
{1/3, 2/6, 3/9, 4/12, 5/15, 6/18}

423-19 $\frac{1}{3}/1 \sim x/180$.
{60}

423-20 $x < 1\frac{1}{5} \wedge \frac{7}{8} + x > 1$.
$\{x \mid x < 1\frac{1}{5} \wedge x > \frac{1}{8}\}$

Checking up

Test 94

423-1 Associative property of addition

423-2 Closure property of multiplication

423-3 Commutative property of addition

423-4 Associative property of multiplication

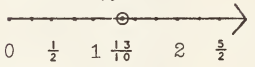
423-5 Closure property of addition

423-6 Distributive property of multiplication over addition

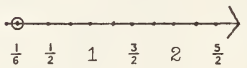
423-7 Commutative property of multiplication

Test 95

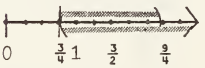
423-8 $\{x \mid x = 1\frac{3}{10}\}$



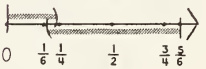
423-9 $\{x \mid x = \frac{1}{6}\}$



423-10 $\{x \mid x > \frac{3}{4} \wedge x < \frac{9}{4}\}$



423-11 $\{x \mid x > \frac{1}{6} \wedge x < \frac{1}{4}\}$



Test 96

424-12 0

424-13 {2.2}

424-14 $\frac{7}{8}$

424-15 50

424-16 $\frac{68}{25}$

424-17 .875

424-18 926

424-19 $.\overline{5}$

424-20 $7(10^2) + 9(10^1) + 2(1) + 3(\frac{1}{10^1}) + 4(\frac{1}{10^2})$

424-21 reduction

424-22 reciprocal

Test 97

424-23 $\frac{8}{25}$

424-24 $1\frac{1}{10}$

424-25 8.39

424-26 $\frac{13}{24}$

424-27 198.112

424-28 25.468443

424-29 $5\frac{7}{9}$

424-30 .039974

424-31 32.65

424-32 $12\frac{4}{7}$

424-33 $11\frac{1}{4}$

424-34 $11\frac{5}{22}$

424-35 231.997

424-36 6.1

424-37 23.85

424-38 $\frac{33}{94}$

Test 98

424-39 (c) $\frac{6}{0}$

424-40 (d) $\frac{20}{85}$

424-41 (b) $ad = cb$.

424-42 (a) $\frac{75}{72}$

424-43 (c) 24

424-44 (d) $4\frac{1}{2}$

424-45 (b) $x + 1 = 1$.

424-46 (c) $\frac{2}{5}$

424-47 (d) $\frac{5}{6} \times \frac{5}{2}$

424-48 (b) $\frac{10}{13}$

425-49 (c) an infinite set

Test 99

425-50 F

425-51 T

425-52 F

425-53 F

425-54 F

425-55 T

425-56 T

425-57 F

425-58 F

425-59 T

425-60 T

425-61 T

425-62 T

Test 100

425-63 $\frac{1}{4}/100 \sim 8/x$.
{3200}

425-64 $34/100 \sim x/6.5$.
{2.21}

425-65 $3.6/60 \sim x/100$.
{6}

425-66 $4.05/100 \sim x/3000$.
{121.5}

425-67 $.8/100 \sim 140/x$.
{17500}

425-68 $48/88 \sim x/1$.
$\{\frac{6}{11}\}$

425-69 $14/51 \sim x/100$.
{27.45}

425-70 $\frac{5}{8}/1 \sim 41\frac{1}{4}/x$.
{66}

425-71 $70/100 \sim 64\frac{2}{5}/x$.
{92}

Test 101

425-1 space

425-2 plane

425-3 17

425-4 empty

425-5 multiple

425-6 skew

425-7 vertex

425-8 proportional relation

425-9 5 + 7

425-10 twin primes

Test 102

425-11 9

425-12 453

425-13 708

425-14 142_{five}

425-15 111110_{two}

425-16 1914_{twelve}

425-17 1111011_{two}

425-18 1323_{eight}

425-19 $123\mathcal{X}_{twelve}$

Test 103

426-20 x, number of pictures Richard took.
y, number of exposures left.
$x + y = 12$.
{(1,11), (2,10), (3,9), (4,8), (5,7), (6,6), (7,5), (8,4), (9,3), (10,2), (11,1)}.
Each member of the solution set may be used to obtain an answer to the problem. The answer obtained from (5,7) is: Richard took 5 pictures and had 7 exposures left.

426-21 x, number of dollars Mr. Frazer made.
y, number of dollars that was the total cost of the stock, including expenses.
$810 - y = x \wedge y = 745 + 22$.
{(43,767)}. \$43
Another way that you might work the problem is given below.
x, number of dollars the price of the stock increased.
y, number of dollars Mr. Frazer made.
$810 - 745 = x \wedge x - 22 = y$.
{(65,43)}. \$43

426-22 x, first component of the per cent of the rebounds that Terry got.
y, total number of rebounds Terry's team got.
$x/100 \sim 5/y \wedge y = 20 + 5$.
{(20,25)}. 20%

426-23 x, number of hours Mr. Holmes worked on Saturday afternoon.
$4\frac{2}{3} + x = 6\frac{3}{4}$.
$\{2\frac{1}{12}\}$. $2\frac{1}{12}$ hr.

426-24 x, number of hours it took Bert to paint the whole fence.
$\frac{1}{4}/9 \sim x/183$.
$\{5\frac{1}{12}\}$. $5\frac{1}{12}$ hr.

426-25 x, number of times a movie must be shown.
$1/385 \sim x/1078$.
{2.8}. 3 times

426-26 x, fraction the distance Jim can throw a softball is of the distance he can throw a baseball.
$180/195 \sim x/1$.
$\{\frac{12}{13}\}$. Jim can throw a softball $\frac{12}{13}$ as far as he can throw a baseball.

Test 104

426-1 T

426-2 T

426-3 F

426-4 F

426-5 F

426-6 T

426-7 F. Point C is in neither half-plane.

Test 105

426-8 $9(10^2) + 3(10^1) + 4(1)$

426-9 $7(10^3) + 0(10^2) + 6(10^1) + 5(1)$

426-10 $1(2^3) + 0(2^2) + 1(2^1) + 1(1)$

426-11 $2(12^2) + 11(12^1) + 10(1)$

426-12 $8(10^1) + 5(1) + 0(\frac{1}{10^1}) + 1(\frac{1}{10^2})$

426-13 $6(10^2) + 0(10^1) + 1(1) + 3(\frac{1}{10^1}) + 1(\frac{1}{10^2}) + 7(\frac{1}{10^3})$

426-14 7×10^1

426-15 4×10^3

426-16 9×10^6

426-17 2×10^5

Test 106

427-18 (d) 3000

427-19 (b) 970

427-20 (b) B

427-21 (a) $\overleftrightarrow{GR}$

427-22 (d) quotient

427-23 (b) 7

427-24 (a) 11

427-25 (c) quotient

427-26 (a) an average

427-27 (b) 8

Test 107

427-28 x, number of games the Centre High School football team lost in the last two seasons.
y, number of games they

won.
$x + y = 20 \wedge x < 7$.
{(0,20), (1,19), (2,18), (3,17), (4,16), (5,15), (6,14)}. From 14 through 20 games

427-29 x, number of logs Mr. Graham burned.
$x > 10 \wedge x < 25$.
{11, 12, 13, ..., 24}. From 11 through 24 logs

427-30 x, number of skirts Patricia has.
y, number of sweaters Patricia has.
$x + y < 11 \wedge x = y$.
{(1,1), (2,2), (3,3), (4,4), (5,5)}. Each member of the solution set can be used to obtain an answer to the problem. The answer obtained from (4,4) is 4 skirts and 4 sweaters.

427-31 x, number of rooms the newer building has.
y, number of rooms the older building has.
$x + y = 82 \wedge y - x = 6$.
{(38,44)}. The newer building has 38 rooms and the older building has 44 rooms.

427-32 x, number of miles that Sam rode his bicycle on the trip.
$2387.4 + x = 2397.1$.
{9.7}. 9.7 mi.

427-33 x, number of pounds Sally weighed before she gained the 8 lb.
y, number that is the first component of the per cent of increase in Sally's weight.
$y/100 \sim 8/x \wedge x + 8 = 104$.
{(96,8$\frac{1}{3}$)}. 8$\frac{1}{3}$%

427-34 $6/10 \sim x/y \wedge y < 9 + x$.
{3/5, 6/10, 9/15, 12/20}

427-35 $\frac{1}{3}/1 \sim x/58$.
{19$\frac{1}{3}$}

427-36 $18/48 \sim x/100$
{37$\frac{1}{2}$}

427-37 $40/100 \sim 7.08/x$.
{17.7}

427-38 $49/84 \sim x/1$.
{$\frac{7}{12}$}

427-39 $\frac{5}{8}/1 \sim x/200$.
{125}

427-40 $\frac{5}{12}/1 \sim 39\frac{1}{6}/x$.
{94}

2 3 4 5 6 7 8 9
72 71 70 69 68 67 66 65

Date Due

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DEC 18 '69			
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